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A scheme for conditional quantum phase gate via bimodal cavity and a Λ-type three-level atom*

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We propose a scheme to implement a two-qubit conditional quantum phase gate for the intracavity field via a single three-level Λ-type atom driven by two modes in a high-Q cavity. The quantum information is encoded on the Fock states of the bimodal cavity. The gate's averaged fidelity is expected to reach 99.8%.

Keywords: quantum phase gate, Λ-type three-level atom, bimodal cavity, fidelity

PACC: 0367, 4250, 3280

1. Introduction

Since Shor[1] proved that a quantum computer (QC) has the unique features of working more efficiently than a classical one, and Cirac and Zoller[2] proposed a scheme to perform quantum computation with trapped cold ions, much attention has been attracted to the field of quantum computation. Through the last decade, a lot of other schemes have been proposed to realize quantum computer, such as quantum computer using trapped cold ions or hot ions,[3,4] superconducting quantum interference devices (SQUIDs),[5] quantum dots,[6] linear optical devices,[7] NMR system,[8] and cavity quantum electro-magnetic dynamics(CQED).[9−16] etc. The task of looking for a system to realize a quantum computer is far from ending.[17−19] The task of designing a QC is equivalent to finding a physical implementation of quantum logic gate between qubits, where a qubit refers to a two-state system $|0\rangle$, $|1\rangle$. It is well known that any unitary quantum operation can be decomposed into a single qubit rotation and CNOT gate operations between two qubits; while a CNOT gate operation can also be decomposed into single qubit rotations and a conditional phase gate operation between two qubits. A conditional two qubit phase gate operation is defined by $\hat{U}$:$|a,b\rangle \rightarrow (-1)^{ab}|a,b\rangle$, with $a,b=0,1$, so the task of quantum engineering reduces to designing a two-qubit conditional phase gate.

Among the CQED schemes, there are three kinds of methods to encode quantum information. The first one uses two atoms and a cavity mode or two cavity modes, usually assisted by two classical fields, in which quantum information is encoded on the atomic states.[14] The second one employs a bimodal cavity and only one atom, where information is encoded on both of the atomic states and the Fock states of one of the bimodal cavity modes,[11] where the classical field is not necessary. The last one also utilizes two cavity modes and one atom, but quantum information is encoded on both of the Fock states of the two cavity modes.[9,10] In the last one, a ladder-type or V-type atom has been used to entangle two qubits, but the scheme using a Λ-type atom to entangle those two qubits has not been reported yet.

In this paper, we propose a scheme to implement a conditional quantum phase gate (CQPG), following the third method via a Λ-type three-level atom, a bimodal cavity and a classical $\pi/2$ pulse, which is used to excite the atom from the ground state to the excited state. Control qubit and target qubit are associated with the zero- and single-photon states of modes a and b, respectively, and the two-photon resonance condition is employed. The atom is prepared in the ground state and made to pass through a bimodal cavity. When it comes into the cavity, switch on the classical field with Rabi frequency Ω for a short time of $\tau_1=\pi/2\Omega$, then switch on the cavity modes. After a time of τ_2 given in the Table 1, the atom comes out

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Table 1. The optimal m, n, gt and fidelity.

m	n	k	l	Δ/g	gt	fidelity
3	27	33	37.066	0.3162	59.58	0.991
9	29	34.943	39.012	0.9234	61.28	0.989
14	36	42.965	47.949	1.1939	85.79	0.989
21	41	47.969	53.046	1.6868	89.71	0.998

from the cavity and is still in the state $|e\rangle$, the phase gate operation is then completed. The gate operation time, $\tau_1+\tau_2$, is of the order of tenths of a millisecond and the fidelity is expected to reach 99.8%.

2. Model

Let us consider a Λ-type three-level atomic configuration as in Fig.1. The atom has two ground states, $|g\rangle$ and $|f\rangle$, and an excited state $|e\rangle$, let it pass through a high Q bimodal cavity with two mode angular frequencies ω_a and ω_b, the modes with annihilation operators a and b interact with the atomic transitions $|e\rangle \leftrightarrow |g\rangle$ and $|e\rangle \leftrightarrow |f\rangle$, respectively. These two atomic transitions couple to the cavity modes with coupling strength $g_1 = g_2 = g$ and detuning $\Delta_1 = \omega_a - \omega_{eg}$, $\Delta_2 = \omega_b - \omega_{ef}$, $\Delta_1 = \Delta_2 = \Delta$. The

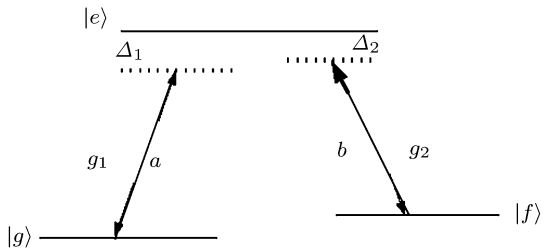


Fig.1. Λ-type three-level atomic configuration with levels $|e\rangle$, $|g\rangle$ and $|f\rangle$ interacting with both of the cavity modes, described by annihilation operators a and b. Here g_1 and g_2 represent the atom–cavity coupling strength of the modes a and b with the corresponding transitions, Δ_1 and Δ_2 are the corresponding one-photon detunings.

