Vol 15 No 11, November 2006
1009-1963/2006/15(11)/2644-07

Chinese Physics

© 2006 Chin. Phys. Soc.
and IOP Publishing Ltd

Velocity overshoot of start-up flow for a Maxwell fluid in a porous half-space*

Tan Wen-Chang(谭文长)†

State Key Laboratory of Turbulence and Complex Systems, Department of Mechanics and Aerospace Engineering, College of Engineering, Peking University, Beijing 100871, *China*

(Received 20 October 2005; revised manuscript received 28 June 2006)

Stokes' first problem has been investigated for a Maxwell fluid in a porous half-space for gaining insight into the effect of viscoelasticity on the start-up flow in a porous medium. An exact solution was obtained by using the Fourier sine transform. It was found that at large values of the relaxation time the velocity overshoot occurs obviously and the system exhibits viscoelastic behaviours. On the other hand, for short relaxation time the velocity overshoot disappears and the system exhibits viscous behaviours. A critical value of the relaxation time was obtained for the emergence of the velocity overshoot. Furthermore, it was found that the velocity overshoot is caused by both the viscoelasticity of the Maxwell fluid and the Darcy resistance resulting from the structure of the micropore in the porous medium.

Keywords: Maxwell fluid, porous media, velocity overshoot

PACC: 4570

1. Introduction

The motion of a fluid caused by the sudden motion of a flat plate from rest, also named as Stokes' first problem or Rayleigh's problem, is not only of fundamental and theoretical interest, but also can be found in many practical processes. This problem provides insights into the phenomena of boundary layer growth and fluid flow bounded by an infinite flat plate. By using a simple transformation method, an exact solution for a Newtonian fluid was first obtained by Stokes.[1] Although it is difficult to obtain such a solution for a non-Newtonian fluid, Stokes' first problem has been extended to non-Newtonian fluids because of its practical importance. Owing to the difficulties in establishing a single model that possesses all properties of non-Newtonian fluids, many investigators have approached Stokes' first problem for non-Newtonian fluids with different constitutive models.[2−10]

Recently, interest in flows through porous media has also grown considerably, due largely to the demands in diverse areas such as biorheology, geophysics, chemical and petroleum industries.[11−17] It is well known that in the flow of a Newtonian fluid through a porous medium the pressure drop caused by the friction drag is directly proportional to the velocity. This is the familiar Darcy's law:

$$\nabla p = -\frac{\mu}{K}\boldsymbol{V}_{\rm D}, \tag{1}$$

where $V_{\rm D}$ is the Darcian velocity, μ the dynamic viscosity of the fluid and K the permeability of the porous medium. However, Darcy's law ignores the boundary effect on the flow. This negligence is not allowed when the boundaries of the porous medium have to be taken into accounted. As such, in 1947 Brinkman proposed an equation to describe the locally averaged flow in a porous medium.[17] The Brinkman equation considers the balance of forces acting on a volume element of the fluid, i.e., the pressure gradient, the divergence of the viscous stress tensor, and the friction force exerted by the porous medium:

$$-\nabla p + \mu_{\rm eff}\nabla^2\boldsymbol{V}_{\rm D} - \frac{\mu}{K}\boldsymbol{V}_{\rm D} = 0, \tag{2}$$

where $\mu_{\rm eff}$ is the effective dynamic viscosity of the medium. Although the Brinkman equation can be used to account for the effect of boundaries, it is only justified for steady-state flows of a Newtonian fluid in porous media. Hence, it cannot be applied to unsteady-state flows. In contrast, only a few mathematical macroscopic filtration models have been proposed for viscoelastic fluid flows in porous media. By analogy with the constitutive equation of a Maxwell

*Project supported by the National Natural Science Foundation of China (Grant Nos 10372007 and 10572006) and the New Century Training Programme Foundation for the Talents by Chinese Ministry of Education.

