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LMI-based output feedback fuzzy control of chaotic system with uncertainties*

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This paper studies the robust fuzzy control for nonlinear chaotic system in the presence of parametric uncertainties. An uncertain Takagi–Sugeno (T–S) fuzzy model is employed for fuzzy modelling of an unknown chaotic system. A sufficient condition formulated in terms of linear matrix inequality (LMI) for the existence of fuzzy controller is obtained. Then the output feedback fuzzy-model-based regulator derived from the LMI solutions can guarantee the stability of the closed-loop overall fuzzy system. The T–S fuzzy model of the chaotic Chen system is developed as an example for illustration. The effectiveness of the proposed controller design methodology is finally demonstrated through computer simulations on the uncertain Chen chaotic system.

Keywords: chaos, output-feedback controller, T–S fuzzy model, linear matrix inequality
PACC: 0545

1. Introduction

Recently, there has been rapidly growing interest in chaos control. Many theories and techniques have been developed, using various control schemes inclusive of intelligent processing methodologies for the control of chaotic systems.[1−10] For uncertain chaotic systems, the parametric uncertainties are principal factors responsible for the degraded stability and performance in general of an unknown nonlinear control system. Therefore, they have to be carefully handled for safe and reliable operations. In order to overcome all kinds of difficulties in the design of a controller for an uncertain chaotic system, intelligent control schemes have been investigated in the last decades, among which a successful approach is fuzzy control.

It is well known that the trend of fuzzy control has been to develop some systematic design algorithms so as to guarantee the control performance and system stability for the T–S fuzzy-model-based controllers.[11]

Meanwhile, it is also very important to consider the robust stability against parametric uncertainties in the T–S fuzzy-model-based control system. On the other hand, the internal states of many industrial plants cannot be directly available for measurement but only their outputs are obtained through the control processes. With this in mind, the output feedback fuzzy-model-based control technique and the fuzzy control scheme are incorporated, and then a sufficient condition for the synthesis of the output feedback fuzzy-model-based controller is derived via the LMI tool. As a result, the desired output feedback fuzzy-model-based controller is obtained through the convex optimization technique.

This paper is organized as follows. In Section 2, we set up the problem considerations and brief for the T–S fuzzy model, and in Section 3, we present the main result. An illustrative example is given in Section 4 and, finally, in Section 5 some conclusions are drawn.

2. Problem formulation[12−14]

In this section, we briefly review the T–S fuzzy model. Considering parametric uncertainties, we propose the T–S fuzzy system in which the ith rule is formulated in the following form:

Rule i:

IF $z_1(t)$ is N_{i1} and ... and $z_n(t)$ is N_{in}

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THEN

$$\dot{\boldsymbol{x}}(t) = (\boldsymbol{A}_i + \Delta\boldsymbol{A}_i)\boldsymbol{x}(t) + (\boldsymbol{B}_i + \Delta\boldsymbol{B}_i)\boldsymbol{u}(t),$$
$$\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x}, i = 1, 2, ..., q, \tag{1}$$

where z_i is the premise variable assumed to be independent of the control input $\boldsymbol{u}$ and $N_{ij}(j = 1, 2, ..., n)$ is a fuzzy set, $\boldsymbol{x}(t) \in \Re^n$ is the state vector, $\boldsymbol{y}(t) \in \Re^p$ is the output vector, $\boldsymbol{u}(t) \in \Re^m$, $\boldsymbol{A}_i \in \Re^{n\times n}$ and $\boldsymbol{B}_i \in \Re^{n\times m}$ are the system matrix and input matrix, respectively, $\boldsymbol{C} \in \Re^{p\times n}$ with full row-rank, $\Delta\boldsymbol{A}_i$ and $\Delta\boldsymbol{B}_i$ represent parametric uncertainties in the system, and q is the number of fuzzy rules.

