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The quantum features of the photon statistics in a semiconductor microcavity

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The quantum features of the temporal photon statistics of an exciton–cavity coupled system in a quantum-well semiconductor microcavity are investigated analytically. Under the secular approximation, if the nonlinear interactions, i.e. the exciton–exciton coupling and the phase-space filling, are much weaker than the exciton–photon interaction, the evolution of the Fano factor shows that the distribution of the photon numbers exhibits the feature of collapses–revivals (CRs), and the relevant revival time may be adjusted by several factors such as the total particle number, the detuning, and the nonlinear coupling strengths, etc. Especially, the ideal maximum antibunching with the minimum value 0 of the Fano factor occurs periodically for such a situation, with the dissipation of exciton–polariton being ignored.

Keywords: antibunching photon, collapses–revivals, quantum well, exciton–polariton
PACC: 4250, 7865H, 7136, 7135

1. Introduction

With the development of semiconductor optical microcavities, the dynamical behaviours of the quantum properties of some exciton–cavity coupled systems have attracted considerable attention. There have been carried out some important experimental and theoretical studies based on these systems in quantum well inside a semiconductor microcavity. The vacuum Rabi oscillation has been observed first by Weisbuch *et al.*[1] Due to the effective exciton–exciton interaction, the exciton emission is found to exhibit the characteristic features of collapse and revival (CR) phenomenon by Jin and Liu,[2] which is regarded as the origin of the granular nature of the quantized field captured by the Jaynes–Cummings model.[3] Tan *et al*[4] have indicated that the entanglement between excitons and cavity field, the squeezing of the exciton and the cavity field can be enhanced via an additional squeezed vacuum control. Though the fluorescent light is theoretically proved being always bunched in the same controlling way as that in Ref.[4] by Erenso *et al*,[5] the antibunching emission from the radiative recombination of the impurity-bound excitons is really achieved experimentally in a ZnSe quantum well structure by Strauf *et al* [6] indeed. For example, the emission that has the well-known and sharp transition energies, not only provides a practical way to make high effective quantum emitters, but also appears highly attractive for cavity-quantum electrodynamics experiments in an all-semiconductor nanostructure. In addition, it should be pointed out, for the exciton photon-coupling systems in quantum point inside a semiconductor microcavity, that highly efficient single photon emissions in a three-level system of semiconductor quantum dots with pulsed excitation can be realized,[7] and a three-mode Gaussian entangled mixed state of the excitons in the three quantum dots placed in a cavity driven by a broadband squeezed vacuum is also generated in a low exciton density regime.[8] In a word, due to the interaction between the photons and the excitons being stronger in both a quantum well and a quantum dot than in the three-dimensional system, the optical behaviours of exciton appear more striking among the reduced dimensional systems, and therefore based on which the respective researches become more and more significant either in fundamental or application field.

In this paper, we further analytically investigate the quantum features of the temporal photon statistics of the exciton–photon system in a quantum well inside a semiconductor microcavity.[2] By studying the evolution of the Fano factor,[9] we first show that the distribution of the photon numbers may exhibit the CR phenomenon, and then analyse those factors adjusting the relevant revival time, and finally manifest the possibility of the existence of antibunching pho-

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tons.

2. The model and the solution

The present work is based on the model in Ref.[2], i.e. the coupled exciton–photon system in a semiconductor quantum well embedded in a high finesse microcavity. The relevant Hamiltonian can be written as ($\hbar = 1$)[2]

$$H = \omega_{\rm c} a^+ a + \omega_{\rm ex} b^+ b + g(a^+ b + a^+ a) + A b^+ b^+ b b - B(b^+ b^+ b a + a^+ b^+ b b), \quad (1)$$

where $a(a^+)$ is the annihilation (creation) operator for the cavity field with frequency $\omega_{\rm c}$, and $b(b^+)$ is the annihilation (creation) operator for the exciton with frequency $\omega_{\rm ex}$. These operators are assumed to obey the bosonic commutation relation $[b, b^+] = [a, a^+] = 1$. The third term on the right-hand side of the expression (1) is for the interaction between the exciton and the cavity field with the coupling strength g, which is larger than the nonlinear interaction coefficients A and B. The fourth term describes the effective exciton–exciton interaction due to Coulomb interaction, and the fifth term represents the phase-space filling effects.