Hamiltonian under rotating-wave approximation can be written as

$$H = \omega_{eg}|e\rangle\langle e| + \omega_{ef}|f\rangle\langle f| + \omega_a a^+ a + \omega_b b^+ b + g\left(|e\rangle\langle g|a + |e\rangle\langle f|b + \text{H.c.}\right), \tag{1}$$

where $\hbar = 1$, $\omega_{el}(l \in g, f)$ is the atomic transition frequency, $\omega_i(i \in a, b)$ is the frequency of the cavity modes a and b. The Hamiltonian in the interaction picture is

$$H_I = g\left(|e\rangle\langle g|a\mathrm{e}^{\mathrm{i}\Delta t} + |e\rangle\langle f|b\mathrm{e}^{\mathrm{i}\Delta t} + \text{H.c.}\right). \tag{2}$$

If the initial photon numbers in mode a and b are n_a and n_b, respectively, then the state vector of the atom–cavity system can be expanded in terms of the possible basis states in the following way:

$$|\Psi(t)\rangle = A|e\rangle|n_a, n_b\rangle + B|g\rangle|n_a+1, n_b\rangle + C|f\rangle|n_a, n_b+1\rangle, \tag{3}$$

where A, B and C are the probability amplitudes of the corresponding states. The probability amplitude equations can be obtained from the Schrödinger equation; they are

$$\mathrm{i}\dot{A}_{n_a,n_b} = g\mathrm{e}^{\mathrm{i}\Delta t}(\sqrt{n_a+1}B_{n+1,n_b} + \sqrt{n_b+1}C_{n_a,n_b+1}), \tag{4}$$

$$\mathrm{i}\dot{B}_{n_a+1,n_b} = g\mathrm{e}^{-\mathrm{i}\Delta t}\sqrt{n_a+1}A_{n_a,n_b}, \tag{5}$$

$$\mathrm{i}\dot{C}_{n_a,n_b+1} = g\mathrm{e}^{-\mathrm{i}\Delta t}\sqrt{n_b+1}A_{n_a,n_b}. \tag{6}$$

The probability amplitude of the state $|e\rangle|n_a, n_b\rangle$ found from the Eqs.(4)–(6) is

$$A_{n_a,n_b} = \mathrm{e}^{\mathrm{i}\Delta t/2}(\cos(\frac{\Omega_{n_a,n_b}t}{2}) - \mathrm{i}\frac{\Delta}{\Omega_{n_a,n_b}}\sin(\frac{\Omega_{n_a,n_b}t}{2})), \tag{7}$$

where

$$\Omega_{n_a,n_b} = \sqrt{\Delta^2 + 4(n_a+n_b+2)g^2}, n_a, n_b = 0, 1, \tag{8}$$

and we assumed that the atom is in the state of $|e\rangle$ when the cavity's modes are switched on. Using Eqs.(7) and (8), we can study the realization of CQPG.

3. Quantum phase gate operation

According to the Eq.(7), we can perform two different kinds of phase gate operation:

$$|e\rangle|0,0\rangle \longrightarrow |e\rangle|0,0\rangle, \quad |e\rangle|0,1\rangle \longrightarrow |e\rangle|0,1\rangle, \quad |e\rangle|1,0\rangle \longrightarrow |e\rangle|1,0\rangle, \quad |e\rangle|1,1\rangle \longrightarrow -|e\rangle|1,1\rangle, \tag{9}$$

or

$$|e,0,0\rangle \longrightarrow |e,0,0\rangle, \quad |e,0,1\rangle \longrightarrow -|e,0,1\rangle, \quad |e,1,0\rangle \longrightarrow -|e,1,0\rangle, \quad |e,1,1\rangle \longrightarrow -|e,1,1\rangle. \tag{10}$$

