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Entanglement and decoherence in a quantum dimer

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The dynamical properties of quantum entanglement in an integrable quantum dimer are studied in terms of the reduced-density linear entropy with various coupling parameters and total boson numbers. The characteristic time of decoherence process in the early-time evolution of the linear entropy is obtained, indicating that the characteristic time and the corresponding entropy exhibit a maximum near the position of the corresponding classical separatrix energy.

Keywords: entanglement, linear entropy, decoherence, dimer
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In recent years, the physics of decoherence has been extensively investigated both theoretically and experimentally. It has been recognized that the decoherence process has rather unique properties for systems whose classical analogues are chaotic since the decoherence rate is larger for chaotic systems than for integrable ones.[1] It is suggested that the small-amplitude oscillations of entropy might be regarded as fingerprints of the underlying classical chaos in smooth Hamiltonian systems.[2] It is also suggested that entropy production rate during decoherence could be used as a diagnostic for quantum chaos in an open quantum system,[3] and a novel signature of quantum chaos based on the dynamics of entanglement has been proposed for the kicked top.[4] Moreover, most work relating to the decoherence has been done subject to the interaction of the quantum system with its environment with many degrees of freedom.[1,3,5] Recently, a study of decoherence has been carried out for the systems with a few degrees of freedom.[2,6−9] An important result shows that the rapid decoherence that usually happens in chaotic systems can occur in the integrable Jaynes–Cummings model due to a boundary effect[6] and is strongly influenced by the shapes of the classical orbits in the underlying classical phase space.[7] In the present work, we study how the characteristic time of decoherence[10] depends on the coupling parameter in an integrable quantum dimer. Thereby the properties of decoherence near the corresponding classical separatrix can be explored.

The integrable quantum dimer describes important physical properties in a two-species Bose–Einstein condensate, two modes of electromagnetic fields interacting in Kerr media, and excitons in two coupled semiconductor microcrystallites. For the first two cases, the entanglement dynamics has been studied in detail in terms of both the reduced von Neumann entropy and the reduced linear entropy.[11,12] For the last case, exciton entanglement is quantified in terms of the reduced von Neumann entropy,[13] where it is observed that the nonlinear interaction between excitons increases the maximum value of the entropy of entanglement more than that of the linear coupling model. However, the entanglement near or at the classical separatrix and bifurcation has not yet been considered in Refs.[11–13]. It is shown that the static entanglement described by von Neumann entropy in the ground state of Jahn–Teller models abruptly changes near the classical bifurcation[14] but that displays a maximum in the coupled-tops model.[15] Here we investigate the dynamical entanglement in the dimer model with various coupling parameters and total boson numbers. The entanglement is measured by the reduced-density linear entropy. We determine the characteristic time of decoherence in the model, which is defined as the instant when the slope of the curve of the reduced-density linear entropy decreases appreciably in the early-time evolution.[10] It is demonstrated that the characteristic time and the corresponding entropy exhibit a maximum near the position of the corresponding classical separatrix energy. It should be mentioned that the correspondence between classical and quantum properties of the dimer system was studied in Refs.[16,17]. It is shown that a drastic change

in the splitting of quantum energy levels occurs at the position of the classical separatrix.[16]

The quantum dimer model that we study takes the same Hamiltonian as that in Ref.[16] ($\hbar = 1$ hereafter), namely

$$H = a_1^\dagger a_1 + a_2^\dagger a_2 + \frac{1}{2}[(a_1^\dagger a_1)^2 + (a_2^\dagger a_2)^2] + C(a_1^\dagger a_2 + a_2^\dagger a_1), \quad (1)$$

where a_i and $a_i^\dagger$ ($i = 1, 2$) are the annihilation and creation operators on site i, and C is the linear coupling parameter between site 1 and site 2. The quantum Hamiltonian Eq.(1) conserves the total boson number $N = n_1 + n_2$, where $n_i = a_i^\dagger a_i$ is the boson number on site i, so that together with the energy there exist two integrals of the motion and the quantum system is integrable. Diagonalization of the quantum Hamiltonian Eq.(1) in a basis of number states $|n_1, n_2\rangle$ gives the quantum spectrum with the corresponding eigenvectors. The extended Eq.(1) becomes a potential model for vibrational spectra of polyatomic molecules.[18]

We study the dynamics of entanglement in the quantum model. Several measures of quantum entanglement have been introduced, which include entanglement of formation,[19] entanglement of distillation,[20] relative entropy of entanglement,[21] linear entropy of entanglement,[22] concurrence,[23] negativity,[24] and so on. In our case the entanglement can be described by the linear entropy or the von Neumann entropy. We calculate those two entropies, whose behaviours are similar to each other. Thus we only present the results of the linear entropy. Furthermore, the linear entropy is extensively used in the study of decoherence.[2,3,5–10] It is important that the characteristic time of decoherence[10] can be obtained in the calculation of the linear entropy.

