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Controlling chaos using Takagi–Sugeno fuzzy model and adaptive adjustment*

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In this paper, an approach to the control of continuous-time chaotic systems is proposed using the Takagi–Sugeno (TS) fuzzy model and adaptive adjustment. Sufficient conditions are derived to guarantee chaos control from Lyapunov stability theory. The proposed approach offers a systematic design procedure for stabilizing a large class of chaotic systems in the literature about chaos research. The simulation results on Rössler's system verify the effectiveness of the proposed methods.

Keywords: controlling chaos, adaptive adjustment mechanism, Rössler system, TS fuzzy model
PACC: 0545

1. Introduction

Chaos has been found in many engineering systems. It is irregular, complex and unpredictable. In many practical systems, removing chaos can improve system performance, avoid fatigue failure of system, and lead to a predictable system behaviour. Various methods have been employed to control chaos,[1−15] since Ott, Grebogi and Yorke proposed a method of controlling chaos, i.e. the well-known OGY control.[1] Among those methods, the adaptive adjustment mechanism (AAM) proposed by Huang,[5,6] that was extended by Bu *et al*,[9] and Zheng,[12] is one of the most widely used methods mainly because it utilizes neither priori knowledge of the system itself nor extra external control.

On the other hand, fuzzy logic has received much attention from the theorists in control theory as a powerful tool for the chaos control. An approach to controlling chaos via linear matrix inequality (LMI) based fuzzy control system design has been suggested in Refs.[16–18], where the key idea is to use the well-known Takagi–Sugeno (TS) fuzzy model to represent typical chaotic systems and then design a controller for the fuzzy model. The stabilization of the overall fuzzy system is determined by solving a set of linear matrix inequalities (LMIs). It is required that a common positive-definite matrix P can be found to satisfy the LMIs for all the local models. However, this is a difficult problem to be solved since such a matrix might not exist in many cases, especially for highly nonlinear complex systems.

In the present paper, a novel approach to controlling chaotic systems is proposed that has the advantages of both the fuzzy control and the AAM. The approach represents a chaotic system by linear models in different state space regions based on the TS fuzzy model and then stabilizes the linear models in different state space regions by adaptive adjustment mechanisms. Therefore, it has a convenient analytic structure so that some well-established conventional linear systems theory can be easily applied to the theoretical analysis of the overall closed-loop controlled system. Our approach to choosing the adaptive parameters is different from Huang's method that is based on linearizing the system in the neighbourhood of a fixed point and evaluating the eigenvalues of the corresponding Jacobian matrix.

The rest of this paper is organized as follows. In Section 2, the brief description of the TS fuzzy model is given. Section 3 is devoted to the procedure for controlling chaotic systems via the TS fuzzy model and AAM. Simulation study is given in Section 4, and conclusions are drawn in Section 5.

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2. TS fuzzy model

The TS fuzzy model [18,19] is described by fuzzy IF-THEN rules in which each rule locally represents a linear realization of the system over a certain region in the state space. The overall system is then an aggregation of these local linear system models. Suppose that the TS fuzzy model is given in the following form:

$$\begin{aligned}&\text{Rule } i: \text{IF } z_1(t) \text{ is } M_{i1} \text{ and } \ldots \text{ and } z_p(t) \text{ is } M_{ip}\\ &\quad \text{THEN } \dot{x}(t) = A_i x(t), \quad i = 1, 2, \ldots, r,\end{aligned} \tag{1}$$

where M_{ij} $(j = 1, 2, \ldots, p)$ is the fuzzy set and r is the number of IF-THEN rules, $x(t) \in \Re^n$ is the state vector, $A_i \in \Re^{n\times n}$, $z_1(t) - z_p(t)$ are the premise variables. Each consequent linear equation represented by $A_i x(t)$ is called a subsystem. The final output of the fuzzy system is inferred as follows:

$$\dot{x}(t) = \frac{\sum\limits_{i=1}^{r} w_i(z(t)) A_i x(t)}{\sum\limits_{i=1}^{r} w_i(z(t))}, \tag{2}$$

where $z(t) = (z_1(t), z_2(t), \ldots, z_p(t))$,

$$w_i(z(t)) = \prod_{j=1}^{p} M_{ij}(z_j(t)) \tag{3}$$

for all t, with $M_{ij}(z(t))$ being the grade of membership of $z_j(t)$ in M_{ij}, and $w_i(z(t))$ satisfying the following conditions:

$$\begin{cases} \sum\limits_{i=1}^{r} w_i(z(t)) > 0, \\ w_i(z(t)) \geq 0, \quad i = 1, 2, \ldots, r. \end{cases}$$

By introducing $h_i(z(t)) = w_i(z(t)) / \sum_{i=1}^{r} w_i(z(t))$ instead of $w_i(z(t))$, the expression (2) is rewritten as

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t)) A_i x(t). \tag{4}$$

Note that

$$\begin{cases} \sum\limits_{i=1}^{r} h_i(z(t)) = 1, \\ h_i(z(t)) \geq 0, \quad i = 1, 2, \ldots, r, \end{cases} \tag{5}$$

for all t, where $h_i(z(t))$ can be regarded as the normalized weights of the IF-THEN rules.