Here we shall focus on the first kind of CQPG only. To perform the CQPG defined by Eq.(9), it is easy to find that the following conditions are necessary:

$$\Delta t/2 = 2m\pi, \Omega_{0,0}t/2 = 2n\pi, \Omega_{0,1}t/2 = 2k\pi, \Omega_{1,1}t/2 = (2l+1)\pi, \tag{11}$$

where the integers m, n, k and l should satisfy the following inequality:

$$2l+1 > 2k > 2n > 2m \geq 0. \tag{12}$$

By the definition of $\Omega_{0,0}$, $\Omega_{0,1}$ and $\Omega_{1,1}$, it follows that

$$2k^2 = 3n^2 - m^2, (2l+1)^2 = 8n^2 - 4m^2. \tag{13}$$

For fixed integers m and n, the detuning $\Delta/g = 2\sqrt{2}m/\sqrt{n^2-m^2}$. The key to our problem is to find k and l closest to other two integers which will satisfy the inequality (12). To check the validity of our proposal, we characterize the deviation by the definition of fidelity as the following:

$$F = \Big|\frac{1}{4}\sum_{n_a,n_b=0,1} |A_{n_a,n_b}| \mathrm{e}^{\mathrm{i}\delta\phi_{n_a,n_b}}\Big|^2, \tag{14}$$

where $\delta\phi_{n_a,n_b}$ is the difference between the phase acquired during the gate operation and the exact phase of the gate defined by Eq.(9). Then we find the optimal m, n and the interaction time t as in Table 1. The gate operation time is of the order of tenths of a millisecond with present experimental techniques.[20,21] Figures 2 and 3 show the time evolution of the probability amplitude of the input states, corresponding

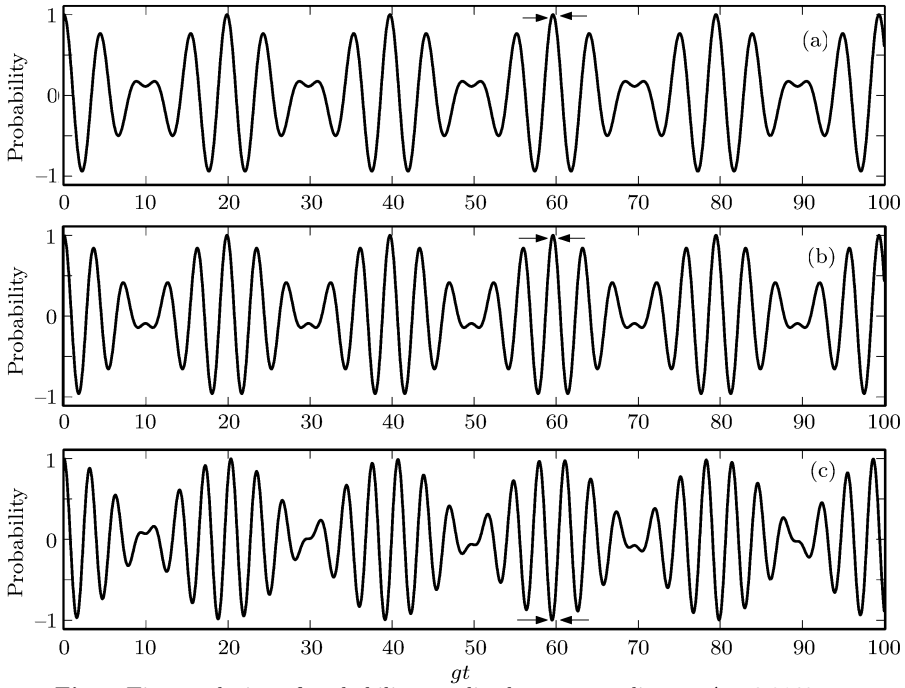


Fig.2. Time evolution of probability amplitude corresponding to $\Delta = 0.3162g$.