In a strong-coupling regime, the exciton–polariton can be generated,[10] and described by the following two polariton operators:

$$p_1(t) = -va(t) + ub(t), \quad p_2(t) = ua(t) + vb(t), \quad (2)$$

where u and v are Hopfield coefficients for the exciton and cavity modes, respectively. Both of the polariton operators are still bosonic ones, i.e. they obey the relation

$$[p_i, p_j^+] = \delta_{i,j} \quad \text{for} \quad i, j = 1, 2, \quad (3)$$

provided that the relation $u^2 + v^2 = 1$ holds. Naturally, the following definitions

$$u = \sin\theta, \quad \text{and} \quad v = \cos\theta \quad (4)$$

are appropriate options.

The inverse transformations of Eq.(2) are

$$b(t) = up_1(t) + vp_2(t), \quad a(t) = up_2(t) - vp_1(t). \quad (5)$$

Therefore the total Hamiltonian (1) can be written in terms of the polariton operators as

$$H_{\rm eff} = \omega_1 p_1^+ p_1 + \omega_2 p_2^+ p_2 + A_{11} p_1^+ p_1^+ p_1 p_1 + A_{22} p_2^+ p_2^+ p_2 p_2 + 2A_{12} p_1^+ p_2^+ p_2 p_1, \quad (6)$$

where

$$\omega_j = \frac{1}{2}(\omega_{\rm ex} + \omega_{\rm c} - (-1)^j \Delta), \quad \text{for} \quad i, j = 1, 2, \quad (7)$$

are the lower-branch ($j = 1$) and upper-branch ($j = 2$) polariton energies, respectively, and $\Delta = \omega_2 - \omega_1 = \sqrt{\delta^2 + 4g^2}$ is the splitting energy of the two polaritons. The detuning between the cavity mode and the exciton mode is $\delta = \omega_{\rm c} - \omega_{\rm ex}$, and obeys

$$\delta uv = g(u^2 - v^2), \quad (8)$$

namely, $\mathrm{tg}2\theta = -2g/\delta$, which makes Hamiltonian (1) diagonal. The other three coefficients in Hamiltonian (6) are

$$A_{11} = u^3(Au + 2Bv), \quad A_{22} = v^3(Av - 2Bu),$$
$$A_{12} = A_{21} = 2uv[Auv - B(u^2 - v^2)]. \quad (9)$$

In Hamiltonian (6), some terms proportional to $p_1^{+2} p_2^2$, $p_1^{+2} p_1 p_2$ and their Hamiltonian conjugate terms in Hamiltonian (6), which describe the particle-number-nonconservation scattering channels,[11] are ignored, which is the so-called secular approximation, and is the same as the rotating-wave approximation (RWA) in physical essence.[12]

From Hamiltonian (6), it is easy to find out that the polariton number operators $p_j^+(t)p_j(t)$ of each branch are the constant of motion, i.e. $p_j^+(t)p_j(t) = p_j^+(0)p_j(0) = \text{const}$. Thus the total particle number operator $N = p_1^+ p_1 + p_2^+ p_2 = a^+ a + b^+ b$ is also time-independent. The formal solutions of the Heisenberg equations for the polariton operators $p_j(t)$ are

$$p_j(t) = \exp\Big[-\mathrm{i}\Big(\omega_j - \mathrm{i}\gamma_j/2 + 2\sum_{k=1}^{2} A_{jk} p_k^+ p_k\Big)t\Big] p_j, \quad (10)$$

where p_j represents the initial time operator $p_j(0)$, and γ_j is the natural linewidth of the lower-(upper-) branch of the polariton.

