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Nonideal effects in quantum field-effect directional coupler*

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The nonideal effects in a quantum field-effect directional coupler where two quantum wires are coupled through a finite potential barrier are studied by adopting the lattice Green function method. The results show that the electron energy distribution, asymmetric geometry and finite temperature all have obvious influence on the electron transfer of the coupler. Only for the electrons with energies in a certain region, can the complete periodic transfer between two quantum wires take place. The conductance of these electrons as a function of the barrier length and potential height exhibits a fine periodic or quasi-periodic pattern. For the electrons with energies beyond the region, however, the complete periodic transfer does not hold any more since many irregular oscillations are superimposed on the conductance profile. In addition, the finite temperature and asymmetric geometry both can reduce the electron transfer efficiency.

Keywords: nonideal effects, directional coupler, Green function
PACC: 7320D, 7340G

1. Introduction

Electronic transport in mesoscopic systems[1] has attracted much attention due to the improvement of micro-fabrication technology and material growth techniques. The dimensions of these mesoscopic systems are smaller than the inelastic mean free path of electrons at sufficiently low temperatures; as a consequence the phase of the electron wavefunction is preserved throughout the structure and quantum effects dominating the electron transport properties. Based on these effects, a quantum interference device called quantum field-effect directional coupler (QFED) has been proposed[2] and experimentally realized.[3]

In general, a QFED consists of two one-dimensional quantum wires (QWRs)[4,5] coupled by a potential barrier whose height can be controlled by an externally applied voltage.[3] The QWRs can be formed by constricting the two-dimensional electron gas of a GaAs/AlGaAs heterostructure with mesa etching or the split gate technique. Many studies have been carried out on the structure.[2,3,6−14] Based on the coupled mode theory directly transplanted from optics, Alamo and Eugster[2] found that an electron wave packet initially confined in one of the waveguides could transfer to the other waveguides while propagating along the device. A complete transfer could occur if the electrons had just travelled a certain distance defined as transfer length.[2,6,7,15] The gate voltage can be modulated to achieve a complete transfer at either of the two QWRs at the output of the device. Therefore the device behaves as a current switch. Xu *et al*[6] accurately calculated the transfer length by using the transfer matrix method, and revealed that the complete transfer effect was an intrinsic property of the parity symmetry of the system. The investigation on the multimodedness of the device indicated that the complete transfer occurred only in the case of single mode while the correlation among different modes reduced the transfer efficiency.[6] However, the previous studies omitted many nonideal effects which are important to the real device, including the finite energy distribution of electrons, finite temperature, geometric asymmetry, etc.

In this paper, we focus our attention on the influence of the nonideal effects on the electron transfer in a QFED. The conductances as a function of potential height, barrier width or thickness for different electron

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energies are calculated. It is found that the complete periodic transfer occurs only for the electrons with energies in a certain region, while the energy region is associated with the thickness of potential barrier. For the electrons with energies beyond the region, the complete periodic transfer will not hold any more. We also calculate the conductance for the structure with asymmetric geometry or at a finite temperature. The results indicate that the nonideal factors have considerable effects on the electron transfer of a QFED and cannot be omitted.

2. Calculation and results

The model device is schematically illustrated in Fig.1. Two parallel QWRs, with widths W_1 and W_2 respectively, are coupled via a finite potential barrier III with a width W and length L. The boundaries of

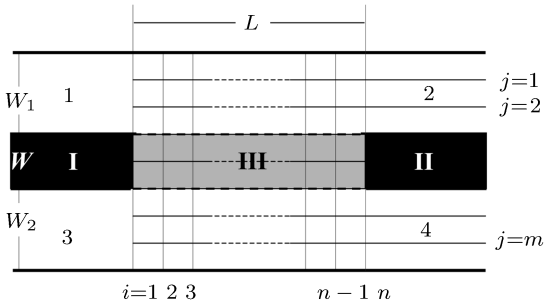


Fig.1. Schematic illustration of a QFED where two parallel QWRs coupled by a finite potential barrier III with a height P. $W_1(W_2)$ is the width of the upper (bottom) QWR. L and W respectively represent the length and width of the potential barrier.

