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Synchronization and coherence resonance in chaotic neural networks*

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Synchronization and coherence of chaotic Morris–Lecar (ML) neural networks have been investigated by numerical methods. The synchronization of the neurons can be enhanced by increasing the number of the shortcuts, even though all neurons are chaotic when uncoupled. Moreover, the coherence of the neurons exhibits a non-monotonic dependence on the density of shortcuts. There is an optimal number of shortcuts at which the neurons' motion is most ordered, i.e. the order parameter (the characteristic correlation time) that is introduced to measure the coherence of the neurons has a maximum. These phenomena imply that stochastic shortcuts can tame spatiotemporal chaos. The effects of the coupling strength have also been studied. The value of the optimal number of shortcuts goes down as the coupling strength increases.

Keywords: synchronization, coherence, chaotic neuron, complex networks

PACC: 0547, 6460C, 8730E

1. Introduction

The study of synchronization of coupled oscillatory elements is an intensively developing branch of nonlinear science[1−3] for its relevance to many problems of physics, chemistry, and life science, and in particular, to neuroscience. Cooperative behaviour is a major focus of study in neural systems. The selective synchronization of coupled neurons has been regarded as a mechanism for binding spatially distributed features into a coherent object and could play a key role in intercommunication between neurons.[4] Synchronization behaviour is also important for signal encoding and transduction in the information processing of neurons.[5] Coherence and synchronization of coupled neurons may elucidate how the coherent spontaneous synchronized oscillations are established in many neural systems, which have been observed in brain cortex.[6−8]

So far, most studies of synchronization and coherence are between two coupled neurons, or neurons on completely local regular networks and completely random networks. But many biological networks lie somewhere between these two extremes. Recently, the investigation of complex networks has attracted much attention,[9−10] for its importance in mimicking the real systems of the world, such as social,[11] biological, and electronic communication networks. In fact, nearly any complex system in nature can be viewed as a network or graph in which vertices represent the dynamic elements of the system, and the edges denote the interactions or couplings between them. Two distinct types of topology: small-world network[12] (SWN) and scale-free network[13] (SFN), have been studied extensively for their properties seem to be a quantifiable characteristic of many real-world structures. Thus, it is important to study how the interplay between the intrinsic dynamics of the constituent elements and their complex connectivity can influence dynamic features of the system. It has been shown that any spreading rate can lead to complete infection of disease in SFN,[14,15] stochastic resonance[16] and synchronization[17,18] can be considerably improved in SWN, and SWN can greatly enhance the probability of spiral wave formation in excited media,[19] ordering chaos,[20] and oscillation death.[21] It has also been shown that the topology of the FitzHugh–Nagumo neuron network has an important impact on the effective frequency range of the system.[22] All these work shows that the complex network connectivity plays a

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crucial role in the dynamics of the system.

In biology, neural networks are always a research subject in this area. It is known that a single neuron in the vertebrate cortex is connected to more than 10^4 postsynaptic neurons via synapses, forming complex networks.[23] Therefore, adding a number of long-range connections among them is reasonable and feasible. It is rather interesting and also of great significance to study how the topology of networks a effects the synchronization and coherence of neural systems. In the present work, we study the synchronization and coherence of chaotic ML neurons in complex networks. On one hand, the neurons can be more and more synchronous as the shortcuts of the complex increases. On the other hand, there is an optimal number of the shortcuts at which the coherence of the neuron has a maximum, which means that the neurons show their best performance. The effects of the coupling strength have also been studied, and it is found that the value of the optimal number of shortcuts goes down as the coupling strength increases.