Using centre-average defuzzification, product inference, and singletone fuzzifier, the global T–S fuzzy system (1) is formulated as

$$\begin{aligned}\dot{\boldsymbol{x}}(t) =& \sum_{i=1}^{q} \mu_i(z(t))((\boldsymbol{A}_i + \Delta\boldsymbol{A}_i)\boldsymbol{x}(t) \\ &+ (\boldsymbol{B}_i + \Delta\boldsymbol{B}_i)\boldsymbol{u}(t)) \\ \boldsymbol{y} =& \boldsymbol{C}\boldsymbol{x},\end{aligned} \tag{2}$$

where

$$\mu(z(t)) = \frac{\omega_i(z(t))}{\sum\limits_{i=1}^{q} \omega_i(z(t))}, \quad \omega_i(z(t)) = \prod_{j=1}^{n} N_{ij}(z_j(t)),$$

and $N_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in N_{ij}.

Assumption 1 The parameter uncertainty matrices $\Delta\boldsymbol{A}_i$ and $\Delta\boldsymbol{B}_i$ are norm-bounded in the form

$$[\Delta\boldsymbol{A}_i \quad \Delta\boldsymbol{B}_i] = \boldsymbol{D}_i\boldsymbol{F}_i(t)[\boldsymbol{E}_{1i} \quad \boldsymbol{E}_{2i}],$$

where $\boldsymbol{D}_i$, $\boldsymbol{E}_{1i}$, and $\boldsymbol{E}_{2i}$ are known real constant matrices of appropriate dimensions, and $\boldsymbol{F}_i(t) \in \Re^{i\times j}$ is an unknown matrix function with Lebesgue-measurable elements, and satisfies

$$\boldsymbol{F}_i^{\mathrm{T}}(t)\boldsymbol{F}_i(t) \le \boldsymbol{I},$$

where $\boldsymbol{I}$ is the identity matrix of appropriate dimension.

3. Stabilization of the T–S fuzzy model

This section presents some sufficient conditions that guarantee the global asymptotic stability of the controlled T–S fuzzy system subject to parametric uncertainties.

Consider the following T–S fuzzy model with parametric uncertainties, which is used to represent a complex, multi-input multi-output unknown nonlinear system:

$$\begin{aligned}\dot{\boldsymbol{x}}(t) =& \sum_{i=1}^{q} \mu_i(z(t))((\boldsymbol{A}_i + \Delta\boldsymbol{A}_i)\boldsymbol{x}(t) + (\boldsymbol{B}_i \\ &+ \Delta\boldsymbol{B}_i)\boldsymbol{u}(t)), \\ \boldsymbol{y} =& \boldsymbol{C}\boldsymbol{x}.\end{aligned} \tag{3}$$

The objective is to design a T–S fuzzy-model-based output feedback controller for robust stabilization of the system (3) in the form

$$\begin{aligned}\boldsymbol{u}(t) =& -\sum_{i=1}^{q} \mu_i(\boldsymbol{x}(t))\boldsymbol{K}_i\boldsymbol{y} \\ =& -\sum_{i=1}^{q} \mu_i(\boldsymbol{x}(t))\boldsymbol{K}_i\boldsymbol{C}\boldsymbol{x}(t),\end{aligned} \tag{4}$$

where $\boldsymbol{K}_i$ is the local output feedback gain.

Thus, the closed-loop system of (3) and (4) is

$$\begin{aligned}\dot{\boldsymbol{x}} =& \sum_{i=1}^{q} \mu_i(\boldsymbol{A}_i - \boldsymbol{B}_i\boldsymbol{K}_i\boldsymbol{C} + \boldsymbol{D}_i\boldsymbol{F}_i(\boldsymbol{E}_{1i} - \boldsymbol{E}_{2i}\boldsymbol{K}_i\boldsymbol{C}))\boldsymbol{x}, \\ \boldsymbol{y} =& \boldsymbol{C}\boldsymbol{x}.\end{aligned} \tag{5}$$

The main result on global asymptotic stability of the T–S fuzzy model with uncertainties is summarized in the following theorem.