†E-mail: tanwch@mech.pku.edu.cn

fluid, the following phenomenological model has been introduced[13]

$$(1+t_{\rm m}\frac{\partial}{\partial t})\nabla p=-\frac{\mu}{K}\boldsymbol{V}_{\rm D}, \tag{3}$$

where $t_{\rm m}$ is the relaxation time of the Maxwell fluid. As Darcy's law does, Eq.(3) also ignores the boundary effect on the flow. So it cannot be directly used to investigate Stokes' first problem for a Maxwell fluid in a porous half-space. In this paper, based on the local volume averaging technique,[18−21] a modified Darcy's law is presented and Stokes' first problem is extended to Maxwell fluids in a porous half-space. An exact solution of the velocity is obtained using the Fourier sine transform. Numerical simulations are carried out to investigate the effects of the relaxation time and the porous parameter on the velocity field. This work has applications in the design of novel half-plate flow chamber for examining the effect of fluid shear stress on cultured cell monolayers, blood flow in tissue and other bioengineering problems.[9,22,23]

2. Basic equations

Consider now the flow of a Maxwell fluid in a porous half-space. Taking the positive y-axis of a Cartesian coordinate system to be in the upward direction, a Maxwell fluid flow through a porous half-space ($y>0$) above and in contact with a flat plate occupying the xz-plane. Both the fluid and plate are at a resting state at time $t=0$. At time $t=0^+$, the plate suddenly starts to slide in its plane with a constant speed U_0. Under these conditions, no flow occurs in the y- and z- directions and the flow velocity at a given point in the porous half-space depends only on its y-coordinate and time t, that is, the velocity field is of the form

$$\boldsymbol{V}=u(y,t)\boldsymbol{i}, \tag{4}$$

where $\boldsymbol{i}$ is the unit vector in the x-direction, $\boldsymbol{V}$ is the intrinsic velocity, which is averaged over the pore space. The relation between the intrinsic velocity and Darcian velocity is given by $\boldsymbol{V}_{\rm D}=\phi\boldsymbol{V}$, where ϕ is the porosity of the porous medium. Based on a local volume averaging technique,[8−21] a more rigorous modified Darcy's law will be considered for Stokes' first problem with a Maxwell fluid in a porous half-space in this work. In consideration of the balance of forces acting on a volume element of the fluid, the local volume-averaged balance of linear momentum can be given by

$$\rho\frac{\partial\boldsymbol{V}}{\partial t}=-\nabla p+\nabla\boldsymbol{\tau}+\boldsymbol{r}, \tag{5}$$

where ρ is the density of the fluid, $\boldsymbol{\tau}$ represents the viscous stress tensor, $\boldsymbol{r}$ is Darcy resistance, i.e., the viscous damping force caused by the micropore structure of a porous medium. Here we describe the viscoelastic properties of the fluid using the upper convected derivative Maxwell fluid,[24] assuming

$$\boldsymbol{\tau}+t_{\rm m}\left[\frac{\partial\boldsymbol{\tau}}{\partial t}+(\boldsymbol{V}\cdot\nabla)\boldsymbol{\tau}-(\nabla\boldsymbol{V})^{\rm T}\boldsymbol{\tau}-\boldsymbol{\tau}(\nabla\boldsymbol{V})\right]=\mu[\nabla\boldsymbol{V}+(\nabla\boldsymbol{V})^{\rm T}]. \tag{6}$$

It should be noted that this model includes the classical viscous Newtonian fluid as a special case at $t_{\rm m}=0$. Substituting Eq.(4) into Eq.(6) and keeping in mind the initial condition $\boldsymbol{\tau}(y,0)=\boldsymbol{0}$, we have $\tau_{xz}=\tau_{yy}=\tau_{yz}=\tau_{zz}=0$ and

$$\tau_{xx}+t_{\rm m}\frac{\partial\tau_{xx}}{\partial t}=2\tau_{xy}\frac{\partial u}{\partial y}, \tag{7}$$

$$\tau_{xy}+t_{\rm m}\frac{\partial\tau_{xy}}{\partial t}=\mu\frac{\partial u}{\partial y}. \tag{8}$$

Since the pressure gradient given in Eq.(3) for a Maxwell fluid can also be taken as a measure of the same resistance to flow in the bulk of the porous medium,[18] and $\boldsymbol{r}$ in Eq.(5) is also in effect a measure of the flow resistance offered by the solid matrix, thus $\boldsymbol{r}$ can be inferred from Eq.(3) to satisfy the following equation:

$$(1+t_{\rm m}\frac{\partial}{\partial t})\boldsymbol{r}=-\frac{\mu}{K}\boldsymbol{V}_{\rm D}. \tag{9}$$

