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A scheme for transferring an unknown atomic entangled state via cavity quantum electrodynamics*

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In this paper, we propose a scheme for transferring an unknown atomic entangled state via cavity quantum electrodynamics (QED). This scheme, which has a successful probability of 100 percent, does not require Bell-state measurement and performing any operations to reconstruct an initial state. Meanwhile, the scheme only involves atom–field interaction with a large detuning and does not require the transfer of quantum information between the atoms and cavity. Thus the scheme is insensitive to the cavity field states and cavity decay. This scheme can also be extended to transfer ring an entangled state of n-atom.

Keywords: transfer, cavity QED, entangled state, quantum teleportation

PACC: 0365, 0367, 4250

1. Introduction

Quantum entanglement is very important in quantum mechanics related to the Einstein–Podolsky–Rosen (EPR) paradox[1] and Bell's theorem.[2] Meanwhile, the quantum entanglement plays a key role in quantum information processing, such as quantum teleportation,[3] quantum dense coding,[4] and quantum cryptography.[5] And much attention has been paid to the quantum teleportation that was first proposed by Bennett *et al.*[3] The quantum teleportation is the process that allows a sender, Alice, to transfer an unknown state to a receiver, Bob. Usually, we know that three steps are required to finish the transfer process. The first step is to prepare two particles in a maximally entangled state as a quantum channel. The second step is to perform a Bell-state measurement on the two particles. The last step is that the result of the joint measurement is sent to Bob through a classical channel and Bob performs a corresponding unitary operation to reconstruct the initial state on his particle. Based on the quantum teleportation protocol, a number of protocols of quantum teleportation have been presented, quantum teleportation of a photon has been demonstrated experimentally by Bouwmeester *et al*[6] and Boschi *et al*[7] using parametric down-conversion. Rydberg atoms crossing superconductive cavities are another almost ideal system to perform the quantum information processing.[8] Davidovich *et al*[9] have presented a scheme for the teleportation of an unknown atomic state between two high-Q cavities initially prepared in an entangled photon-number state. Cirac and Parkins[10] and other authors[11−16] have made some proposals for the quantum teleportation of an atomic state or field state. Zhan[17] has proposed a scheme for teleporting an unknown N-particle entangled W state via entanglement swapping. Very recently, several protocols[18,19] to teleport an arbitrary unknown two-particle state and a there-particle state have been presented.

Recently Pires *et al*[20] proposed a scheme for teleporting a two-mode entanglement of zero- and one-photon states from one cavity to another with a successful probability of 100 percent. The scheme can discriminate each of the four Bell states. Soon after that Cardoso *et al*[21] modified the proposal to teleport a two-mode entanglement of zero- and one-photon states from one cavity to another without using Bell-state measurements. Biswas and Agarwal[22] presented a scheme for transferring an unknown quantum state. The proposal is based on two-mode cavity interacting dispersively with three-level atoms in the

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Λ configuration. Furthermore, a quantum memory of arbitrary superposition of two modes of a cavity containing one photon through quantum networking is realized. Ye and Guo[23] proposed to transfer a two-mode entangled state between two cavities. In their scheme, the entangled state of an atom and two modes in a cavity is employed as the quantum channel, and via atom–cavity field interaction the entanglement swapping can occur. Zheng[24] presented a scheme for transferring the quantum information between two atoms. The scheme only required a single resonant interaction of the atoms with a cavity mode but did not use the cavity as the memory, and it transferred the quantum information directly from one atom to another atom. Recently Zhang[25] proposed a scheme for transferring an unknown single-qubit atomic state from one atom to another by letting two atoms interact simultaneously with a cavity quantum electrodynamics (QED).

In this paper, we present a scheme for transferring an unknown two-atom entangled state. The distinct advantage of our scheme is that the cavity is only virtually excited during the atoms passing through the cavity field. Meanwhile, no quantum information transfers between the atoms and the cavity. In addition our scheme does not require the Bell-state measurement and performing any transformations to reconstruct the initial state. And the transfer can occur at a successful probability of 100 percent in a simple manner.

2. Transferring a two-atomic entangled state to other two atoms via cavity QED

Consider a system composed of two identical two-level atoms simultaneously interacting with a single-mode cavity field, and the Hamiltonian for the system will be given by

$$H = H_0 + H_i \tag{1}$$

with

$$H_0 = \omega a^+ a + \omega_0 \sum_{j=1}^{2} S_{z,j}, \tag{2}$$

$$H_i = g \sum_{j=1,2} (a^+ S_j^- + a S_j^+), \tag{3}$$

where $S_{z,j} = \frac{1}{2}(|e_j><e_j| - |g_j><g_j|)$, $S_j^+ = |e_j><g_j|$, $S_j^- = |g_j><e_j|$, and $|e_j>$ and $|g_j>$ are, respectively, the excited and ground states of the jth atom, a and a^+ are the annihilation and creation operators for the cavity mode respectively, g is the atom–cavity coupling strength, ω is the cavity frequency, ω_0 is the atomic transition frequency. In the case $\delta \gg g$, there is no energy exchange between the atomic system and the cavity, where δ is the detuning between the atomic transition frequency ω_0 and the cavity frequency ω. Then the effective Hamiltonian for the system is[26]

$$H_{\rm e} = \lambda \left[\sum_{j=1}^{2} |e_j><e_j| a a^+ - |g_j><g_j| a^+ a + \sum_{i,j=1, i\neq j}^{2} S_j^+ S_i^- \right], \tag{4}$$

where $\lambda = g^2/\delta$. The first and second terms on the right-hand side of the above expression describe the photon-number-dependent Stark shifts, and the third term describes the dipole coupling between the two atoms induced by the cavity mode. Consider that the cavity field is initially in a vacuum state. In an interaction picture, the effective interaction Hamiltonian reduces to[27]

$$H_{\rm e} = \lambda \left[\sum_{j=1}^{2} |e>_j {}_j<e| + \sum_{i,j=1, i\neq j}^{2} S_j^+ S_i^- \right]. \tag{5}$$

Assume that an entangled atom-pair, which consists of two identical two-level atoms (1 and 2), is initially in the state

$$|\Psi>_{12} = (\alpha|ge> + \beta|eg>)_{12}, \tag{6}$$

where α and β are unknown coefficients, and $|\alpha|^2 + |\beta|^2 = 1$.