2. Model

The Morris–Lecar (ML) neuron model[24] is a simplified version of the Hodgkin–Huxley model. The model developed by Morris and Lecar was first used to describe the firing and refractory properties of real neurons. Because it has biophysically meaning and measurable parameters, the model became quite popular in the computational neuronscience community. Although there are many different neuron models, the ML model has been frequently employed even in recent years.[25] The ML equations represent an electrical circuit equivalent to a cellular membrane crossed by three different transmembrane currents: the voltage-gated Ca^{2+} current, the voltage-gated delayed-rectifier K^+ and the leak current. Dynamics of a isolated ML neuron are given by the following differential equations:

$$\begin{aligned} C\frac{\mathrm{d}V}{\mathrm{d}t} &= g_{\mathrm{Ca}} m_\infty(V)(V_{\mathrm{Ca}} - V) + g_{\mathrm{K}}\omega(V_{\mathrm{K}} - V) \\ &\quad + g_{\mathrm{l}}(V_{\mathrm{l}} - V) + I, \\ \frac{\mathrm{d}\omega}{\mathrm{d}t} &= \lambda_\infty(V)(\omega_\infty(V) - \omega), \end{aligned} \tag{1}$$

with

$$m_\infty(V) = 0.5\left(1 + \tanh\frac{V - V_a}{V_b}\right),$$

$$\omega_\infty(V) = 0.5\left(1 + \tanh\frac{V - V_c}{V_d}\right),$$

$$\lambda_\infty(V) = \frac{1}{3}\cosh\left(\frac{V - V_c}{2V_d}\right),$$

where t is the time variable that is in units of millisecond. V represents for the membrane action potential, C denotes the capacity of the membrane, while ω represents the activation of the K^+ current. The parameters g_{Ca}, g_{K} and g_{l} are the maximal conductances associated with the three transmembrane currents, V_{Ca}, V_{K} and V_{l} are the corresponding reversal potentials. V_a, V_b, V_c and V_d are constants. For a detailed explanation of the parameters, see Ref.[24]. The parameter set used in our simulations is as follows: $C = 1\,\mu\mathrm{F}\cdot\mathrm{cm}^{-2}$, $g_{\mathrm{Ca}} = 1.2\,\mathrm{mS}\cdot\mathrm{cm}^{-2}$, $g_{\mathrm{K}} = 2.0\,\mathrm{mS}\cdot\mathrm{cm}^{-2}$, $g_{\mathrm{l}} = 0.5\,\mathrm{mS}\cdot\mathrm{cm}^{-2}$, $V_{\mathrm{K}} = -1.1\,\mathrm{mV}$, $V_{\mathrm{l}} = -0.5\,\mathrm{mV}$, $V_a = -0.01\,\mathrm{mV}$, $V_b = 0.15\,\mathrm{mV}$, $V_c = 0.1\,\mathrm{mV}$ and $V_d = 0.05\,\mathrm{mV}$. I is the external stimulus current. To generate rich dynamics in the ML neuron model, an external linear slow subsystem is introduced as follows:[26]

$$\frac{\mathrm{d}I}{\mathrm{d}t} = -\mu(\frac{1}{5} + V), \tag{2}$$

where μ is a constant and we set $\mu = 0.005$. In this case μ is a small quantity, thus I is a slow variable and the system (1) is a double-time scale system now.

Especially the parameter V_{Ca} is selected as the control parameter. When V_{Ca} is changed, the system exhibits a rich dynamics including chaotic behaviour. Figure 1 shows the bifurcation structure of the inter-spike interval (ISI) versus the parameter V_{Ca} of a single ML neuron.

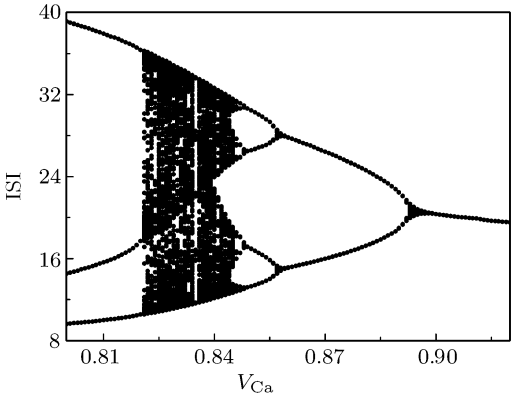


Fig.1. The bifurcation diagram of inter-spike interval (ISI) versus the parameter V_{Ca} in a single ML neuron.