Based on Schwinger's angular momenta: $J_z = (b^+ b - a^+ a)/2$ and the ladder operator $J_+ = (J_-)^+ = b^+ a$, any quantum state of the coupled exciton–photon system can be written in terms of the angular momentum states

$$|j, m\rangle = (b^+)^{j+m} (a^+)^{j-m} / \sqrt{(j+m)!(j-m)!}\,|0\rangle, \quad (11)$$

which is a direct product of two number states with $j + m$ excitons in the quantum well and $j - m$ photons in the cavity, respectively. Thus the initial state of the exciton–photon system $|\Psi(0)\rangle = |N\rangle_{\rm ex} \otimes |0\rangle_{\rm c}$, namely, the excitons being initially in a number state and the photons in vacuum states, can be written as $|j, j\rangle$ with $j = N/2$, consequently the average photon number can be expressed as

$$\langle\,\Psi(0)|a^{+}(t)a(t)|\,\Psi(0)\rangle = \frac{N}{2}\sin^2(2\theta) - \frac{N}{2}\Big\{\frac{\sin^2(2\theta)}{2}\mathrm{e}^{\mathrm{i}\Delta t}\mathrm{e}^{-(\gamma_1+\gamma_2)t/2}\mathrm{e}^{2\mathrm{i}(A_{11}-A_{12})(N-1)t} \times [\sin^2\theta + \cos^2\theta\mathrm{e}^{2\mathrm{i}(2A_{12}-A_{11}-A_{22})t}]^{N-1} + \mathrm{c.c.}\Big\}. \tag{12}$$

In the resonant case $\delta = 0$ (i.e. $\theta = \pi/4$), Eq.(12) may reduce to

$$\langle\,\Psi(0)|a^{+}(t)a(t)|\,\Psi(0)\rangle = \frac{N}{2}\Big\{1 - \cos\{[2g + B(N-1)t]\} \times \Big[\cos\Big(\frac{At}{2}\Big)\Big]^{N-1}\mathrm{e}^{-(\gamma_1+\gamma_2)t/2}\Big\}. \tag{13}$$

Here, some relations

$$p_j^{+}p_j = \frac{N}{2} + (-1)^j e^{-2\mathrm{i}\theta J_y}J_z\mathrm{e}^{2\mathrm{i}\theta J_y},\ p_1^{+}p_2 = -\mathrm{e}^{-2\mathrm{i}\theta J_y}J_{-}\mathrm{e}^{2\mathrm{i}\theta J_y},\ \mathrm{e}^{2\mathrm{i}\theta J_y}\exp[\lambda\mathrm{e}^{-2\mathrm{i}\theta J_y}J_z\mathrm{e}^{2\mathrm{i}\theta J_y}]\mathrm{e}^{-2\mathrm{i}\theta J_y} = \exp[\lambda J_z]\,,$$
$$p_1^{+}(t)p_2(t) = f_1\exp(\mathrm{i}\Delta t)\exp[-2\mathrm{i}(A_{22}-A_{11})t] \times \exp[2\mathrm{i}(A_{22}-A_{12})p_2^{+}p_2t]p_2^{+}p_1\exp[2\mathrm{i}(A_{21}-A_{11})p_1^{+}p_1t] \tag{14}$$

are all used in the process of Eqs.(12) and (13) being reduced. Up to now, all of these statements above originates from Ref.[2] except for the last equation in Eqs.(14).

We further consider the properties of photon statistics characterized by the Fano factor F as follows:

$$F = \frac{\bar{N}_{\mathrm{ph}}^2 - \bar{N}_{\mathrm{ph}}^2}{\bar{N}_{\mathrm{ph}}}\begin{cases} < 1 & \mathrm{sub-Poisson} \quad \mathrm{distribution}, \\ = 1 & \mathrm{Poisson} \quad \mathrm{distribution}, \\ > 1 & \mathrm{super-Poisson} \quad \mathrm{distribution}, \end{cases} \tag{15}$$

where N_{ph} is the number of emitted photons in a certain time interval. The super-Poissonian distribution corresponds to the classical bunching behaviour of photons; but the quantum sub-Poissonian distribution exhibits the nonclassical antibunching property of photons, namely, single photons can be emitted in short time intervals and deterministic dwell time between them. Especially, the maximum antibunching effect is in correspondence with $F = 0$.