Eliminating $\boldsymbol{\tau}$ and $\boldsymbol{r}$ in Eq.(5) by using Eqs.(8) and (9) and keeping in mind Eq.(4) and $\boldsymbol{V}_{\rm D}=\phi\boldsymbol{V}$, the momentum equation of Stokes' first problem for a Maxwell fluid in the porous half-space can be given by

$$\rho(1+t_{\rm m}\frac{\partial}{\partial t})\frac{\partial u}{\partial t}=-(1+t_{\rm m}\frac{\partial}{\partial t})\frac{\partial p}{\partial x}+\mu\frac{\partial^2 u}{\partial y^2}-\frac{\mu\phi}{K}u. \tag{10}$$

The corresponding initial and boundary conditions are respectively

$$u(y,0)=0, \tag{11}$$

$$u(0,t)=U_0. \tag{12}$$

Moreover, the natural conditions are

$$u(y,t)\to 0;\quad \frac{\partial u(y,t)}{\partial y}\to 0,\quad \text{as}\quad y\to\infty. \tag{13}$$

It deserves to note that the partial differential equation (10) is one order higher than that of a Newtonian fluid. In order to obtain an exact solution, we need an additional initial condition. As Fetecau did,[8] we also assume

$$\frac{\partial u(y,t)}{\partial t}\to 0\qquad \text{as}\quad t\to 0. \tag{14}$$

The equations are made dimensionless by scaling the velocity by U_0, length by v/U_0 and time by v/U_0^2. For the Stokes' first problem, the pressure gradient in the x-direction should be ignored. Then we obtain the dimensionless equation of motion with the initial and boundary conditions as follows

$$(1+\alpha\frac{\partial}{\partial t})\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}-\beta^2 u, \tag{15}$$

$$u(y,0)=0, \tag{16}$$

$$u(0,t)=1, \tag{17}$$

$$u(y,t)\to 0;\quad \frac{\partial u(y,t)}{\partial y}\to 0\quad \text{as}\quad y\to\infty, \tag{18}$$

$$\frac{\partial u(y,t)}{\partial t}\to 0\quad \text{as}\quad t\to 0, \tag{19}$$

where $\alpha=t_{\rm m}U_0^2/v$, with kinematic viscosity $v=\mu/\rho$, is a dimensionless relaxation time, which is also called the Deborah number;[14] $\beta=v/(U_0\sqrt{K/\phi})$ is a porous parameter describing the property of the porous medium.

3. Exact solution

If we multiply both sides of Eqs.(15), (16) and (19) by $\sin(sy)$, integrate them with respect to y from 0 to ∞, keeping in mind the conditions (17) and (18), we obtain

$$\alpha\frac{\partial^2\overline{u}(s,t)}{\partial t}+\frac{\partial\overline{u}(s,t)}{\partial t}+(s^2+\beta^2)\overline{u}(s,t)=s, \tag{20}$$

$$\overline{u}(s,0)=0, \tag{21}$$

$$\overline{u}_t(s,0)=0, \tag{22}$$

where $\overline{u}(s,t)$ denotes the Fourier sine transform of $u(y,t)$ with respect to the variable y.

The solution to the ordinary differential equation (20) subject to the initial conditions (21) and (22) has one of the following forms:

(i) If $4\alpha\geq\dfrac{1}{\beta^2}$.

$$\overline{u}(s,t)=\frac{s}{s^2+\beta^2}\left(1-{\rm e}^{\frac{-t}{2\alpha}}\left(\cos\left(\frac{\eta t}{2\alpha}\right)+\frac{1}{\eta}\sin\left(\frac{\eta t}{2\alpha}\right)\right)\right),\qquad s\in(0,\infty). \tag{23}$$

(ii) If $4\alpha<\dfrac{1}{\beta^2}$.

$$\overline{u}(s,t)=\begin{cases}\dfrac{s}{s^2+\beta^2}\left(1-\dfrac{r_1{\rm e}^{r_2 t}-r_2{\rm e}^{r_1 t}}{r_1-r_2}\right), & s\in(0,a),\\ \dfrac{s}{s^2+\beta^2}\left(1-{\rm e}^{\frac{-t}{2\alpha}}\left(\cos\left(\dfrac{\eta t}{2\alpha}\right)+\dfrac{1}{\eta}\sin\left(\dfrac{\eta t}{2\alpha}\right)\right)\right), & s\in(a,\infty),\end{cases} \tag{24}$$