Now we consider the dynamics of a complex network of N coupled chaotic ML neurons. The complex neural network in the present work is constructed as follows. We start from a one-dimensional regular ring that comprises N identical chaotic ML neurons, and each neuron has two nearest neighbours. Links

are then randomly added between non-nearest vortices. Figure 2 shows one realization of the networks for $N = 12$. In the case of globally coupled neural network, i.e. all neurons are coupled to each other, the network contains $N(N-1)/2$ edges. If the number of added random shortcuts is denoted by M, then the fraction of random shortcuts, which is the ratio of the number of random shortcuts to the number of all the possible edges among the neurons, is given by $p = M/(N(N-1)/2)$. Since the structural properties of complex networks, e.g. characteristic length and clustering coefficient, vary together with p, thus it is a key to study the effects of p on the dynamics of complex networks. Note that for a given p, there could be a lot of network realizations.

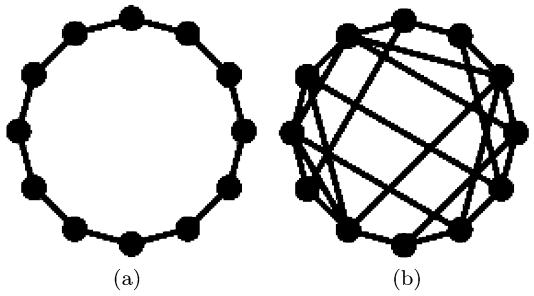


Fig.2. Sketches of the network graph with $N = 12$: (a) ring regular connection, (b) 12 random shortcuts are added.

Now the dynamics of the ML neuron network can be described by the following equations:

$$C\frac{\mathrm{d}V_i}{\mathrm{d}t} = g_{\mathrm{Ca}} m_\infty(V_i)(g_{\mathrm{Ca}} - V_i) + g_{\mathrm{K}}\omega_i(V_{\mathrm{K}} - V_i) + g_{\mathrm{l}}(V_{\mathrm{l}} - V_i) + I_i + \sum_j \varepsilon_{ij}(V_j - V_i),$$

$$\frac{\mathrm{d}\omega_i}{\mathrm{d}t} = \lambda_\infty(V_i)(\omega_\infty(V_i) - \omega_i),$$

$$\frac{\mathrm{d}I_i}{\mathrm{d}t} = -\mu(\frac{1}{5} + V_i), \qquad (3)$$

where $i = 1, 2, \ldots, N$; $N = 60$ is the number of neurons. V_{Ca} is fixed at 0.84 as shown in the bifurcation diagram. All the ML neurons are chaotic when uncoupled. ε_{ij} is the coupling strength between the two neurons i and j, which is determined by the coupling pattern of the system. If these two neurons are coupled to each other, we have $\varepsilon_{ij} = \varepsilon$, and otherwise $\varepsilon_{ij} = 0$. Numerical integration of Eq.(3) is carried out by explicit Euler method with a time step of 0.001 ms. In each calculation, the time evolution of the system lasts 6000 ms after the transient is discarded.

3. Results and discussion

The spatiotemporal evolutions of the neurons membrane action potentials are displayed in Fig.3. In the bright regions, the neurons fire, while in the dark ones they are quiescent. The thin structures in the brighter regions correspond to the firing patterns of individual spikes inside a burst. Time passes from top to bottom. The left panel in Fig.2 corresponds to the regular lattice case ($p = 0$). Spatiotemporal chaos is observed, and the ML neurons show rather irregular and non-synchronization bursts/spikes. However, when certain random shortcuts are added (e.g., $p = 0.12$, i.e. $M \approx 212$), the synchronization is more enhanced and the performance of temporal regularity becomes best. If the number of the random shortcuts is further increased, the spikes are irregular in time although the neurons are still spatially synchronized (see the panel for $p = 0.4$).