In this work, the initial state of the exciton–photon system is still assumed to be $|\,\Psi(0)\rangle = |j,j\rangle$, and then the average of the squared photon number can be further obtained as follows:

$$\begin{aligned}\langle(a^{+}a(t))^2\rangle = {} & v^4\langle(p_1^{+}p_1)^2\rangle + u^4\langle(p_2^{+}p_2)^2\rangle + (uv)^2\langle 2p_1^{+}(t)p_1(t)p_2^{+}(t)p_2(t) + p_2^{+}(t)p_1(t)p_1^{+}(t)p_2(t) \\ & + p_1^{+}(t)p_2(t)p_2^{+}(t)p_1(t)\rangle - 2u^3v\mathrm{Re}[\langle p_1^{+}(t)p_2(t)p_2^{+}(t)p_2(t) + p_2^{+}(t)p_2(t)p_1^{+}(t)p_2(t)\rangle] \\ & - 2uv^3\mathrm{Re}[\langle p_1^{+}(t)p_2(t)p_1^{+}(t)p_1(t) + p_1^{+}(t)p_1(t)p_1^{+}(t)p_2(t)\rangle] + 2\mathrm{Re}[(uv)^2\langle(p_2^{+}(t)p_1(t))^2\rangle]\,.\end{aligned} \tag{16}$$

Here we do not think that some terms such as $p_1^{+}(t)p_2(t)p_2^{+}(t)p_2(t)$, $p_2^{+}(t)p_2(t)p_1^{+}(t)p_2(t)$ etc in the expression (16) can lead to the particle-number-nonconservation scattering processes in the system, because Hamiltonian (6) is already based on the secular approximation. With the help of the expression (14), we can obtain these terms in the expression (16) as follows:

$$\begin{aligned}&\langle(p_1^{+}(t)p_1(t))^2\rangle = N^2\sin^4\theta + N\sin^2\theta\cos^2\theta\,, \\ &\langle(p_2^{+}(t)p_2(t))^2\rangle = N^2\cos^4\theta + N\sin^2\theta\cos^2\theta\,, \\ &\langle p_2^{+}(t)p_1(t)p_1^{+}(t)p_2(t)\rangle = N\cos^4\theta + N^2\sin^2\theta\cos^2\theta\,, \\ &\langle p_1^{+}(t)p_2(t)p_2^{+}(t)p_1(t)\rangle = N\sin^4\theta + N^2\sin^2\theta\cos^2\theta\,, \\ &\langle(p_2^{+}(t)p_1(t))^2\rangle = N(N-1)\sin^2\theta\cos^2\theta d_1^2d_2^*d_3^{*2}d_4^{*N-2}\,, \\ &\langle p_1^{+}(t)p_2(t)p_2^{+}(t)p_2(t)\rangle = N\sin\theta\cos\theta(\sin^2\theta + N\cos^2\theta d_2)d_1d_3d^{N-2}\,, \\ &\langle p_2^{+}(t)p_2(t)p_1^{+}(t)p_2(t)\rangle = N(N-1)\cos^3\theta\sin\theta d_1d_2d_3d^{N-2}\,, \\ &\langle p_1^{+}(t)p_2(t)p_1^{+}(t)p_1(t)\rangle = N(N-1)\sin^3\theta\cos\theta d_1d_3d^{N-2}\,, \\ &\langle p_1^{+}(t)p_2(t)p_2^{+}(t)p_2(t)\rangle = N\sin\theta\cos\theta(N\sin^2\theta + \cos^2\theta d_2^*)d_1d_3^*d^{*(N-2)}\,,\end{aligned} \tag{17}$$

where

$$d_1 = \exp[-(\gamma_1+\gamma_2)t/2], \quad d_2 = \exp[2\mathrm{i}\alpha t], \quad d_3 = \exp[\mathrm{i}\Omega' t],$$
$$d_4 = \sin^2\theta + \cos^2\theta d_2^2, \quad \text{and} \quad d = \sin^2\theta + \cos^2\theta d_2, \tag{18}$$

with parameters

$$\alpha = 2A_{12} - A_{11} - A_{22}, \quad \beta = A_{12} - A_{11}, \quad \text{and} \quad \Omega' = 2\beta(1-N) - \Delta\,. \tag{19}$$