In the above relations we have used the notations:

$$r_{1,2}=\frac{-1\pm\sqrt{1-4\alpha(s^2+\beta^2)}}{2\alpha},\quad \eta=\sqrt{4\alpha(s^2+\beta^2)-1},\quad a=\sqrt{\frac{1}{4\alpha}-\beta^2}.$$

Inverting these results by means of Fourier's sine formula, we have

(i) If $4\alpha \geq \dfrac{1}{\beta^2}$.

$$u(y,t) = \mathrm{e}^{-\beta y} - \frac{2}{\pi}\int_0^{\infty} \frac{s}{s^2+\beta^2}\mathrm{e}^{\frac{-t}{2\alpha}}\left(\cos\left(\frac{\eta t}{2\alpha}\right) + \frac{1}{\eta}\sin\left(\frac{\eta t}{2\alpha}\right)\right)\sin(ys)\mathrm{d}s. \tag{25}$$

(ii) If $4\alpha < \dfrac{1}{\beta^2}$.

$$\begin{aligned} u(y,t) = \mathrm{e}^{-\beta y} & -\frac{2}{\pi}\int_0^{a} \frac{s}{s^2+\beta^2}\frac{r_1\mathrm{e}^{r_2 t} - r_2\mathrm{e}^{r_1 t}}{r_1 - r_2}\sin(ys)\mathrm{d}s \\ & -\frac{2}{\pi}\int_0^{\infty} \frac{s}{s^2+\beta^2}\mathrm{e}^{\frac{-t}{2\alpha}}\left(\cos\left(\frac{\eta t}{2\alpha}\right) + \frac{1}{\eta}\sin\left(\frac{\eta t}{2\alpha}\right)\right)\sin(ys)\mathrm{d}s. \end{aligned} \tag{26}$$

In a special case, when $\beta = 0$, the above results can be simplified to the solution of Stokes' first problem for a clear Maxwell fluid (nonporous case). Equation (26) reduces to

$$\begin{aligned} u(y,t) = 1 & -\frac{2}{\pi}\int_0^{\frac{1}{2\sqrt{\alpha}}} \frac{r_1\mathrm{e}^{r_2 t} - r_2\mathrm{e}^{r_1 t}}{r_1 - r_2}\frac{\sin(ys)}{s}\mathrm{d}s \\ & -\frac{2}{\pi}\int_{\frac{1}{2\sqrt{\alpha}}}^{\infty} \mathrm{e}^{\frac{-t}{2\alpha}}\left(\cos\left(\frac{\eta t}{2\alpha}\right) + \frac{1}{\eta}\sin\left(\frac{\eta t}{2\alpha}\right)\right)\frac{\sin(ys)}{s}\mathrm{d}s. \end{aligned} \tag{27}$$

Equation (27) is the dimensionless form of the result obtained by Jordan.[10]

In another limiting case, when $\alpha = 0$, corresponding to Stokes' first problem for a viscous Newtonian fluid in a porous-half space, the solution can be easily obtained as follows:

$$u(y,t) = \mathrm{e}^{-\beta y} - \frac{2}{\pi}\int_0^{\infty} \frac{s}{s^2+\beta^2}\mathrm{e}^{-(s^2+\beta^2)t}\sin(ys)\mathrm{d}s. \tag{28}$$

When $\alpha = \beta = 0$, corresponding to the classical version of Stokes' first problem for a clear viscous Newtonian fluid, the solution can be obtained from Eq.(28) as follows:

$$u(y,t) = 1 - \frac{2}{\pi}\int_0^{\infty} \mathrm{e}^{-s^2 t}\frac{\sin(ys)}{s}\mathrm{d}s. \tag{29}$$

Using some standard mathematical identities, this solution can be rewritten as:

$$u(y,t) = 1 - \mathrm{erf}\left(\frac{y}{2\sqrt{t}}\right), \tag{30}$$

where $\mathrm{erf}(x)$ is the error function of Gauss. This is the well-known solution to Stokes' first problem for a clear Newtonian fluid in the dimensionless form.