Theorem 1 If there exists a symmetric and positive definite matrix $\boldsymbol{P}_i$, some matrices $\boldsymbol{M}_i$ and $\boldsymbol{N}_i$, and some scalars ε_i such that the following LMIs are satisfied, then the uncertain T–S fuzzy model (4) is asymptotically stable via the T–S fuzzy model-based output feedback controller (4)

$$\begin{bmatrix} \Psi_i & * \\ \boldsymbol{E}_{1i}\boldsymbol{P}_i - \boldsymbol{E}_{2i}\boldsymbol{N}_i\boldsymbol{C} & -\varepsilon_i\boldsymbol{I} \end{bmatrix} < 0, \tag{6}$$

$$\boldsymbol{M}_i\boldsymbol{C} = \boldsymbol{C}\boldsymbol{P}_i, \quad i = 1, 2, ...q, \tag{7}$$

where

$$\begin{aligned}\Psi_i =& (\boldsymbol{A}_i\boldsymbol{P}_i - \boldsymbol{B}_i\boldsymbol{N}_i\boldsymbol{C})^{\mathrm{T}} \\ &+ (\boldsymbol{A}_i\boldsymbol{P}_i - \boldsymbol{B}_i\boldsymbol{N}_i\boldsymbol{C}) + \varepsilon_i\boldsymbol{D}_i\boldsymbol{D}_i^{\mathrm{T}}, \\ \boldsymbol{K}_i =& \boldsymbol{N}_i\boldsymbol{M}_i^{-1}.\end{aligned}$$

In order to easily determine $\boldsymbol{M}_i$ in (7), the equation (7) in Theorem 1 may be converted into the following inequality:

$$(\boldsymbol{M}_i\boldsymbol{C} - \boldsymbol{C}\boldsymbol{P}_i)^{\mathrm{T}}(\boldsymbol{M}_i\boldsymbol{C} - \boldsymbol{C}\boldsymbol{P}_i) < \lambda\boldsymbol{R}_i, \tag{8}$$

where λ is a sufficiently small positive scalar and $\boldsymbol{R}_i$ are positive definite matrices such that $\boldsymbol{R}_i^{\mathrm{T}} \cdot \boldsymbol{R}_i < \boldsymbol{I}$. Only if the value λ is small enough, the condition (7)

is closely satisfied. One thus obtain the following LMI condition through the Schur complement:[15]

$$\begin{bmatrix} \lambda \boldsymbol{R}_i & * \\ \boldsymbol{M}_i\boldsymbol{C} - \boldsymbol{C}\boldsymbol{P}_i & \boldsymbol{I} \end{bmatrix} > 0, \quad i = 1, 2, ..., q. \tag{9}$$

Accordingly, the output feedback fuzzy-model-based controller design may proceed as follows.

Minimize λ subject to

$$\begin{bmatrix} \lambda \boldsymbol{R}_i & * \\ \boldsymbol{M}_i\boldsymbol{C} - \boldsymbol{C}\boldsymbol{P}_i & \boldsymbol{I} \end{bmatrix} > 0, \quad i = 1, 2, ..., q \tag{10}$$

$$\begin{bmatrix} \boldsymbol{I} & \boldsymbol{R}_i \\ \boldsymbol{R}_i & \boldsymbol{I} \end{bmatrix} > 0, \quad i = 1, 2, ..., q \tag{11}$$

$$\begin{bmatrix} \Psi_i & * \\ \boldsymbol{E}_{1i}\boldsymbol{P}_i - \boldsymbol{E}_{2i}\boldsymbol{N}_i\boldsymbol{C} & -\varepsilon_i \boldsymbol{I} \end{bmatrix} < 0. \tag{12}$$

where * denotes the transposed elements in the symmetric positions.

4. Numerical simulations

In this section, to demonstrate the effectiveness of the proposed controller design scheme, we simulate the control of the chaotic Chen system with parametric uncertainties. The control objective is to drive its chaotic trajectory to the origin.

4.1. Fuzzy modelling of the Chen chaotic system

The Chen system is expressed by

$$\begin{cases} \dot{x}_1(t) = -a(x_2(t) - x_1(t)), \\ \dot{x}_2(t) = (c-a)x_1(t) + cx_2(t) - x_1(t)x_3(t), \\ \dot{x}_3(t) = x_1(t)x_2(t) - bx_3(t). \end{cases} \tag{13}$$

Figure 1 shows the phase portrait of its attractor, with (a, b, c)=(35, 3, 28), time interval from 0s to 20s, where the initial condition is (x_1, x_2, x_3)=(1, 1, 1).

