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Practical scheme for remote preparation of a photon–photon entangled state*

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We present a scheme to remotely prepare a photon–photon entangled state via entanglement swapping in cavity QED. Using two successive processes of appropriate atom–cavity interaction and subsequent measurements, we obtain the entangled state with certain probability.

Keywords: remote state preparation, entanglement swapping, cavity QED, photon–photon entangled state

PACC: 0367, 0365

Entanglement is recognized nowadays as a key ingredient for fundamental tests of quantum mechanics[1] and as a basic resource of quantum information processing.[2−5] It is essential that entangled states are created and manipulated for quantum information application. One of the interesting applications of entanglement is remote state preparation(RSP),[6−9] which has the same goal as quantum teleportation, but the sender Alice is given complete classical knowledge on the state that is to be prepared by Bob. Since Lo[7] introduced the concept of 'remote preparation' of a quantum state, more theoretical and experimental protocols on the generation of RSP have been presented.[10−16] Recently, Zhang *et al*[17] presented a scheme for remote preparation of a two-atom entangled state by Alice and Bob sharing two two-atom entangled states via entanglement swapping in cavity QED. But the initial pairs of entangled states in their scheme requires relatively complex techniques, and it is unlikely that quantum information stored exclusively in material particles (atoms and ions) will ever be effectively distributed to remote locations as required for most quantum communication and distributed computing applications. Meanwhile, one of the most promising systems to store and manipulate quantum information is the polarization state or spatial mode of single photons. This is due to the individual polarization states of photons are easily controlled, and their quantum coherence can be preserved over many kilometers of an optical fibre.[18] Here we present a scheme for probabilistic remote preparation of a photon–photon entangled state by entanglement swapping in cavity QED.

First we consider two identical two-level atoms simultaneously interacting with a single-mode quantum cavity field and driven by a classical field. In the rotating-wave approximation,[17,19] the Hamiltonian is (assuming $\hbar$=1)

$$H = \omega_0 S_z + \omega_a a^+ a + \sum_{j=1}^{2} [g(a^+ S_j^- + a S_j^+) + \Omega(S_j^+ e^{-i\omega t} + S_j^- e^{i\omega t})], \quad (1)$$

where $S_z = \frac{1}{2}\sum_{j=1}^{2} |e\rangle_j\langle e| - |g\rangle_j\langle g|, S_j^+ = |e\rangle_j\langle g|, S_j^- = |g\rangle_j\langle e|$ and $|e\rangle_j$, $|g\rangle_j$ are the excited and ground states of the jth atom, respectively. a^+, a are the creation and annihilation operators for the cavity mode, g is the atom–cavity coupling strength, Ω is the Rabi Frequency, ω_0 is the atomic transition frequency, ω_a is the cavity frequency, and ω is the frequency of the classical field. Assuming $\omega_0 = \omega$, in the interaction picture, the interaction Hamiltonian is[19]

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$$H_{\rm I} = \Omega \sum_{j=1}^{2}(S_j^+ + S_j^-) + g\sum_{j=1}^{2}({\rm e}^{-{\rm i}\delta t}a^+ S_j^- + {\rm e}^{{\rm i}\delta t}aS_j^+), \tag{2}$$

where δ is the detuning between the atomic transition frequency ω_0 and cavity frequency ω_a.

In the strong driving regime $2\Omega \gg \delta$, g and in the case $\delta \gg g/2$, there is no energy exchange between the atomic system and the cavity. Then in the interaction picture,[17,19] the effective interaction Hamiltonian can be described as(assuming $h = 1$)

$$H_{\rm e} = \lambda\left[\frac{1}{2}\sum_{j=1}^{2}(|e_j\rangle\langle e_j| + |g_j\rangle\langle g_j|) + \sum_{j,l=1}^{2}(S_j^+ S_l^+ + S_j^+ S_l^- + {\rm H.C.})\right], \quad j \neq l, \tag{3}$$

where $\lambda = g^2/2\delta$. We note that the effective Hamiltonian is independent of the cavity field state, allowing it to be in a thermal state. Then the evolution operator of the system is given by

$$\begin{aligned} U(t) &= {\rm e}^{-{\rm i}H_0 t}{\rm e}^{-{\rm i}H_e t}, \\ H_0 &= \Omega\sum_{j=1}^{2}(S_j^+ + S_j^-). \end{aligned} \tag{4}$$