3. Controlling chaotic systems via TS fuzzy model and AAM

Suppose that a chaotic system can be exactly represented by the TS fuzzy model (1). In order to stabilize the TS fuzzy model (1), by AAM we denote the following adjusted system:

$$\begin{aligned}&\text{Rule } i: \text{IF } z_1(t) \text{ is } M_{i1} \text{ and } \ldots \text{ and } z_p(t) \text{ is } M_{ip}\\ &\quad \text{THEN } \dot{x}(t) = (1 - \gamma_i) A_i x(t) - \gamma_i x(t),\\ &\quad i = 1, 2, \ldots, r.\end{aligned} \tag{6}$$

The final output of the fuzzy system (6) is inferred as follows:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))[(1 - \gamma_i) A_i x(t) - \gamma_i x(t)], \tag{7}$$

where γ_i is the positive controlling parameter, and referred to as the adaptive parameter named after Refs.[5, 6]. It is easy to see that the system (6) has the same equilibrium point $x = 0$ as that of the system (1).

The following theorem gives a sufficient condition for the stability of the adjusted system (7) and a convenient guide to the choice of the adaptive parameters.

Theorem 1 Let I be an $n \times n$ unit matrix and $\{\lambda_j^{(i)}\}_{j=1}^{n} (i = 1, 2, \ldots, r,)$ be the eigenvalues of the symmetric matrix

$$\frac{(1 - \gamma_i) A_i^{\mathrm{T}} + (1 - \gamma_i) A_i}{2} - \gamma_i I, \qquad (i = 1, 2, \ldots, r).$$

If $\max\limits_{1\leq i\leq r} \{\lambda_j^{(i)}\}_{j=1}^{n} < 0$, then the system (7) is globally asymptotically stable about the equilibrium point $x = 0$, that is, the chaotic orbits of the original system (1) can be stabilized to the equilibrium point $x = 0$ by AAM.

Proof Assume that the symmetric matrix

$$\frac{(1 - \gamma_i) A_i^{\mathrm{T}} + (1 - \gamma_i) A_i}{2} - \gamma_i I$$

is similar to

$$\Lambda_i = \mathrm{diag}(\lambda_1^{(i)}, \lambda_2^{(i)}, \ldots, \lambda_n^{(i)})$$

by the operation of an orthogonal matrix U_i, that is,

$$U_i^{\mathrm{T}} \left[\frac{(1 - \gamma_i) A_i^{\mathrm{T}} + (1 - \gamma_i) A_i}{2} - \gamma_i I \right] U_i = \Lambda_i.$$

Choose the Lyapunov function

$$V = \frac{1}{2} x^{\mathrm{T}}(t) x(t), \tag{8}$$

and the derivative of V along trajectories of (7) will be

$$\begin{aligned}\dot{V} =&\frac{1}{2}[\dot{x}^{\mathrm{T}}(t)x(t)+x^{\mathrm{T}}(t)\dot{x}(t)]\\ =&\sum_{i=1}^{r}h_i(z(t))x^{\mathrm{T}}(t)\Bigg[\frac{(1-\gamma_i)A_i^{\mathrm{T}}+(1-\gamma_i)A_i}{2}\\ &-\gamma_i I\Bigg]x(t)\\ =&\sum_{i=1}^{r}h_i(z(t))x^{\mathrm{T}}(t)U_i\Lambda_i U_i^{\mathrm{T}}x(t)\\ =&\sum_{i=1}^{r}h_i(z(t))(U_i^{\mathrm{T}}x(t))^{\mathrm{T}}\Lambda_i U_i^{\mathrm{T}}x(t)\\ \le&\max_{1\le i\le r}\{\lambda_j^{(i)}\}_{j=1}^{n}\sum_{i=1}^{r}h_i(z(t))(U_i^{\mathrm{T}}x(t))^{\mathrm{T}}U_i^{\mathrm{T}}x(t)\\ \le&\max_{1\le i\le r}\{\lambda_j^{(i)}\}_{j=1}^{n}x^{\mathrm{T}}(t)x(t)<0. \end{aligned}\tag{9}$$

Furthermore, $\dot{V}=0$ if and only if $x=0$. So $x(t)\to 0$ as $t\to\infty$. The system (7) is globally asymptotically stable about the equilibrium point $x=0$, that is, the chaotic orbits of the original system (1) can be stabilized to the equilibrium point $x=0$ by AAM.

Remark 1 In order to stabilize a chaotic system represented by the TS fuzzy model (1), we only need to choose γ_i such that the eigenvalues of the symmetric matrix

$$\frac{(1-\gamma_i)A_i^{\mathrm{T}}+(1-\gamma_i)A_i}{2}-\gamma_i I$$

are less than zero by Theorem 1. Fortunately, it can be achieved by Hurwitz's Theorem.