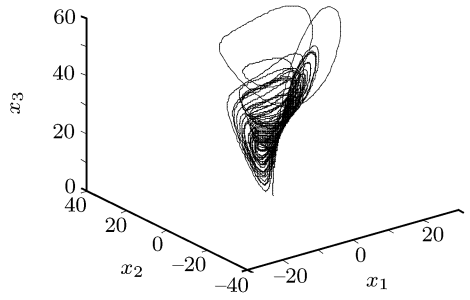


Fig.1. Phase portrait of Chen chaotic attractor with initial condition $x(0) = (1, 1, 1)$.

To construct a T–S fuzzy model for the Chen system, the quadratic nonlinear terms x_1x_2 and $-x_1x_3$ in (13) must be expressed as linear weighted sums of linear functions.

Fact 1 The nonlinear term $f(\boldsymbol{x}(t)) = x_1(t)x_2(t)$ can be represented as

$$f(\boldsymbol{x}(t)) = \sum_{i=1}^{2} \mu_i g_i(\boldsymbol{x}(t)) x_2(t),$$

where

$$g_1(\boldsymbol{x}(t)) = M_1, \ g_2(\boldsymbol{x}(t)) = M_2$$

and

$$\mu_1(\boldsymbol{x}(t)) = \frac{-x_1 + M_2}{M_2 - M_1}, \qquad \mu_2(\boldsymbol{x}(t)) = \frac{x_1 - M_1}{M_2 - M_1}. \tag{14}$$

The membership functions for the plant rules are shown in Fig.2.

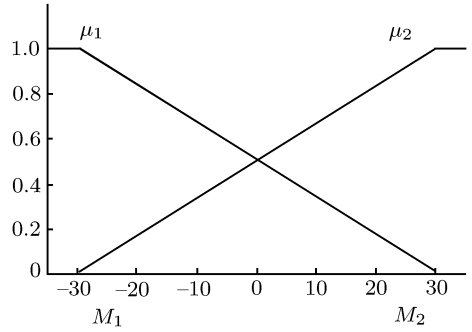


Fig.2. The membership functions for the T–S fuzzy model of Chen system.

Since all nonlinear terms in (13) are function of $x_1(t)$, thus, from Fact 1 we can construct an exact T–S fuzzy model of the system (13) as follows.

Plant Rules:

Rule1 : IF $x_1(t)$ is about M_1, THEN $\dot{\boldsymbol{x}} = \boldsymbol{A}_1\boldsymbol{x}$;

Rule1 : IF $x_1(t)$ is about M_2, THEN $\dot{\boldsymbol{x}} = \boldsymbol{A}_2\boldsymbol{x}$; (15)

where

$$\boldsymbol{A}_1 = \begin{bmatrix} -a & a & 0 \\ c-a & c & -M_1 \\ 0 & M_1 & -b \end{bmatrix},$$

$$\boldsymbol{A}_2 = \begin{bmatrix} -a & a & 0 \\ c-a & c & -M_2 \\ 0 & M_2 & -b \end{bmatrix}.$$

We can use (14) as membership functions by choosing $[M_1, M_2]$ as [–30, 30], because $x_1(t)$ is bounded within [–30, 30] in Fig.1.

The defuzzified output of the T–S fuzzy model (15) with membership functions (14) can be obtained by some manipulations. Figure 3 is the phase portrait of the T–S fuzzy model (15), which shows the equivalence to the original model. Thus, the T–S fuzzy model of the controlled system with parametric uncertainties is

Rule 1: IF $x_1(t)$ is about M_1, THEN $\dot{\boldsymbol{x}} = (\boldsymbol{A}_1 + \Delta\boldsymbol{A}_1)\boldsymbol{x} + (\boldsymbol{B}_1 + \Delta\boldsymbol{B}_1)\boldsymbol{u}, \boldsymbol{y} = \boldsymbol{C}\boldsymbol{x}$;

