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Calculation of plasma characteristics of the sun*

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The ionization level and free electron density of most abundant elements (C, N, O, Mg, Al, Si, S, and Fe) in the sun are calculated from the centre of the sun to the surface of the photosphere. The model and computations are made under the assumption of local thermodynamic equilibrium (LTE). The Saha equation has been used to calculate the ionization level of elements and the electron density. Temperature values for calculations along the solar radius are taken from references.

Keywords: Saha equation, ionization balance, plasma characteristics
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1. Introduction

Calculation of ionic populations for a given temperature and density of a plasma is the first step toward its modelling, due to the strong influence these quantities have on most of the plasma properties.[1] So, the quantitative study of ionization equilibrium plays the key role in plasma spectroscopy, astroplasma studies, and high energy density physics (e.g., controlled nuclear fusion, gas phase nuclear reactors, interaction of lasers and shock waves with matter, etc.). The ionization in laser heated targets comprising different materials is used to diagnose the interaction. This kind of data are also needed for the study of spectral classes of stars in astronomy. It is for this reason that such data are needed for modelling of high temperature laboratory and stellar plasmas.

There are several methods referenced in the literature to perform these calculations for plasmas, both in local thermodynamic equilibrium (LTE) and in non local thermodynamic equilibrium (NLTE). In this work, we will focus on the commonly used Saha equations for LTE plasmas. LTE is based on the assumption that collision-induced transitions and reactions are more frequent than radiative ones, and that there exists a micro-reversibility between the collision processes. If LTE prevails, then only the total mass density and temperature are required for a constant volume to determine the particle densities (neutral and charged) and the Saha equation can be used to evaluate the plasma composition in this case.

It is a fairly good approximation to assume local thermodynamic equilibrium in our calculations from the centre of the sun to the surface of the photosphere. Our data show a gradual change in temperature from one atmospheric level to another, but it is small for moderate changes in height (less than 5 degrees per km or 0.05 percent per km over small distances). We are thus justified in considering the temperature to be constant over small distances. Furthermore, the thermodynamic-equilibrium concept is applied to relatively small volumes of the model's photosphere of dimensions of the order of unity in optical depth. The photosphere may then be characterized by the same physical temperature at each depth.[2] The excitation, ionization, source function and thermal velocity distributions in the vicinity of one point are all described by the unique temperature. Hence the Saha equation may be applied to study the plasma characteristics inside the sun as well as in the photosphere.[3]

In this work, ionization levels, abundance of ions and free electron density of most abundant elements in the interiors and the photosphere of the sun are calculated by using the Saha equation under LTE. The different ionization levels are calculated from the centre to the surface of the sun. The electron density has

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also been calculated as a function of temperature from the centre of the sun to its surface. Both the electron density and ion abundance are plotted as a functions of solar radius. The results suggest that stripping out of electrons in all of the most abundant elements is different at different distances from the centre of the sun to its surface due to temperature variations with distance.[4]

2. Theory and method

For plasmas in which the assumption of local thermodynamic equilibrium (LTE) is applicable, the populations of atoms, ions and electrons follow the Saha equation.[5] Taking into account the ionization potentials, the Saha equation for a single element species can be written as

$$\frac{N(m+1)N_{\mathrm{e}}}{N(m)} = 2\frac{U_{m+1}}{U_m}\left[\frac{2\pi m_{\mathrm{e}} k_{\mathrm{B}} T}{h^2}\right]^{3/2} \times \exp\left(-\frac{I(m+1)}{k_{\mathrm{B}} T}\right), \quad m = 0, 1 \ldots\ldots\ldots\ldots Z-1, \tag{1}$$

where N_{e} is the number density of free electrons, N_m is the number density of all m-fold ionized atoms, U_m is the state-dependent partition function of m-fold ionized atoms, m_{e} is the mass of an electron, k_{B} is the Boltzmann constant, T is the absolute temperature, Z is the highest-fold ionization to be reached in the plasma under consideration (i.e., the atomic number), h is Planck's constant and I_m is the ionization energy for the ionization process $m \rightarrow$ (m+1).