Hence the expression (16) can be written as

$$\begin{aligned}\langle (a^+a(t))^2\rangle =& 2N\sin^2\theta\cos^2\theta(3N\sin^2\theta\cos^2\theta + \sin^4\theta + \cos^2\theta\cos(2\theta)) \\ &+ 2N(N-1)\sin^4\theta\cos^4\theta d_1^2 \mathrm{Re}(d_2 d_3^2 d_4^{N-2}) \\ &- 2N\sin^2\theta\cos^2\theta d_1 \mathrm{Re}[2N(1+d_2)\sin^2\theta\cos^2\theta d_3 d^{N-2}] \\ &- 2N\sin^2\theta\cos^2\theta d_1 \mathrm{Re}[(d_2\cos^2\theta - \sin^2\theta)\cos(2\theta) d_3 d^{N-2}]\,. \end{aligned} \tag{20}$$

In the resonant case $\delta = 0$ (i.e., $\theta = \pi/4$), Eq.(20) reduces to

$$\begin{aligned}\langle (a^+a(t))^2\rangle =& \frac{N}{8}(3N+1) + \frac{N}{8}(N-1)d_1^2\cos^{(N-2)}(2\alpha t)\cos(2\Omega t) \\ &- \frac{N^2}{4} d_1 \cos^{(N-2)}(\alpha t)[\cos[(\alpha-\Omega)t] + \cos[(\alpha+\Omega)t]] \end{aligned} \tag{21}$$

with

$$\Omega = B(1-N) + 2g\,. \tag{22}$$

It should be noted that there are some terms such as $[\cos(2\alpha t)]^{(N-2)}$ and $[\cos(\alpha t)]^{(N-2)}$ in the expression (21) as well as $[\cos(At/2)]^{(N-1)}$ in the expression (13), which will exhibit the revival phenomena of the distributions corresponding to squared photon number and photon number, respectively. Therefore the relevant revival times are, in the final analysis, conditional on the nonlinear interactions, i.e. the exciton–exciton interaction constant A and the phase-space filling effects B (see expressions (9) and (19)) in the case $\delta = 0$.

3. Discussion and conclusion

Using the expressions (12) and (20), we schematically show the whole evolution of the Fano factor (15) in Figs.1, 2 and 3, obviously, in which there appears the feature of CR phenomenon. From the viewpoint of pure theoretical consideration, we take $A \sim 0.01g$ to satisfy the requirement for the weak-nonlinearity as same in Ref.[2], and $\gamma_1 = \gamma_2 = 0.0005g$. Comparing with Figs.1(a) and 1(b), we first find that the temporal decay of the polaritons results in a reduction in the revival amplitude but does not change the revival time, which is due to the fact that revival times of the distributions corresponding to photon number as well as the squared photon number are related only to the exciton–exciton interaction constant A (here $B = 0$ and $\delta = 0$) as mentioned above. With the increase of the total particle number of the system N, the whole evolution is evidently divided into two parts shown in Fig.1(c): one has the feature of the CRs, and the other is in the periodic oscillation. Especially for the latter, the lowest points of the oscillation may reach the minimum value of the Fano factor, i.e. $F = 0$ for the case $\gamma_1 = \gamma_2 = 0$. In addition, CR phenomenon becomes more prominent, the revival time becomes smaller, and the level of central line around which Mandel's Q parameter oscillates up and down is elevated. Especially, comparing Fig.1(c) with Fig.1(d), we may see that the temporal decay of the polaritons not only decrease the revival amplitudes, but also even may modulate the periodic oscillations that makes them transform into a quasi-revival oscillations in the case $\gamma_1 = \gamma_2 = 0.5$