The linear entropy $s(t)$ of quantum entanglement is defined as[22]

$$s(t) = 1 - \mathrm{Tr}_1 \rho_1(t)^2, \quad (2)$$

where Tr_1 denotes a trace over the first subsystem, and $\rho_1(t)$ is the reduced-density matrix, $\rho_1(t) = \mathrm{Tr}_2 |\psi(t)\rangle\langle\psi(t)|$, where indices 1 and 2 represent the degrees of freedom on sites 1 and 2, respectively, and $|\psi(t)\rangle$ is the quantum state of the full system, which evolves in time under the action of Hamiltonian Eq.(1). The entropy $s(t)$ describes the degree of purity of the subsystem in a scale from 0 (pure state) to 1 (statistical mixture).

In order to study the dynamical properties of quantum entanglement, we take state $|0, N\rangle$ or $|N, 0\rangle$ as the initial state since its quantum eigenenergy is $N + \frac{1}{2}N^2$, which is a maximal energy subject to no coupling existing between two sites in both classical and quantum cases. Moreover, the eigenenergy of that state is equal to the classical separatrix energy as the coupling increases up to $C = N/4$.[16] Here we refer to that coupling as the separatrix coupling $C_s = N/4$ correspondingly. With such an initial state, Fig.1 shows the entanglement dynamics described by the linear entropy with three coupling parameters: $C = 0.6$, 9.6, and 18.6, where the total boson number is taken as 20 for example. A number of observations in Fig.1 are as follows. First, for a weak coupling $C = 0.6$, the entropy is periodic and its period decreases as C increases. Such a periodicity is also found in other integrable systems.[25] However, for the larger coupling parameters $C = 9.6$ and 18.6, the entropy is not periodic any more. Secondly, for a fixed N, the maximum entropy increases up to a saturation value as the coupling increases. The larger the coupling, the larger the oscillatory frequency of the entropy is. A similar behaviour is also observed in an integrable Jaynes–Cummings model.[6] Finally, the increasing rate of the entropy becomes faster as C increases in the early-time evolution. That property will be discussed in detail below.

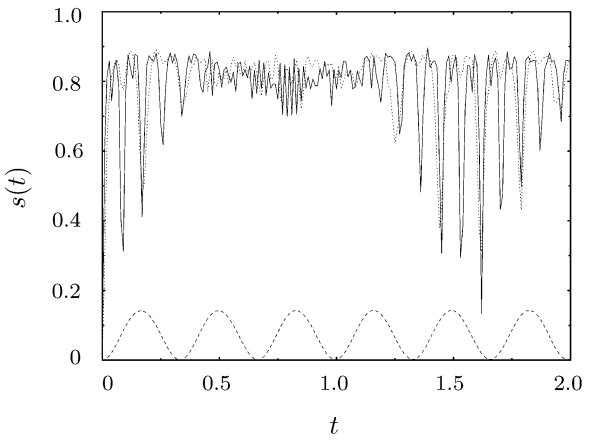


Fig.1. The linear entropy $s(t)$ for the initial state $|0, 20\rangle$ or $|20, 0\rangle$ with three different coupling parameters: $C = 0.6$ (dotted line), $C = 9.6$ (dashed line), and $C = 18.6$ (solid line).

We determine the characteristic time t_d of the model, which is defined as the instant when the slope of the curve of the entropy $s(t)$ decreases appreciably

in the early-time evolution:[10] that is, $t_{\rm d}$ is given by

$$\left.\frac{{\rm d}s(t)}{{\rm d}t}\right|_{t=t_{\rm d}} = 0. \tag{3}$$

The characteristic time $t_{\rm d}$ of the decoherence process is numerically calculated with the initial state $|0, N\rangle$ or $|N, 0\rangle$. Figure 2 shows the characteristic time $t_{\rm d}$ of decoherence vs the coupling parameter C with three total boson numbers: N =10, 20, and 35, as an example, where C changes from 0.001 to 20. A number of observations are as follows. In the regime of coupling $C \leq 3$, the characteristic time $t_{\rm d}$ decreases with the total boson number N increasing. In the regime of weak coupling, the time $t_{\rm d}$ has a little variation with the coupling increasing. The larger the total boson number, the larger the regime of coupling parameter is. In the regime of strong coupling $C \geq 17.5$, the time $t_{\rm d}$ remains nearly unchanged for different N. But that value has a little variation as the coupling increases further. The time $t_{\rm d}$ increases to a maximum as the coupling increases up to a value, and then decreases with the coupling increasing further. The values of the coupling parameter for different N can be obtained numerically, and they are 2.2, 4.1, and 7.2 for the total boson number N =10, 20, and 35, respectively. Those values are close to the corresponding separatrix coupling $C_s = N/4$. More calculations can be performed for other values of N.