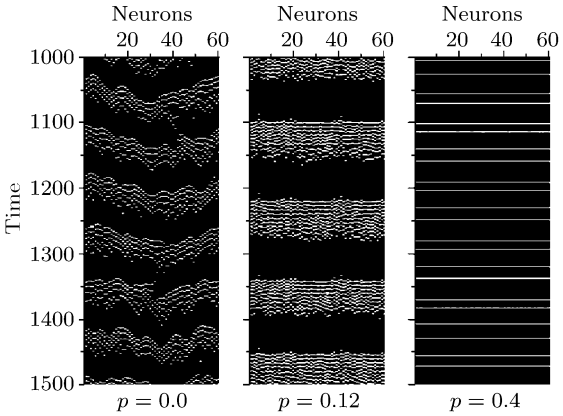


Fig.3. Typical spatiotemporal evolutions of membrane action potentials of 60 coupled ML neurons in the network. Time increases from top to bottom. From left to right, p is 0.0, 0.12 and 0.2. The spatial synchronization is enhanced with increasing p, and temporal coherence becomes the most prominent for an intermediate p. The coupling strength is $\varepsilon = 0.03$.

To qualitatively show the regularity of the neurons in time, the interspike interval histograms (ISIHs) for different p are displayed in Fig.4. For moderate p ($p = 0.12$) the ISI is relatively narrow, i.e. the neurons shows that the burst synchronization and the bursts are almost periodical in time. For small p, however, the distribution is scattered that corresponds to incoherence bursts of the neurons. For large p, the neurons show complete synchronization spikes. Thus the neurons behaviour is the same a that of a single neuron, which shows chaotic spikes (see the ISI distribution of Fig.4.(c)). This means that the motion of the neurons for moderate p is more regular in time.

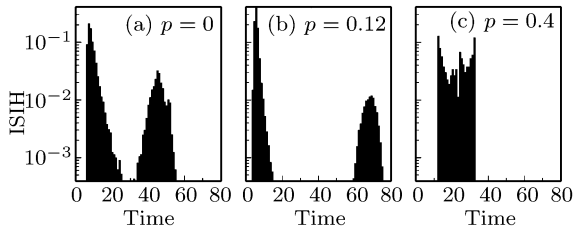


Fig.4. Normalized interspike interval histograms of all neurons in the network for different p. The coupling strength is $\varepsilon = 0.03$.

To further quantitatively characterize this behaviour, we introduce the standard deviation σ and characteristic time τ to measure the spatial synchronization and the temporal regularity of the system, respectively. σ is defined as

$$\sigma = [\langle \sigma(t) \rangle]$$

with

$$\sigma(t) = \sqrt{\frac{\left[\frac{1}{N}\sum_{i=1}^{N} V_i(t)^2 - \left\{\frac{1}{N}\sum_{i=1}^{N} V_i(t)\right\}^2\right]}{(N-1)}}$$

where $\langle \bullet \rangle$ represents averaging over time and $[\bullet]$ denotes averaging over 50 different network realizations for each p. $\sigma(t)$ is a good quantity for measuring the spatial synchronization of the neurons at a fixed time t. Large $\sigma(t)$ represents large deviation between various elements, and small $\sigma(t)$ demonstrates stronger collective motion and, consequently, better synchronization. The extreme of $\sigma(t) = 0$ shows complete synchronization. In this paper, we use σ to quantitatively measure the synchronization of the neurons membrane action potential over a long time. Characteristic correlation time τ is used to quantitatively measure the temporal coherence of the system, and it is defined as

$$\tau = [\langle \tau_i \rangle] \qquad \text{and} \quad \tau_i = \frac{1}{T}\int_0^T C_i^2(s)\mathrm{d}s,$$

where τ_i is the characteristic correlation time of the ith ML neuron, $\langle \bullet \rangle$ now represents averaging over all the ML neurons, $[\bullet]$ has the same meaning as that mentioned above, and $C_i(s)$ is the normalized autocorrelation function,[27] defined as

$$C_i(s) = \frac{\left\langle \tilde{V}_i(t)\tilde{V}_i(t+s) \right\rangle}{\left\langle \tilde{V}_i^2 \right\rangle} \quad \text{with} \qquad \tilde{V}_i = V_i - \langle V_i \rangle ,$$

where $V_i(t)$ is the membrane action potential of the ith ML neuron at time t. In the present case of limited and discrete sampling with N_0 data points for each neuron, the characteristic correlation time is given by $\tau_i = \frac{1}{N_0 \Delta t}\sum_{k=1}^{N_0} C_i^2(\tau_k)\Delta t$, where $\tau_k = k\Delta t$ with Δt being the length of the sampling time. The more ordered the motion of a neuron is, the longer is its characteristic correlation time and hence its contribution to τ. Such a measure has been used successfully to characterize the coherence of coupled chaotic elements in Ref.[20]. Therefore, this quantity can be readily used to measure the degree of spatiotemporal order in the present system.