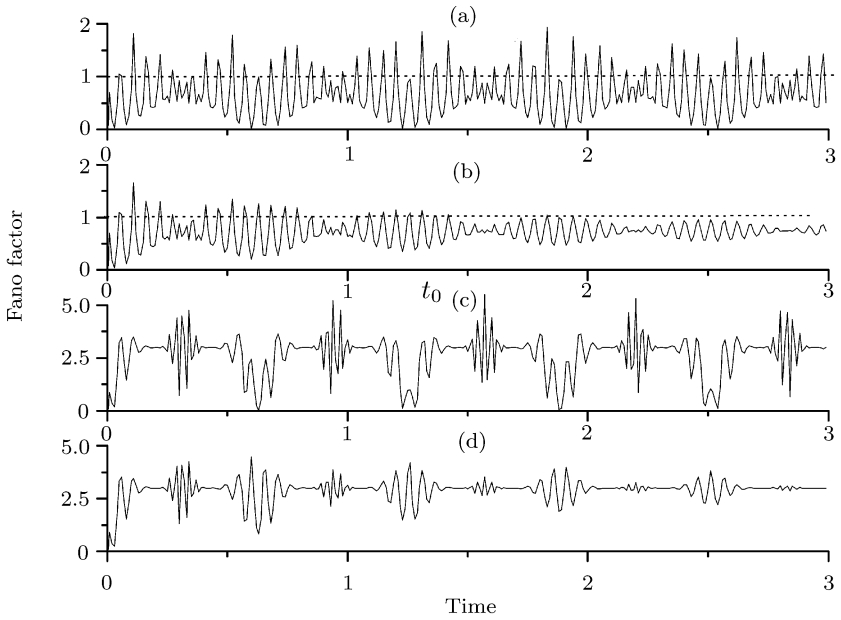


Fig.1. The Fano factor for the number state as a function of time t. Time is in units of $1/\gamma$. The parameters are $g = 2000\gamma$, $A = 0.01g$, $B = 0$, and $\delta = 0$. From top to bottom (a) $N = 2$, $\gamma_1 = \gamma_2 = 0$; (b) $N = 2$, $\gamma = 0.5$; (c) $N = 11$, $\gamma_1 = \gamma_2 = 0$; (d) $N = 11$, $\gamma = 0.5$.

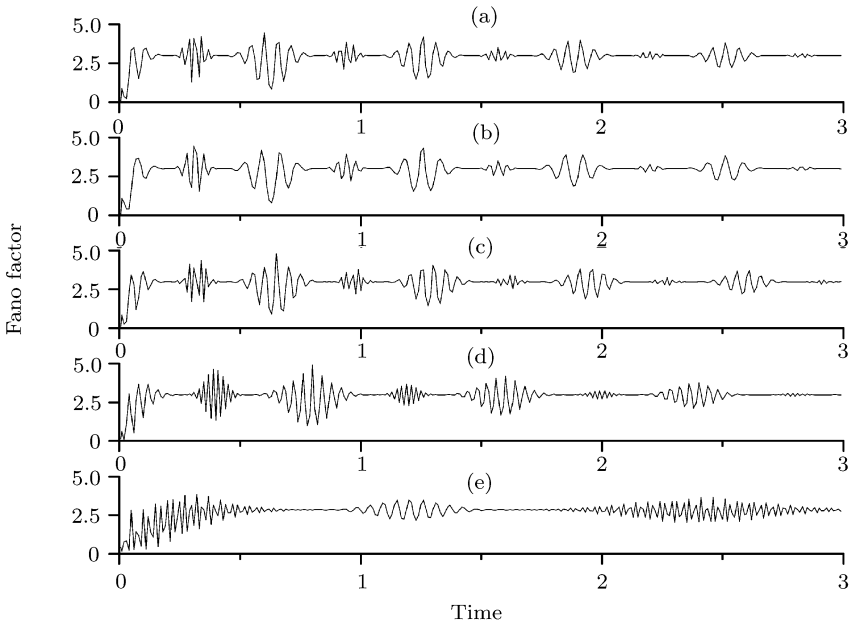


Fig.2. The Fano factor for the number state as a function of time t. Time is in units of $1/\gamma$. The parameters are $g = 2000\gamma$, $A = 0.01g$, $N = 11$, $\gamma = 0.5$. From top to bottom (a) $\delta = 0$, $B = 0$ (b) $\delta = 0.3A$, $B = 0$; (c) $\delta = 0.2g$, $B = 0$; (d) $\delta = 0.2g$, $B = 0.3A$; (e) $\delta = 0.6g$, $B = 0.3A$.