4. Results and discussion

By setting in Eq.(15) $\dfrac{\partial}{\partial t} = 0$ and combining with the boundary conditions (17) and (18), we can obtain the steady-state solution of Stokes' first problem for a Maxwell fluid in a porous half-space as follows:

$$u(y) = \mathrm{e}^{-\beta y}. \tag{31}$$

By setting $t \to \infty$, this solution can also be obtained from Eqs.(25) and (26). The solution, which is a damping exponential function with respect to y, is different from that of Stokes' first problem for a clear fluid (nonporous cases). It is well known that in a pure viscous fluid, the velocity input is responded throughout the fluid, with $u(y,t) \to 1$, $0 < y < \infty$ as $t \to \infty$. In addition, it is interesting to note that the steady-state solutions for a Maxwell fluid and a Newtonian fluid are identical. If we define the boundary-layer

thickness δ to be the distance over which the velocity decreases to a given level, say 0.01, it is well known that $\delta = O(\sqrt{vt})$ for a clear Newtonian fluid. However, for a fluid (Newtonian fluid or non-Newtonian fluid) in a porous half-space, it is easy to find that the boundary-layer thickness tends to a limited value, that is $\delta \to 4.6/\beta$.

In order to investigate the viscoelastic effect on the velocity field, we numerically simulated the exact solution with the following set of parameters: $\alpha = 2$ and $\beta = 1$. The numerical results are presented in Fig.1, in which the velocity at a fixed distance $(y = 0.5, 1.2)$ above the flat plate is given as a function of time. It can be seen that it needs not too long time for the flow to get to the steady state. Although there is somewhat delay in starting up the flow, once the flow starts up, the velocity overshoot occurs clearly. Because the velocity overshoot has not been observed for a Newtonian fluid in the porous half-space, it is an important phenomenon for Stokes' first problem for a Maxwell fluid in the porous half-space.

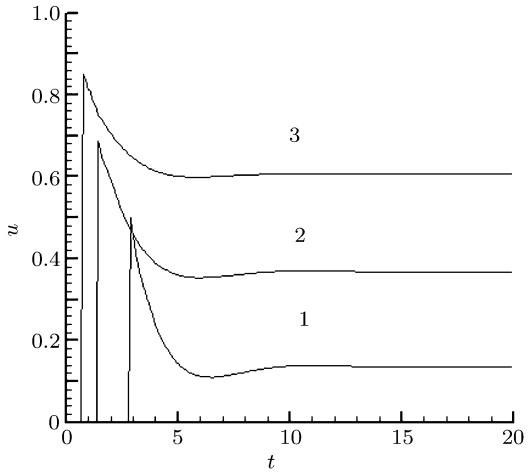


Fig.1. The velocity at different distances from the flat plate as a function of time for a Maxwell fluid with $\alpha = 2$ and $\beta = 1$. Curve 1: $y = 2$, Curve 2: $y = 1$, Curve 3: $y = 0.5$.

It is known that the viscoelastic fluids, described by the Maxwell model, have different flow regimes depending on the value of the relaxation time α. For high values of α, the system exhibits viscoelastic behaviour. On the other hand, for small α, the system is dissipative, and conventional viscous effects dominate. Now we consider the above solution again. If $\alpha > 1/(4\beta^2)$, the solution to Stokes' first problem is Eq.(25). In this case, the viscoelasticity of the Maxwell fluid plays an important role in the system's behaviours, and the velocity overshoot occurs as shown in Fig.2.

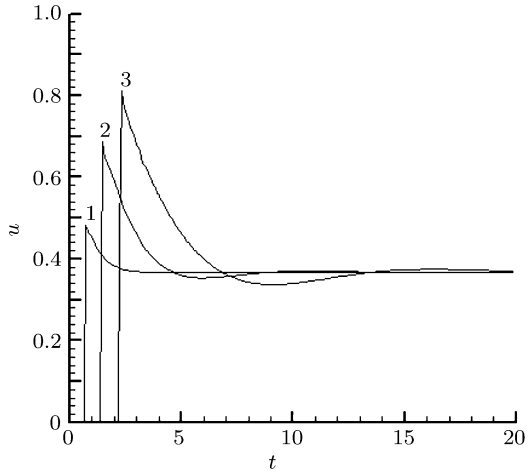


Fig.2. The velocity at a fixed distance $y = 1$ from the flat plate for Maxwell fluids with different relaxation times in the $\alpha > 1/(4\beta^2)$ case. $\beta = 1$ for all curves. Curve 1: $\alpha = 0.5$, Curve 2: $\alpha = 2$, Curve 3: $\alpha = 5$.

In addition, the longer the relaxation time, the stronger the velocity overshoot, and it needs longer time to start up the flow with the increase of the relaxation time. The effect of the porous parameter on the velocity overshoot is shown in Fig.3.