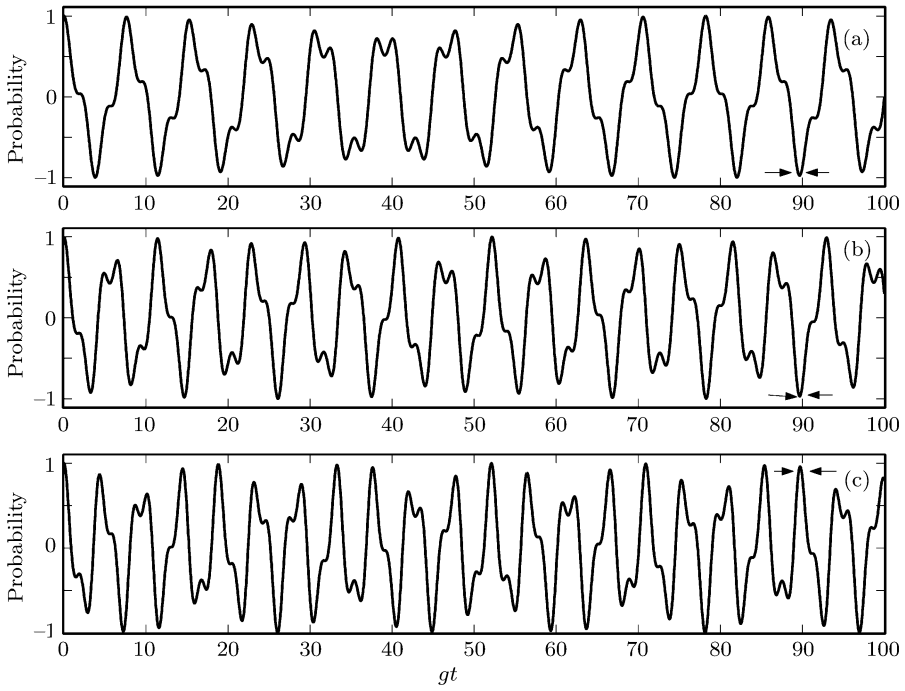


Fig.3. Time evolution of probability amplitude corresponding to $\Delta = 1.6868g$

to the detuning $\Delta = 0.3162\,g$ and $\Delta = 1.6868\,g$, respectively. (a), (b) and (c) in Fig.2 and Fig.3 correspond to the input states $|e\rangle|0,0\rangle$, $|e\rangle|0,1\rangle$ or $|e\rangle|1,0\rangle$, and $|e\rangle|1,1\rangle$, respectively. From Fig.3, we can see that the output states have a global phase difference π from the states defined by Eq.(10). All of the probability amplitudes are close to 1 at the time given in Table 1. Figure 4 shows the time evolution of the fidelity for different values of cavity detuning ($\Delta = 0.3162\,g$ in Fig.4(a) and $\Delta = 1.6868\,g$ in Fig.4(b)). The fidelity oscillates with peak values close to 1 at the time given in Table 1. We also find by numerical calculation that the averaged fidelity will be higher than that given in Table 1, but the gate operation time will be close to or longer than one millisecond, then the dissipations of the system cannot be ignored. As a result, the actual fidelity cannot be higher than that given in Table 1, so the gate operation time given in Table 1 is optimal.

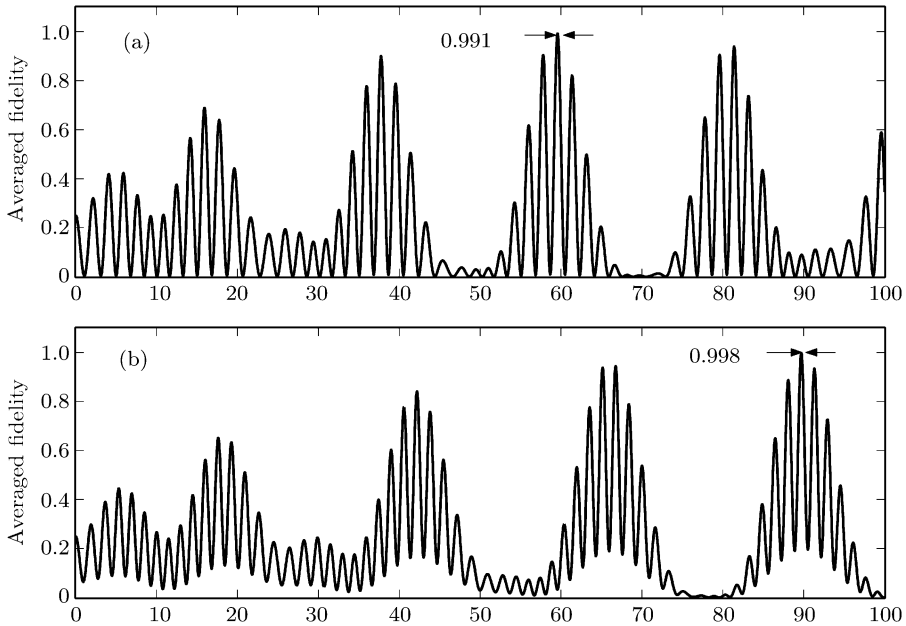


Fig.4. Time evolution of the averaged fidelity. (a) $\Delta = 0.3162g$, (b) $\Delta = 1.6868g$

4. Summary

We have proposed a scheme to realize a two-qubit quantum phase gate with CQED technique, in which a single three-level Λ-type atom and two cavity modes are used. The gate operation time is less than 5×10^{-4} s, and the fidelity can reach 99.8%. In our scheme, the atom is initially prepared in the ground state. When the coupling strength g is much larger than the atomic decay rate κ and cavity's loss rate γ, the gate operation time will be much shorter than the decay time of the atom and the life time of the photon, so the atomic decay and cavity's loss may be ignored. The fidelity will not be obviously reduced by the atomic spontaneous emission and cavity loss. With present experimental technology, the conditions that the coupling strength of $2\pi \times 50 \times 10^3$ kHz and cavity with quality factor of 10^8 are easy to reach,[20,21] so our proposal is feasible.

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