The dependences of standard deviation σ on p for different coupling strengths are depicted in Fig.5. The

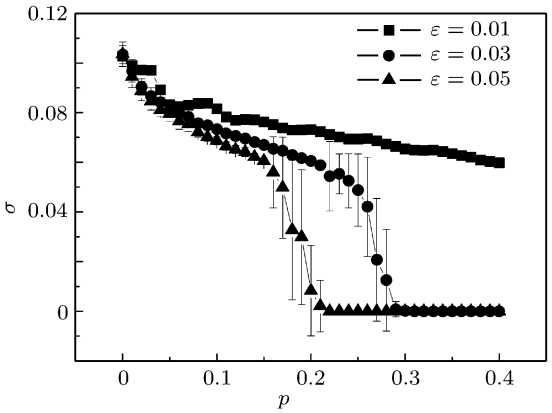


Fig.5. Dependences of standard deviation σ on p for different coupling strengths.

σ value decreases with increasing p, which means that the synchronization of the ML neurons is increasingly enhanced as p increases. The curves of characteristic correlation time τ versus p for different coupling strengths are displayed in Fig.6. There is an optimal value of p at which all the chaotic neurons run nearly synchronously in space and periodically in time. This

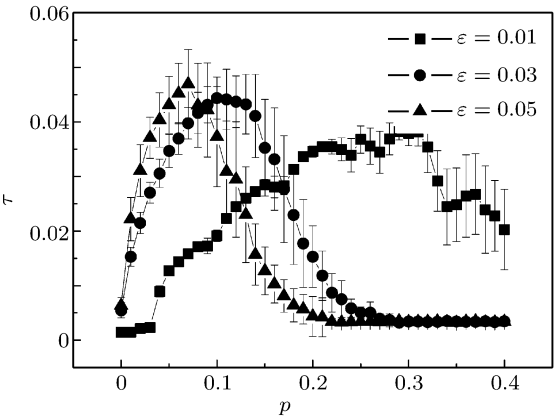


Fig.6. The curves of characteristic correlation time τ versus p for different coupling strengths.

shows that the system dynamics show somewhat resonant behaviour with an optimal number of shortcuts. Moreover, from Figs.5 and 6, we know the coupling strength also has important effects on the dynamics of the neurons. The neurons are better synchronous with larger coupling strength in the same networks. The optimal value of p decreases as the coupling strength increases, and this demonstrates that when the coupling strength is larger, a smaller number of random shortcuts can effectively tame the spatiotemporal chaos.

On one hand, the neural systems are more feasible to be described as a complex network rather than a regular one. On the other hand, synchronous neural activities in the central nervous system have been observed.[28,29] In additional, real neurons display rather rich dynamics including chaotic behaviour;[30] and spatiotemporal chaos is a waiting state for the cortex.[31] Thus, our findings may be instructive in understanding both the collective motions in coupled neurons and the exchange of information between different neuron states.

4. Conclusion

In this paper, we have studied the synchronization and coherence of chaotic Morris–Lecar (ML) neurons in complex networks. We find that the synchronization of the neurons, which are absent in a regular network, can be enhanced by increasing the number of the shortcuts, even though all neurons are chaotic when uncoupled. Interestingly, the coherence of the neurons exhibits a non-monotonic dependence on the density of shortcuts. There is an optimal number of shortcuts at which the neurons motion is most ordered, corresponding to an ordered state which is periodic in time and synchronous in space. These phenomena imply that shortcuts can tame spatiotemporal chaos. The effects of the coupling strength have also been studied, and stronger coupling strength will make it easier to tame the chaos.

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