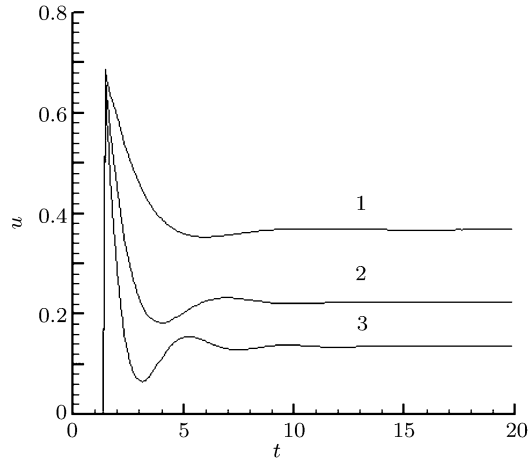


Fig.3. The velocity at a fixed distance $y = 1$ from the flat plate for Maxwell fluids with different values of the porous parameter in the $\alpha > 1/(4\beta^2)$ case. $\alpha = 2$ for all curves. Curve 1: $\beta = 1$, Curve 2: $\beta = 1.5$, Curve 3: $\beta = 2$.

With the increase of β the amplitude of the overshoot becomes larger and larger. If $\alpha < 1/(4\beta^2)$, the solution to Stokes' first problem is Eq.(26). In this case, the effect of viscoelasticity of a Maxwell fluid becomes weaker. As shown in Fig.4, the system exhibits viscous behaviours, and then the velocity overshoot disappears.

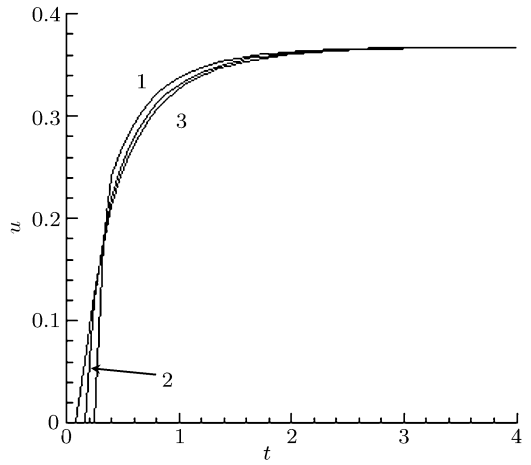


Fig.4. The velocity at a fixed distance $y = 1$ from the flat plate for Maxwell fluids with different relaxation times in the $\alpha < 1/(4\beta^2)$ case. $\beta = 1$ for all curves. Curve 1: $\alpha = 0.1$, Curve 2: $\alpha = 0.05$, Curve 3: $\alpha = 0.01$.

For comparison, in Fig.5, we plot the velocity at a fixed distance $y = 1$ from the flat plate for a Maxwell fluid and a Newtonian fluid in the porous half-space, and also a Maxwell fluid in a nonporous medium.

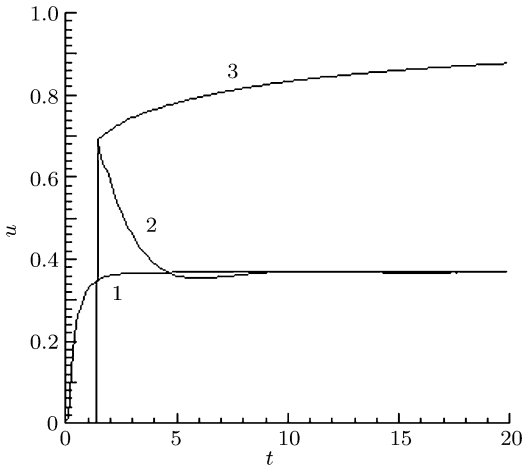


Fig.5. Comparison of the solutions of Stokes' first problem at a fixed distance $y = 1$ from the flat plate. Curve 1: Newtonian fluid in a porous medium ($\alpha = 0$, $\beta = 1$). Curve 2: Maxwell fluid in a porous medium ($\alpha = 2$, $\beta = 1$). Curve 3: Maxwell fluid in a nonporous medium($\alpha = 2$, $\beta = 0$).