From Eqs.(3) and (4), we obtain the following evolution:

$$\begin{aligned} |m\rangle_1|n\rangle_2 \xrightarrow{U(t)} {\rm e}^{-{\rm i}\lambda t}[&\cos(\lambda t)(\cos(\Omega t)|m\rangle_1 - {\rm i}\sin(\Omega t)|\overline{m}\rangle_1) \times (\cos(\Omega t)|n\rangle_2 - {\rm i}\sin(\Omega t)|\overline{n}\rangle_2) \\ &-{\rm i}\sin(\lambda t)(cos(\Omega t)|\overline{m}\rangle_1 - {\rm i}\sin(\Omega t)|m\rangle_1) \times (\cos(\Omega t)|\overline{n}\rangle_2 - {\rm i}\sin(\Omega t)|n\rangle_2)], \end{aligned} \tag{5}$$

where $|m\rangle_j$ and $|n\rangle_j$ are the excited or ground states, with $|\overline{m}\rangle_j(|\overline{n}\rangle_j)$ being complementary states of $|m\rangle_j(|n\rangle_j)$.

Next, we will discuss the RSP process. Suppose that the sender Alice wishes to help the receiver Bob remotely prepare a photon–photon entangled state

$$|\phi\rangle = \alpha|00\rangle + \beta|11\rangle, \tag{6}$$

where the superposition coefficients are all real numbers, and satisfy $|\alpha|^2 + |\beta|^2 = 1$. Meanwhile, Alice is given complete classical knowledge on the states in Eq.(6). We also suppose Alice and Bob share two pure nonmaximally atom–photon entangled states

$$\begin{aligned} |\phi\rangle_{A1} &= a|0\rangle_A|e\rangle_1 + b|1\rangle_A|g\rangle_1, \\ |\phi\rangle_{B2} &= c|0\rangle_B|e\rangle_2 + d|1\rangle_B|g\rangle_2, \end{aligned} \tag{7}$$

with $|a| < |b|$, $|c| < |d|$, $|a|^2 + |b|^2 = 1$ and $|c|^2 + |d|^2 = 1$. Without loss of generality, we can assume that the parameters a, b, c and d are all real numbers. $|e\rangle_j$ and $|g\rangle_j$ $(j = 1, 2)$ are the excited and ground states of the jth atom, respectively. The cavities A and B belong to Bob, atoms 1, 2 belong to Alice. Furthermore the atom 1, 2 are identical.

Alice lets atoms 1, 2 simultaneously pass through a single-mode thermal cavity, and the system evolution is described by Eq.(4). From Eq.(5), we obtain the following evolution of the total system:

$$\begin{aligned} |\phi\rangle_{A1}|\phi\rangle_{B2} \rightarrow\ & ac{\rm e}^{-{\rm i}\lambda t}|00\rangle_{AB}[\cos(\lambda t)(\cos(\Omega t)|e\rangle_1 - {\rm i}\sin(\Omega t)|g\rangle_1) \times (\cos(\Omega t)|e\rangle_2 - {\rm i}\sin(\Omega t)|g\rangle_2) \\ & -{\rm i}\sin(\lambda t)(\cos(\Omega t)|g\rangle_1 - {\rm i}\sin(\Omega t)|e\rangle_1) \times (\cos(\Omega t)|g\rangle_2 - {\rm i}\sin(\Omega t)|e\rangle_2)] \\ & ad{\rm e}^{-{\rm i}\lambda t}|01\rangle_{AB}[\cos(\lambda t)(\cos(\Omega t)|e\rangle_1 - {\rm i}\sin(\Omega t)|g\rangle_1) \times (\cos(\Omega t)|g\rangle_2 - {\rm i}\sin(\Omega t)|e\rangle_2) \\ & -{\rm i}\sin(\lambda t)(\cos(\Omega t)|g\rangle_1 - {\rm i}\sin(\Omega t)|e\rangle_1) \times (\cos(\Omega t)|e\rangle_2 - {\rm i}\sin(\Omega t)|g\rangle_2)] \\ & bc{\rm e}^{-{\rm i}\lambda t}|10\rangle_{AB}[\cos(\lambda t)(\cos(\Omega t)|g\rangle_1 - {\rm i}\sin(\Omega t)|e\rangle_1) \times (\cos(\Omega t)|e\rangle_2 - {\rm i}\sin(\Omega t)|g\rangle_2) \end{aligned}$$