In order to transfer the unknown state $|\Psi>_{12}$, we need other two identical two-level atoms (3 and 4) and two identical single-mode cavity-fields (A and B). Here we describe how to transfer the state (6) to the other atoms (3 and 4).

Assume that atom 3 and atom 4 are initially in the state $|g>_3 |g>_4$, and the two cavity-fields (A and

B) are in a vacuum state. The state of the four-atom system can be written as

$$|\Psi>_{1234}=(\alpha|ge>+\beta|eg>)_{12}|g>_3\,g>_4\,. \qquad (7)$$

Let atom 1 and atom 3 interact simultaneously with the cavity A governed by the Hamiltonian (5) for an appropriate interaction time; and at the same time, let atom 2 and atom 4 interact simultaneously with the cavity B governed by the Hamiltonian (5) for the same interaction time, the state (7) evolution of the system is

$$\begin{aligned}|\Psi>_{1234} = {} & \alpha e^{-i\lambda t}|g>_1|g>_3(\cos(\lambda t)|e>_2|g>_4 - i\sin(\lambda t)|g>_2|e>_4)\\ & +\beta e^{-i\lambda t}(\cos(\lambda t)|e>_1|g>_3 - i\sin(\lambda t)|g>_1|e>_3)|g>_2|g>_4\,. \end{aligned} \qquad (8)$$

If we choose $\lambda t=\pi/2$, the state (8) becomes

$$|\Psi>_{1234}=(-ie^{-i\lambda t}|gg>_{12})(\alpha|ge>_{34}+\beta|eg>_{34}). \qquad (9)$$

From Eq.(9), we note that the entanglement of atom 1 and atom 2 has been transferred to the state of atom 3 and atom 4. Thus the state of atom 3 and atom 4 can be written as

$$|\Psi>_{34}=\alpha|ge>_{34}+\beta|eg>_{34}\,. \qquad (10)$$

This means that the entanglement transferring has happened. Thus we can obtain atom 3 and atom 4 in the initial state like atom 1 and atom 2 without performing any measurements. In this case we implement the transfer with a successful probability of 100 percent in a simple manner.

We note that the above idea can also be extended to transferring an entangled state of n-atom. Assume that n atoms $(1,2,\cdots,n)$ are prepared in an entangled state. To transfer the state, we need other n atoms $(1',2',\cdots,n')$ as a receiver of the teleported state and n single-mode cavity-fields $(A_1,A_2,\cdots,A_n)$. Similar to the above two-atom process, let atom 1 and atom $1'$ interact simultaneously with the cavity A_1 for an interaction time, and at the same time, let atom 2 and atom $2'$, atom 3 and atom $3'$, $\cdots$, atom n and atom n', respectively, interact simultaneously with the cavity-fields $(A_1,A_2,\cdots,A_n)$ governed by the Hamiltonian (5) for the same interaction time, with the interaction time chosen appropriately, we can obtain the other n atoms $(1',2',\cdots,n')$ staying in the initial state like the n atoms (1, 2, $\cdots$, n) without performing any measurements. Thus we implement the transfer with a successful probability of 100 percent in a simple manner.

Now we give a brief discussion on something related to the experiment. For the Rydberg atoms with principal quantum numbers being 49, 50, and 51, the radiative time is about $T_r=3\times10^{-2}\,s$, and the coupling constant is $g=2\pi\times24\,kHz$.[28] With the choice of $\delta=10g$, the essential atom-cavity-field interaction time is approximately $\pi\delta/g^2\approx2\times10^{-4}\,s$. Then the time requisite for completing the whole procedure is approximately $10^{-3}\,s$, much shorter than T_r. A cavity with a quality factor $Q=10^8$ is experimentally achievable.[28] In the case the cavity field frequency is about 50 GHz. The corresponding photon lifetime is $T_c=Q/2\pi v\approx3.0\times10^{-4}\,s$.[27] In the present scheme, the cavity can be excited only at a probability of about one percent during the atoms passing through the cavity. Thus, the efficient decay time of the cavity is about $3.0\times10^{-2}\,s$ on the order of the atomic radiative time. Therefore, the proposed scheme might be realizable with cavity QED techniques.

3. Conclusions

We have proposed a simple scheme for transferring an unknown atomic state in cavity QED. The scheme does not require Bell-state measurement and performing any transformations to reconstruct an initial state. In our scheme the transfer can be achieved at a successful probability of 100 percent in a simple manner. Our scheme involves only atom-field interaction with a large detuning but does not require transferring quantum information between the atoms and cavity. During the passage of the atoms the cavity is only virtually excited; thus the requirement for the quality factor of the cavity is greatly loosened. As a result the scheme is insensitive to cavity decay and may be realizable with cavity QED technology.

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