Rule 2: IF $x_1(t)$ is about M_2, THEN $\dot{\boldsymbol{x}} = (\boldsymbol{A_2} + \Delta\boldsymbol{A_2}(\boldsymbol{x} + (\boldsymbol{B_2} + \Delta\boldsymbol{B_2})\boldsymbol{u}, \boldsymbol{y} = \boldsymbol{C}\boldsymbol{x}$;

where

$$\boldsymbol{A}_1 = \begin{bmatrix} -35 & 35 & 0 \\ -7 & 28 & 30 \\ 0 & -30 & -3 \end{bmatrix},$$

$$\boldsymbol{A}_2 = \begin{bmatrix} -35 & 35 & 0 \\ -7 & 28 & -30 \\ 0 & 30 & -3 \end{bmatrix},$$

$$\boldsymbol{C} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix},$$

and the input matrix $\boldsymbol{B}_i$ is randomly selected, for example, $\boldsymbol{B}_i = \boldsymbol{I}_3, i = 1, 2$ such that $\{\boldsymbol{A}_i, \boldsymbol{B}_i\}$ is controllable.

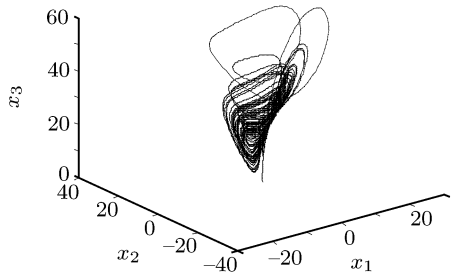


Fig.3. Phase portrait of T–S fuzzy model of Chen chaotic attractor with initial condition $x(0) = (1, 1, 1)$.

4.2. Output feedback fuzzy-model-based control

This section gives a controller design example of the T–S fuzzy model to show the effectiveness of the proposed stabilization technique. Then the chaotic Chen system with parametric uncertainties is used as a test bed. In this paper, we choose the nominal values of (a, b, c)=(35, 3, 28) for chaos to emerge. Also, we assume all system parameters are uncertain but bounded within 30% of their nominal values. Based on Assumption 1, we define

$$\boldsymbol{D}_1 = \boldsymbol{D}_2 = \mathrm{diag}[\,-0.3 \quad 0.3 \quad -0.3\,]$$

$$\boldsymbol{E}_{11} = \boldsymbol{E}_{12} = \begin{bmatrix} a & -a & 0 \\ c-a & c & 0 \\ 0 & 0 & b \end{bmatrix} = \begin{bmatrix} 35 & -35 & 0 \\ -7 & 28 & 0 \\ 0 & 0 & 3 \end{bmatrix},$$

$$\boldsymbol{E}_{21} = \boldsymbol{E}_{22} = \boldsymbol{0}.$$

By applying Theorem 1 and solving the corresponding LMIs with $\varepsilon_i = 1$, we obtain the following controller parameter matrices:

$$\boldsymbol{M}_1 = \begin{bmatrix} 0.6250 & -0.0494 \\ 0.9883 & 0.0157 \end{bmatrix},$$

$$\boldsymbol{M}_2 = \begin{bmatrix} 0.2945 & -0.0229 \\ 0.4441 & 0.0190 \end{bmatrix},$$

$$\boldsymbol{N}_1 = \begin{bmatrix} 0.1349 & 4.6662 \\ -17.0069 & 16.0826 \\ 23.5672 & -8.3406 \end{bmatrix},$$

$$\boldsymbol{N}_2 = \begin{bmatrix} 3.2993 & -0.1852 \\ -1.4115 & 2.6895 \\ 4.8292 & -0.9493 \end{bmatrix},$$

$$\boldsymbol{P}_1 = \begin{bmatrix} 0.1889 & 0.1523 & 0.1879 \\ 0.1523 & 0.2031 & 0.1202 \\ 0.1879 & 0.1202 & 0.2662 \end{bmatrix},$$

$$\boldsymbol{P}_2 = \begin{bmatrix} 0.1012 & 0.0743 & 0.0721 \\ 0.0743 & 0.0964 & 0.0556 \\ 0.0721 & 0.0556 & 0.1444 \end{bmatrix}.$$

and $\lambda = 0.001$. Thus, the controller gain matrices are

$$\boldsymbol{K}_1 = \begin{bmatrix} -78.6370 & 49.8708 \\ -275.7262 & 157.1574 \\ 146.9570 & -69.0966 \end{bmatrix},$$

$$\boldsymbol{K}_2 = \begin{bmatrix} 9.1936 & 1.3333 \\ -77.4636 & 48.1886 \\ 32.5620 & -10.7174 \end{bmatrix}.$$

In the process of simulation, the system parameters a, b, c are arbitrarily varied within 30% of their nominal values. The initial values are $x^{\mathrm{T}}(0) =$

$[1,1,1]^{\mathrm{T}}$. The simulation time is 20s. To explicitly show the control effect of the proposed method, the time interval is divided into two parts: at the first stage of the simulation, no control input is applied; the control input is then activated thereafter.