The set of Saha equations (1) gives the relation and ratio between the densities of ions in two successive ionization states. However, it does not enable one to determine the detailed plasma composition unless supplemented by two additional conservation equations: (a) conservation of electric charge (the condition of electro-neutrality),

$$\sum_{m=0}^{Z} m N_m = N_{\mathrm{e}}, \tag{2}$$

and (b) conservation of nuclei (no nuclear reactions) which requires a constant number of heavy particles (conservation of nuclei in the ionization and recombination processes),

$$\sum_{m=0}^{Z} N_m = N, \tag{3}$$

where N is the total number density of heavy particles (nuclei).

The system of Saha equations (1) supplemented by the assumption of electro-neutrality (2) and requirement of constant number of heavy particles (3) is sufficient for the calculation of the detailed composition of plasma of a single chemical species. Equations (1)–(3) can alternatively be written in terms of the proportions of the gas at the ionization state m,

$$b_m = \frac{N_m}{N} \quad (0 \le b_m \le 1, m = 0, 1, ..., Z), \tag{4}$$

$$b_{\mathrm{e}} = \frac{N_{\mathrm{e}}}{N} \quad (0 \le b_{\mathrm{e}} \le Z). \tag{5}$$

From equations (4) and (5), we obtain the following equations:

$$\sum_{m=0}^{Z} b_m = 1,$$

$$\sum_{m=0}^{Z} m b_m = b_{\mathrm{e}},$$

$$\frac{b_{m+1} b_{\mathrm{e}}}{b_m} = \frac{Z_{\mathrm{e}}}{N}\frac{U_{m+1}}{U_m} \mathrm{e}^{-I(m+1)/k_{\mathrm{B}}T}, \quad m = 0, 1, 2 \ldots Z-1. \tag{6}$$

Equation (6) consists of Z+2 nonlinear algebraic equations and has $Z + 2$ unknowns (b_0, b_1 .... b_Z, b_{e}), where Z_{e} is the free electrons partition function

$$Z_{\mathrm{e}} = 2 \times \left[\frac{2\pi m_{\mathrm{e}} k_{\mathrm{B}} T}{h^2}\right]^{3/2}.$$

Another quantity necessary for solving the Saha equation is the partition function for an ion m given by

$$U_m = \sum_i g_{i,m} \exp(-E_{i,m}/k_{\mathrm{B}} T),$$

where i runs over all the bound states of the ion m, the symbol i is a shorthand for the complete set of quantum numbers identifying an excited state, $E_{i,m}$ is the energy state i of ion m above the ground state, and $g_{i,m}$ is the statistical weight for state i of ion m given by $g_{i,m} = 2J_{i,m} + 1$, $J_{i,m,}$ being the total angular momentum quantum number for state i of ion m. In our calculations, we have taken $U_{m+1}/U_m = 1$ because it is close to 1 in our calculations.[6]

Equation (6) simplifies to

$$K_{m+1} = \frac{Z_{\mathrm{e}}}{N} \mathrm{e}^{-I(m+1)/k_{\mathrm{B}}T}, \tag{7}$$

$$b_{\mathrm{e}} = \sum_{m=0}^{Z} m b_m, \tag{8}$$

$$b_0 = \frac{1}{1 + \sum_{m=1}^{Z} \frac{1}{(b_e)^m} \prod_{j=1}^{m} K_j}, \quad (9)$$

$$b_m = \frac{b_0}{(b_e)^m} \prod_{j=1}^{m} K_j, m = 1, 2 \ldots Z. \quad (10)$$

The set of equations (4)–(10) can be solved by iterative methods. For our case the solution can be obtained with relative simplicity using the following algorithm:

(1) First guess an initial value of b_e ($0 \leq b_e \leq Z$).

(2) Through equation (9), we get b_0.

(3) Using the above two values, we get b_m from equation (10).

(4) Then we calculate a new b_e by equation (8).

(5) If the calculated values for b_e and the guessed one are different then use the calculated value to update the guess for the next cycle or iteration.

(6) Repeat until the difference between the calculated value for b_e and the guessed one meet the desired accuracy.