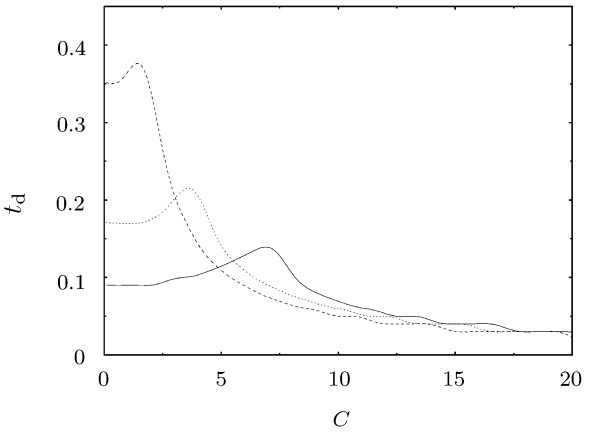


Fig.2. The dependence of the characteristic time $t_{\rm d}$ (in scaled units) of decoherence process in the early-time evolution on the coupling parameter C. Here three total boson numbers are used: $N = 10$ (dashed line), $N = 20$ (dotted line), and $N = 35$ (solid line).

Figure 3 shows the entropy $s_{\rm d}$ corresponding to the $t_{\rm d}$ in Fig.2 with the coupling parameter varying. Some important properties are observed in Fig.3. First, the linear entropy $s_{\rm d}$ in a strong coupling regime increases as the total boson number N increases. The slopes of curves in a weak coupling regime decrease as the total boson number N increases. Second, the entropy $s_{\rm d}$ fluctuates with small amplitudes around its maximum value for $N = 10$. These amplitudes decrease as N increases further. Finally, $s_{\rm d}$ increases to a maximum as the coupling parameter C increases up to a value, and then decreases a little as C increases further. The obtained value of such a coupling C is the same as that in Fig.2. All the above indicates that the maximal entropy and characteristic time can be achieved in early-time evolution as the coupling is near to the separatrix coupling. It is straightforward to obtain the similar curves for large coupling parameters, the large total boson numbers, and other initial states in the model.

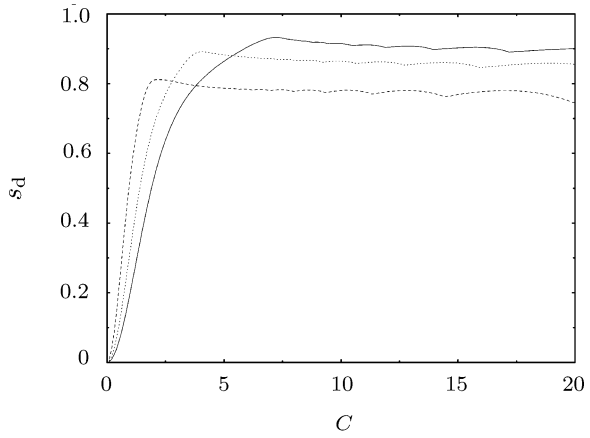


Fig.3. The linear entropy $s_{\rm d}$ corresponding to the characteristic time $t_{\rm d}$ in Fig.2. Here the same total boson numbers as those in Fig.2 are used: $N = 10$ (dashed line), $N = 20$ (dotted line), and $N = 35$ (solid line).

In summary, we have investigated the dynamical entanglement in an integrable quantum dimer with various coupling parameters and total boson numbers, where the entanglement is measured by the linear entropy. The characteristic time of decoherence process is obtained, which is defined as the instant when the slope of the curve of the entropy decreases appreciably in the early-time evolution of the linear entropy. The dependences of both the characteristic time and the corresponding entropy on the coupling parameter show that a maximum value occurs near the position of the corresponding classical separatrix energy. That implies that the maximal entanglement in early-time evolution in the model can be prepared with the quantum state whose quantum eigenvalue is close to the corresponding classical separatrix energy. That might be useful in quantum communication and computation[26] in an integrable quantum dimer.

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