4. Simulation study

Simulation of controlling Rössler's system is performed here to show the effectiveness and feasibility of the proposed method. The chaotic Rössler's system is described by

$$\begin{cases}\dot{x}_1(t)=-x_2(t)-x_3(t),\\ \dot{x}_2(t)=x_1(t)+ax_2(t),\\ \dot{x}_3(t)=bx_1(t)-(c-x_1(t))x_3(t),\end{cases}\tag{10}$$

where a, b and c are the parameters. Assume that $x_1\in[-d,d]$ and $d>0$, let the premise variable be x_1, then Rössler's system can be exactly represented by the TS fuzzy model as follows:[18]

$$\begin{aligned}&\text{Rule 1: IF } x_1(t) \text{ is } F_1(x_1(t)) \text{ THEN } \dot{x}(t)=A_1x(t);\\ &\text{Rule 2: IF } x_1(t) \text{ is } F_2(x_1(t)) \text{ THEN } \dot{x}(t)=A_2x(t),\end{aligned}\tag{11}$$

where $x(t)=[x_1(t),\ x_2(t),\ x_3(t)]^{\mathrm{T}}$,

$$A_1=\begin{bmatrix}0&-1&-1\\1&a&0\\b&0&-d\end{bmatrix},$$

$$A_2=\begin{bmatrix}0&-1&-1\\1&a&0\\b&0&d\end{bmatrix},$$

and the fuzzy sets are $F_1(x_1(t))=(1/2)(1+(c-x_1(t))/d)$, $F_2(x_1(t))=(1/2)(1-(c-x_1(t))/d)$ with $d>0$. In this paper, $a=0.34$, $b=0.4$, $c=4.5$ and $d=10$. The chaotic behaviour is shown in Fig.1.

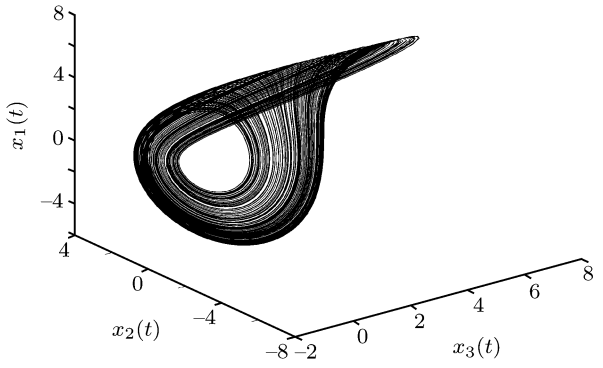


Fig.1. Chaotic behaviour of Rössler fuzzy system.

By AAM we denote the following adjusted system:

$$\begin{aligned}&\text{Rule 1: IF } x_1(t) \text{ is } F_1(x_1(t)) \text{ THEN } \dot{x}(t)=(1-\gamma_1)A_1x(t)-\gamma_1x(t);\\ &\text{Rule 2: IF } x_1(t) \text{ is } F_2(x_1(t)) \text{ THEN } \dot{x}(t)=(1-\gamma_2)A_2x(t)-\gamma_2x(t).\end{aligned}\tag{12}$$

The final output of the fuzzy system (12) is inferred as follows:

$$\dot{x}(t) = \sum_{i=1}^{2} F_i(x(t))[(1-\gamma_i)A_i x(t) - \gamma_i x(t)]. \quad (13)$$

For the above parameter set, we can obtain $0.2537 < \gamma_1 < 1$ and $0.2537 < \gamma_2 < 0.8946$ by Theorem 1 and Hurwitz's Theorem.

Figure 2 shows the simulation result of stabilizing the chaotic Rössler's system for $\gamma_1 = 0.5$ and $\gamma_2 = 0.7$.

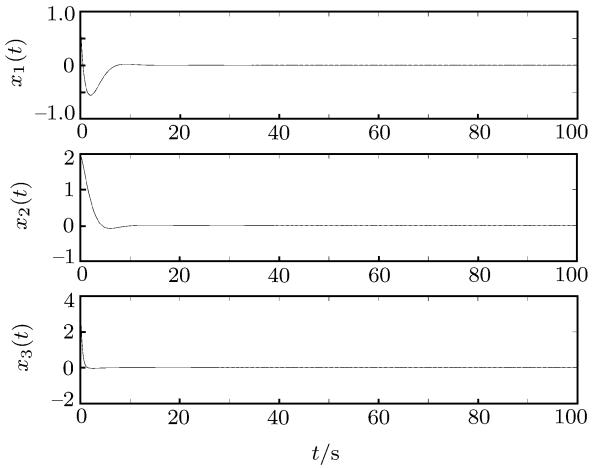


Fig.2. The time series of controlled Rössler fuzzy system via AAM.

5. Conclusions

We have proposed approaches to the stabilization of continuous-time chaotic systems using the TS fuzzy model and adaptive adjustment. Sufficient conditions are derived to guarantee chaos control by Lyapunov stability theory. The proposed approach offers a systematic design procedure for stabilizing a large class of chaotic systems in the literature about chaos research. Simulation of control Rössler's system is made. The simulation result has validated the effectiveness of the proposed approach.

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