It can be seen that the velocity overshoot does not occur for a Newtonian fluid in the porous medium and a Maxwell fluid in the nonporous medium. This implies that the velocity overshoot is caused by both the viscoelasticity of the Maxwell fluid and the Darcy resistance resulting from the micropore structure of the porous medium. Once one of the effects becomes weaker, the velocity overshoot will disappear. In addition, it is also seen when the dimensionless relaxation time is more than 10 there is not any difference between the velocities at a fixed distance for a Newtonian fluid and a Maxwell fluid in the porous medium. It means that for a long time the viscoelastic effect becomes weak and the viscoelastic fluid behaves as if it was a Newtonian fluid.

5. Conclusions

In this paper, Stokes' first problem has been extended to a Maxwell fluid in a porous half-space. The effect of viscoelasticity on the start-up flow in a porous medium was investigated. Owing to the effects of the viscoelasticity of the Maxwell fluids and the Darcy resistance resulting from the micropore structure of the porous medium, the velocity overshoot occurs. It was found that there is a critical value of the relaxation time, say $\alpha_c = 1/(4\beta^2)$. For larger values of the dimensionless relaxation time ($\alpha > \alpha_c$), the velocity overshoot occurs and the system exhibits viscoelastic behaviours. On the other hand, for smaller $\alpha(\alpha < \alpha_c)$, the velocity overshoot disappears and the system exhibits viscous behaviours. There is a y-dependent steady-state solution for a Maxwell fluid in the porous half-space, which is different from that for a clear fluid (nonporous case). It is a damping exponential function with respect to y. Numerical results show that the flow reaches a steady state rapidly. The boundary-layer thickness δ for a clear Newtonian fluid satisfies the equation $\delta = O(\sqrt{vt})$. But for a Maxwell fluid in a porous half-space the boundary-layer thickness tends to a limited value. Some previous and classical results can be considered as particular cases of our results, such as the solutions corresponding to Newtonian fluid in porous or nonporous half-space, and can be easily obtained from our results in different limiting cases.

References

[1] Schlichting H and Gersten K 2000 *Boundary Layer Theory* (Berlin: Springer) p80

[2] Preziosi L and Joseph D D 1987 *J. Non-Newtonian Fluid Mech.* **25** 239

[3] Phan-Thien N and Chew Y T 1988 *J. Non-Newtonian Fluid Mech.* **28** 117

[4] Bandelli R and Rajagopal K R 1995 *Int. J. Non-Linear Mech.* **30** 817

[5] Tan W C and Xu M Y 2002 *Mech. Res. Comm.* **29** 3
[6] Tan W C and Xu M Y 2002 *Acta Mech. Sin.* **18** 342
[7] Tan W C, Pan W X and Xu M Y 2003 *Int. J. Non-Linear Mech.* **38** 645
[8] Fetecau C and Fetecau C 2003 *Int. J. Non-Linear Mech.* **38** 423
[9] Jordan P M and Puri P 2003 *Int. J. Non-Linear Mech.* **38** 1019
[10] Jordan P M, Puri A and Boros G 2004 *Int. J. Non-Linear Mech.* **39** 1371
[11] Rio J A D, Haro M L and Whitaker S 1998 *Phys. Rev.* E **58** 6323
[12] Nield D A and Adrian B 1999 *Convection in Porous Media* (Berlin: Springer) p10
[13] Khuzhayorov B, Auriault J L and Royer P 2000 *Int. J. Eng. Sci.* **38** 487
[14] Tsiklauri D and Beresnev I 2001 *Phys. Rev.* E **64** 036303
[15] Khaled A R A and Vafai K 2003 *Int. J. Heat Mass Transfer* **46** 4989
[16] Younes A 2003 *Int. J. Heat Mass Transfer* **46** 367
[17] Capuani F, Frenkel D and Lowe C P 2003 *Phys. Rev.* E **67** 056306
[18] Vafai K and Tien C L 1981 *Int. J. Heat Mass Transfer* **24** 195
[19] Whitaker S 1999 *The Method of Volume Averaging* (Dordrecht: Kluwer) p1
[20] Derek B I and Ioan P 2002 *Transport Phenomena in Porous Media* (Boston: Elsevier) p231
[21] Tan W C and Masuoka T 2005 *Int. J. Non-Linear Mech.* **40** 515
[22] Zeng Y and Weinbaum S 1995 *J. Fluid Mech.* **287** 59
[23] Tan W C and Masuoka T 2005 *Phys. Fluids* **17** 023101
[24] Kaloni P N and Lou J X 2002 *J. Non-Newtonian Fluid Mech.* **103** 167