The influences of the detuning δ and the phase-space filling B are also investigated. Comparing Fig.2(a) with Fig.2(b), we may see that the resonant coupling between the excitons and the photons, i.e. $\delta = 0$ makes the phase-space filling factor B exert nearly no influence on the revival time, but changes

only the energy oscillation frequency. However, for the case $\delta \neq 0$ the period of CRs obviously increasing with the factor B increasing as shown in Figs.2(c) and 2(d). Besides, the revival time becomes longer with the increase of the detuning δ as shown in Figs.2(b), 2(d) and 2(e). Based on the analyses above, we believe that the revival time may be changed by adjusting the total particle number, the field detuning, the exciton–photon coupling strength, and the nonlinear interaction constants, i.e. the exciton–exciton coupling strength, and the phase-space filling factor.

Finally, we pay close attention to the changes in the distribution of the photon numbers. From Figs.1(a), 1(b) and Fig.3, we find that the transitions between quantum sup-Poissonian and classical super-Poissonian may occur through all of the evolution of the Fano factor for small N. Besides, due to the fact that position of the oscillating central line does not change with the decay of the polaritons, the photon may be always in the state corresponding to sub-Poisson distribution from a time t_0 for the case of weak dissipation as shown in Fig.1(b). Furthermore, we can see that the periodic oscillation even may periodically reach the lowest point of the Fano factor, i.e. $F = 0$ in the case $\gamma_1 = \gamma_2 = 0$ as shown in Fig.1(c), which means that such a system generates the ideal maximum antibunching photons. Actually, if we take $B = (2g - 0.75A)/(N - 1)$, the maximum antibunching effect can be always produced periodically during the evolution as shown in Fig.3. Of course, in this case the exciton number is required to be not too large; otherwise the weak nonlinear interacting approximation mentioned above does not hold.

To conclude, we have studied the quantum properties of photon statistics of a semiconductor microcavity under the secular approximation. By analysing the evolution of the Fano factor, we find that the photon distribution may really reveal the feature of CRs, and several factors such as the total particle number, the exciton–exciton coupling strength, the phase-space filling factor, and the detuning, may influence the relevant revival times. Our result further indicates that the antibunching photons, even the strongest periodic antibunching photons really exist in such a situation for weak, or no dissipation of exciton–polariton. Only if the exciton number is not too large, does the weak nonlinear coupling hold. In short, this work provides a very promising mechanism to make a high effective quantum emitters in semiconductor nanostructure.

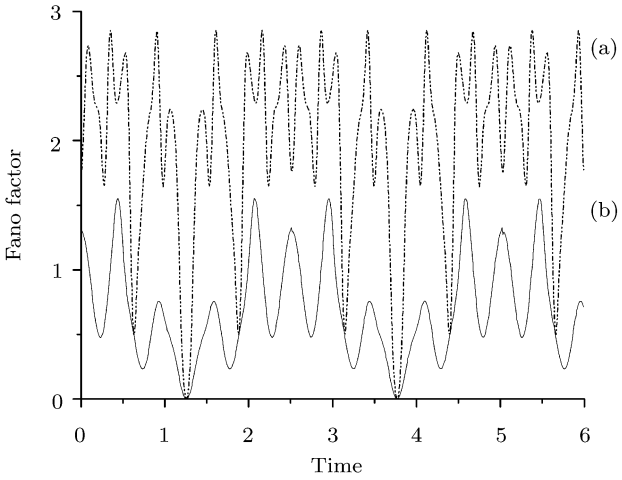


Fig.3. The Fano factor for the number state as a function of time t. Time is in units of $1/\gamma$. The parameters are $g = 2000\gamma$, $A = 0.01g$, $B = (2g - 0.75A)/(N - 1)$, $\delta = 0$ without dissipation. From top to bottom (a) $N = 2$; (b) $N = 5$.

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