$$
\begin{aligned}
&-\mathrm{i}\sin(\lambda t)(\cos(\Omega t)|e\rangle_1 - \mathrm{i}\sin(\Omega t)|g\rangle_1) \times (\cos(\Omega t)|g\rangle_2 - \mathrm{i}\sin(\Omega t)|e\rangle_2)] \\
&bd\mathrm{e}^{-\mathrm{i}\lambda t}|11\rangle_{AB}[\cos(\lambda t)(\cos(\Omega t)|g\rangle_1 - \mathrm{i}\sin(\Omega t)|e\rangle_1) \times (\cos(\Omega t)|g\rangle_2 - \mathrm{i}\sin(\Omega t)|e\rangle_2) \\
&-\mathrm{i}\sin(\lambda t)(\cos(\Omega t)|e\rangle_1 - \mathrm{i}\sin(\Omega t)|g\rangle_1) \times (\cos(\Omega t)|e\rangle_2 - \mathrm{i}\sin(\Omega t)|g\rangle_2)]. \qquad (8)
\end{aligned}
$$

Then Alice measures the state of atoms 1, 2.

Case I If the result of Alice's measurement is $|\mathrm{ee}\rangle_{12}$, the state of cavity A, B collapses into

$$
\begin{aligned}
|\psi\rangle_{AB} = {} & \mathrm{e}^{-\mathrm{i}\lambda t}ac(\mathrm{e}^{-\mathrm{i}\lambda t}\cos^2(\Omega t) + \mathrm{i}\sin(\lambda t))|00\rangle_{AB} \\
& -\mathrm{e}^{-\mathrm{i}\lambda t}bd(\mathrm{e}^{-\mathrm{i}\lambda t}\sin^2(\Omega t) + \mathrm{i}\sin(\lambda t))|11\rangle_{AB} \\
& -\frac{\mathrm{i}}{2}\mathrm{e}^{-2\mathrm{i}\lambda t}ad\sin(2\Omega t)|01\rangle_{AB} \\
& -\frac{\mathrm{i}}{2}\mathrm{e}^{-2\mathrm{i}\lambda t}bc\sin(2\Omega t)|10\rangle_{AB}. \qquad (9)
\end{aligned}
$$

Because Alice knows the parameters α and β, she can select the atomic velocity to meet the condition $\cos(\lambda t) = \alpha$(or $\sin(\lambda t) = \beta$) and modulate the classical field to satisfy $\Omega t = \pi$. After the interaction and measurement, Alice informs Bob of her measurement result through a classical channel. Bob gets the following photon–photon entanglement state

$$
\mathrm{e}^{-\mathrm{i}\lambda t}(\alpha ac|00\rangle_{AB} - \mathrm{i}\beta bd|11\rangle_{AB}). \qquad (10)
$$

After a rotation operation, the state described in Eq.(10) will become

$$
\alpha ac|00\rangle_{AB} + \beta bd|11\rangle_{AB}. \qquad (11)
$$

In order to achieve the goal of RSP, Bob introduces an two-level auxiliary atom a' with the initial state of $|g\rangle_{a'}$. In addition, the transition frequency of the atom is resonant with cavity A's frequency. The joint state of cavity A, B and atom a' will become

$$
(\alpha ac|00\rangle_{AB} + \beta bd|11\rangle_{AB}) \bigotimes |g\rangle_{a'}. \qquad (12)
$$

Bob lets atom a' enter into cavity A, the temporal evolution of atom in cavity A is governed by the Jaynes–Cummings model interaction Hamiltonian

$$
H = \mathrm{i}hg(a|e\rangle\langle g| - a^+|g\rangle\langle e|), \qquad (13)
$$

where $|g\rangle$ and $|e\rangle$ are the ground and excited states of atom a', a and a^+ are the creation and annihilation operators of the resonant field mode in A, and g denotes the coupling strength between atom and cavity. After the interaction time t', the joint state in Eq.(12) evolves to

$$
(\alpha ac|00\rangle_{AB} + \beta bd\cos(gt')|11\rangle_{AB})|g\rangle_{a'} + \beta bd\sin(gt')|01\rangle_{AB}|e\rangle_{a'}. \qquad (14)
$$

Bob can select the auxiliary atomic velocity to meet the condition $\cos(gt') = \dfrac{ac}{bd}$ (or $\sin(gt') = \sqrt{1 - \left(\dfrac{ac}{bd}\right)^2}$). Then Bob will measure the state of atom a'. If he gets the result $|g\rangle_{a'}$, the state of cavity A, B will be

$$
ac(\alpha|00\rangle_{AB} + \beta|11\rangle_{AB}). \qquad (15)
$$

The successful probability of RSP is $|ac|^2$.