The control results are shown in Fig.4. Before the control input is applied, the system is chaotic; yet after the control is activated, the output of the chaotic attractor is quickly guided to the origin, as shown in Fig.4, despite the time-varying uncertainties in the original system. Figure 5 shows the simulation result for the Chen system without parametric uncertainties but with the same controller. The control input is also activated at t=10s. Similarly to the case of the parametric uncertain system, after control input was activated, the system output is rapidly guided to the origin. The two simulation results show that the T–S fuzzy model-based controller, designed by using Theorem 1 not only can stabilize the chaotic system, but has strong robustness against norm-bounded parametric uncertainties.

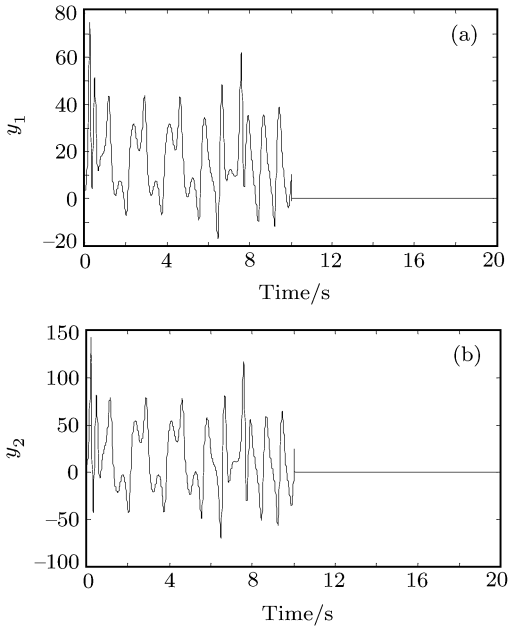


Fig.4. The output responses of the uncertain Chen's chaotic attractor with initial conditions $x(0) = (1,1,1)$: (a) y_1; (b) y_2.

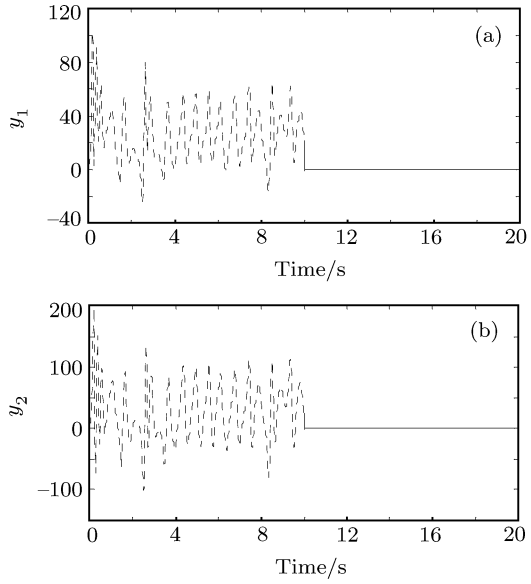


Fig.5. The output responses of the Chen's chaotic attractor, nominal case with initial conditions $x(0) = (1,1,1)$: (a) y_1; (b) y_2.

5. Conclusions

In this paper, a robust controller design method, the output feedback fuzzy-model-based regulator for the uncertain chaotic system was presented. The LMI approach has been utilized to study the fuzzy model with uncertainties. The resulting fuzzy controller can guarantee the globally asymptotically stability of the closed-loop T–S fuzzy system subject to all admissible parametric uncertainties. To demonstrate the effectiveness of the derived fuzzy controller, the uncertain Chen chaotic system is used as a platform and test bed. Firstly, an uncertain T–S fuzzy model representing the unknown Chen system has been constructed. Then the output feedback fuzzy-model-based controller was synthesized by putting the design problem into the typical LMI framework. Simulation results have verified the effectiveness and feasibility of the new method in controlling chaotic system with parametric uncertainties.

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