(7) After desired accuracy is achieved, then we obtain the values of b_e, b_0, $b_1 \ldots b_Z$ and N_e, N_0, $N_{1\ldots}N_Z$ respectively.

3. Results and discussion

The plasma characteristics of the most abundant elements (C, N, O, Mg, Al, Si, S, and Fe) in the sun have been calculated in this paper. Only the results for carbon, aluminium and iron plasmas are shown here. The temperature values along the solar radius are taken from the Ref.[7].

3.1. Carbon plasma

It is well established that carbon is an element of major astrophysical significance. In particular, emission lines of the boron-like C II ion occur frequently in astrophysical objects. For example, lines have been observed in N-type carbon stars,[8] in OB stars,[9] planetary nebulae[10] and in molecular cloud/ H II complexes.[11] It is for this reason that such kind of data are needed for use in modelling.

Figure 1 shows the ionic fractions of carbon ions along the solar radius of the sun. The ionic fractions are sharply peaked functions of the temperature. Figure 1 shows that more carbon atoms are ionized at low temperature, whereas it is the opposite at high temperature. C^{+4} is He-like and C^{+6} is the nude nucleus of C. The curve of He-like C^{+4} stands out above those of adjacent ions due to its stable structure and it appears after half of the solar radius. Near the centre of the sun, all of the electrons in C atoms are stripped and C exists in the form of nude nucleus.

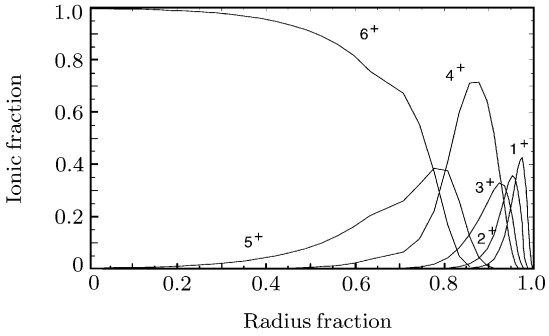


Fig.1. Ionic fraction vs. radius fraction (r/R_0, where R_0=Radius of the sun=6.960×10^{10}cm) for C- plasma.

Figure 2 shows the degree of ionization of C plasma. There is a plateau due to stripping of all the electrons from the C atom. Then there is a rapid decrease in electron density due to decrease in temperature.

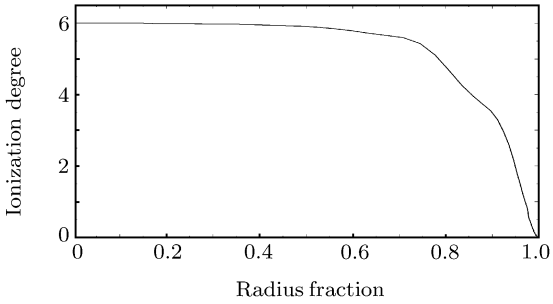


Fig.2. Ionization degree for C- plasma.

3.2. Aluminium plasma

Emission lines of Al XIII have been observed in the solar[12] as well as in the stellar plasmas,[13] particularly in the x-ray region.[14] Additionally, this ion is employed to pump other ions (for example: Li-like iron) in lasing plasmas.[15] With this in view, we are reporting such kind of data.

Figure 3 shows the ionization equilibria of Al ions against the solar radius fraction of the sun. Al^{+3} and Al^{+11} are Ne-like and He-like, respectively, and have full shell structure due to stable atomic structure. Al^{+13} is the nude nucleus. The ionic fraction of Al^{+11} is higher and its peak stands out above and

broader than the adjacent ions due to its stable and closed shell structure.

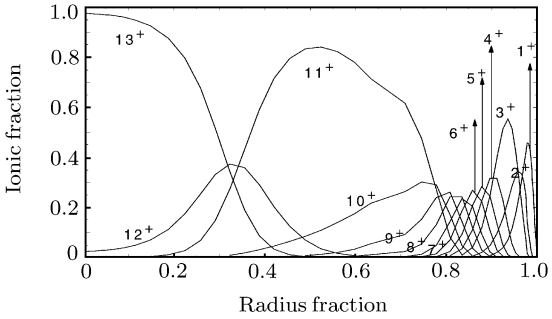


Fig.3. Ionic fraction vs radius fraction for Al-plasma.