Case II If the measurement result is $|gg\rangle_{12}$, the state of A, B becomes

$$
\begin{aligned}
|\psi\rangle_{AB} = {} & -\mathrm{e}^{-\mathrm{i}\lambda t}ac(\mathrm{e}^{-\mathrm{i}\lambda t}\sin^2(\Omega t) + \mathrm{i}\sin(\lambda t))|00\rangle_{AB} \\
& +\mathrm{e}^{-\mathrm{i}\lambda t}bd(\mathrm{e}^{-\mathrm{i}\lambda t}\cos^2(\Omega t) + \mathrm{i}\sin(\lambda t)|11\rangle_{AB} \\
& -\frac{\mathrm{i}}{2}\mathrm{e}^{-2\mathrm{i}\lambda t}ad\sin(2\Omega t)|01\rangle_{AB} \\
& -\frac{\mathrm{i}}{2}\mathrm{e}^{-2\mathrm{i}\lambda t}bc\sin(2\Omega t)|10\rangle_{AB}. \qquad (16)
\end{aligned}
$$

Alice selects the atomic velocity to meet the condition $\sin(\lambda t) = \alpha$ (or $\cos(\lambda t) = \beta$) and modulate the classical field to satisfy $\Omega t = \pi$. After the interaction and measurement, Alice tells Bob her measurement result through a classical channel. Bob gets the following photon–photon entanglement state:

$$
\mathrm{e}^{-\mathrm{i}\lambda t}(-\mathrm{i}\alpha ac|00\rangle_{AB} + \beta bd|11\rangle_{AB}). \qquad (17)
$$

After a rotation operation, the state described in Eq.(17) becomes

$$
\alpha ac|00\rangle_{AB} + \beta bd|11\rangle_{AB}. \qquad (18)
$$

Then the RSP procedure that follows is the same as Case I. We also have the same successful probability $|ac|^2$.

Further analysis shows that for the measurement results $|ee\rangle_{12}$ or $|gg\rangle_{12}$ one can also successfully realize the RSP of the state $\alpha|00\rangle_{AB} + \beta|11\rangle_{AB}$. But for the other measurement results $|ge\rangle_{12}$ or $|eg\rangle_{12}$ one cannot realize RSP in a simple way. So the contribution of the measurement results $|ge\rangle_{12}$, $|eg\rangle_{12}$ is eliminated. The total successful probability for the RSP process is $P_{\rm total} = 2|ac|^2$.

From Eq.(11), it is obvious that if $a = b = c = d = \dfrac{1}{\sqrt{2}}$, Bob need not introduce any auxiliary atom to realize the RSP of the state $\alpha|00\rangle_{AB} + \beta|11\rangle_{AB}$. And the successful probability is $\dfrac{1}{2}$.

Next we give a brief discussion on the experimental aspects. For the Rydberg atoms with principal quantum number 50 and 51, the radiative time is $T_{\rm r} = 3 \times 10^{-2}\,{\rm s}$, and the coupling constant is $g = 2\pi \times 25\,{\rm kHz}$.[20–22] The required atom–cavity-field interaction time is of the order of $T \approx 10^{-4}\,{\rm s}$. Then the time needed to complete the whole procedure is much shorter than $T_{\rm r}$. In recent experiments, the decay time of the cavity is $T_{\rm c} \simeq 1.0 \times 10^{-3}\,{\rm s}$,[20–22] which is longer than the required time. And in the process of preparation of quantum state, our protocol is related to 2 interactions of atoms and cavity. The first interaction is insensitive to both cavity decay and thermal field, which is important from the experimental point of view. The other interaction is based on the resonance of atom with a cavity mode, the required experimental techniques are within the scope of the present experiments. Based on cavity QED techniques presently available, the proposed scheme might be realizable.

In summary, we have proposed a scheme to remotely prepare a photon–photon entangled state via entanglement swapping in cavity QED. Using two successive processes of appropriate atom–cavity interaction and subsequent measurements, we obtain the entangled state with probability of $2|ac|^2$. The experimental implementation of the scheme should demonstrate the power of cavity QED to manipulate entangled states for quantum information processing.

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