Figure 4 shows the ionization degree for Al plasma as a function of radius fraction of the sun. The contribution of electrons to the electron density in the sun from Al decreases as the distance from the centre of the sun to the surface of photosphere increases due to the decrease in temperature. In Fig.4 there are two flat plateaus for the nude ion and the He-like ion.

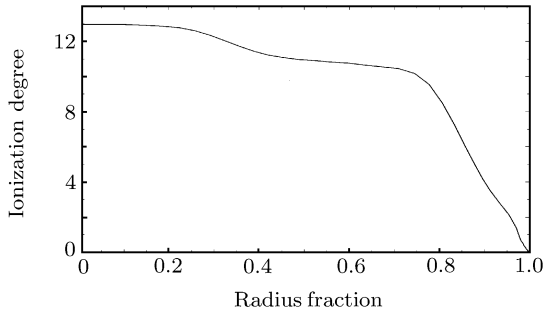


Fig.4. Ionization degree for Al plasma.

3.3. Iron plasma

The ionic fraction for iron plasma as a function of solar radius is shown in Fig.5. More iron atoms are ionized at low temperature, whereas it is the opposite at high temperature. Fe^{+16} and Fe^{+24} are Ne-like and He-like and have full shell structure due to stable atomic structure. Very small amount of iron is fully ionized near the core of the sun but a lot of He-like iron exists near the centre of the sun. Ne-like iron is in the middle of the solar radius.

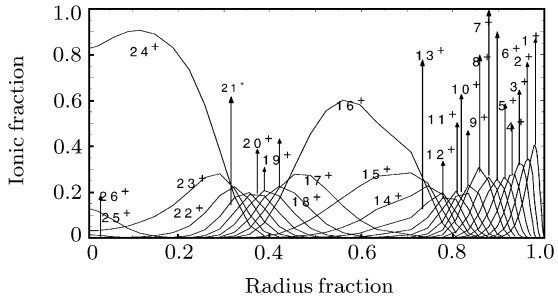


Fig.5. Ionic fraction vs radius fraction for Fe-plasma.

Figure 6 shows the ionization degree of Fe plasma as a function of solar radius. The contribution of electrons to the electron density in the sun from Fe and also from other most abundant elements decreases as the distance from the centre of sun to the surface of photosphere increases due to the decrease in temperature. The two flat plateaus in Fig.6 show the He-like and Ne-like abundance of Fe ions.

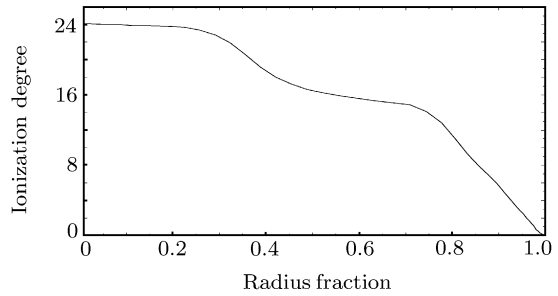


Fig.6. Ionization degree for Fe-plasma.

Plasma characteristics of other elements (N, O, Mg, Si, and S) are calculated in the same way and data for these elements can be acquired through the correspondence address of the author.

4. Conclusions

The plasma characteristics of the sun have been studied by using the well-known Saha equation. The ion abundance, ionization state and electron density of the most abundant elements (C, N, O, Mg, Al, Si, S, and Fe) in the interiors of the sun are calculated from the core of the sun to the surface of the photosphere. The model and computations are carried out under the assumptions of local thermodynamic equilibrium (LTE). The ionic fractions were found to be sharply peaked functions of the temperature. Near the surface of the photosphere, C, Al and Fe were

found to be more clearly ionized. Ionic fraction peaks of He-like C^{+4}, He-like Al^{+11} and Ne-like Fe^{+16} stood out above their adjacent ions due to their full shell atomic structure and very low recombination rates. The contribution of electrons from the most abundant elements to the electron density in the sun decreases as the distance from the centre of sun to the surface of photosphere increases due to the decrease in temperature.

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