the two QWRs are determined by hard walls except region III with a potential height P. We symbolize leads of upper QWR as 1, 2 and those of bottom QWR as 3, 4, respectively. To describe electronic properties of the structure, the tight-binding Hamiltonian of this system is written as[16]

$$\begin{aligned} H = & \sum_{i,j} \varepsilon_{i,j} \left|i,j\right\rangle \left\langle i,j\right| \\ & + \sum_{i,j} V(\left|i,j\right\rangle \left\langle i,j+1\right| + \text{H.c.}) \\ & + \sum_{i,j} V(\left|i+1,j\right\rangle \left\langle i,j\right| + \text{H.c.}), \end{aligned} \tag{1}$$

where the site energy $\varepsilon_{i,j}$ at site (i,j) can be expressed as

$$\varepsilon_{i,j} = \begin{cases} -4V & \text{in two QWRs,} \\ -4V + P & \text{in sregion III,} \\ \infty & \text{in other regions,} \end{cases} \tag{2}$$

with the hopping matrix element $V = -\dfrac{\hbar^2}{2m^* a^2}$ ($m^* = 0.067m_0$ is effective mass of electron and a is a lattice constant) between the nearest neighbouring sites.

In order to calculate transmission amplitude between every two leads, we first regard the four-terminal system as a two-terminal system, i.e. leads 1, 3 and region I (hard wall) are regarded as the left terminal, while the right terminal consists of leads 2, 4 and region II. The Green function of each column (see columns $i = 2-(n-1)$) except the left and right terminals can be obtained from the definition of the Green function $\varsigma = \dfrac{1}{E-H}$ and Eq.(1). The Green function ς_{L} of the left terminal (column i=1) can be calculated by[1]

$$\varsigma_{\text{L}} = (I - \varsigma^0 U)^{-1} \varsigma^0, \tag{3}$$

where I denotes the identity matrix and ς^0 is the Green function of the left terminal without interaction among leads 1, 3 and region I, while U represents the interaction among the three parts.

Similarly, the Green function ς_{R} of the right terminal (column $i = n$) can be found. Then using the recursive Green function (RGF) scheme,[16–20] the Green function $\varsigma_{\text{R,L}}$ from the left terminal to the right one can be calculated. It consists of nine elements:

$$\varsigma_{\text{R,L}} = \begin{pmatrix} \varsigma_{2,1} & \varsigma_{\text{II},1} & \varsigma_{4,1} \\ \varsigma_{2,\text{I}} & \varsigma_{\text{II,I}} & \varsigma_{4,\text{I}} \\ \varsigma_{2,3} & \varsigma_{\text{II},3} & \varsigma_{4,3} \end{pmatrix}, \tag{4}$$

where $\varsigma_{a,b}$ represents the Green function from lead (or region) b to lead (or region) a. Once the Green function $\varsigma_{a,b}$ is obtained, the transmission coefficient T_{ab} from lead b to lead a can be calculated.[19] According to the Launder formula, the conductance from lead b to lead a is

$$G_{ab} = \frac{2e^2}{h} T_{ab}. \tag{5}$$

At finite temperatures, this result must be averaged over energy with the derivative of the Fermi function as a weighing factor[16,20]

$$G = \int G_{\text{E}} \frac{\partial f(E)}{\partial T} \text{d}E. \tag{6}$$

In Fig.2, we show the calculated conductances as a function of the electron energy for a QFED with different lengths L of the potential barrier. The solid line represents the parallel conductance G_{21} (from lead 1 to lead 2), while the dashed and dotted lines represent transfer conductance G_{41} (from lead 1 to 4) and

G_{31} (from lead 1 to 3), respectively. As L is small (L=0.25W_1), the conductances G_{41} and G_{31} are almost zero due to the threshold of region III being larger than that of the QWR [see Fig.2(a)]. So the electrons with energies below the first threshold of the QWR cannot tunnel through the potential (coupling) region, and then completely move along horizontal QWR. As the length L increases to W_1, i.e. $L = W_1$, the electrons can tunnel through the coupling region due to $G_{41} > 0$ and $G_{31} > 0$ as shown in Fig.2(b). Consequently the transfer between two QWRs occurs. With the further increase of L, the transfer further increases while the oscillations on conductance profiles become more complicated (see Fig.2(c) and 2(d)). Especially, at E=0.0506 in Fig.2(d), G_{21}=0 while G_{41}=1($2e^2/h$), which marks a complete transfer from the upper QWR to the bottom one.

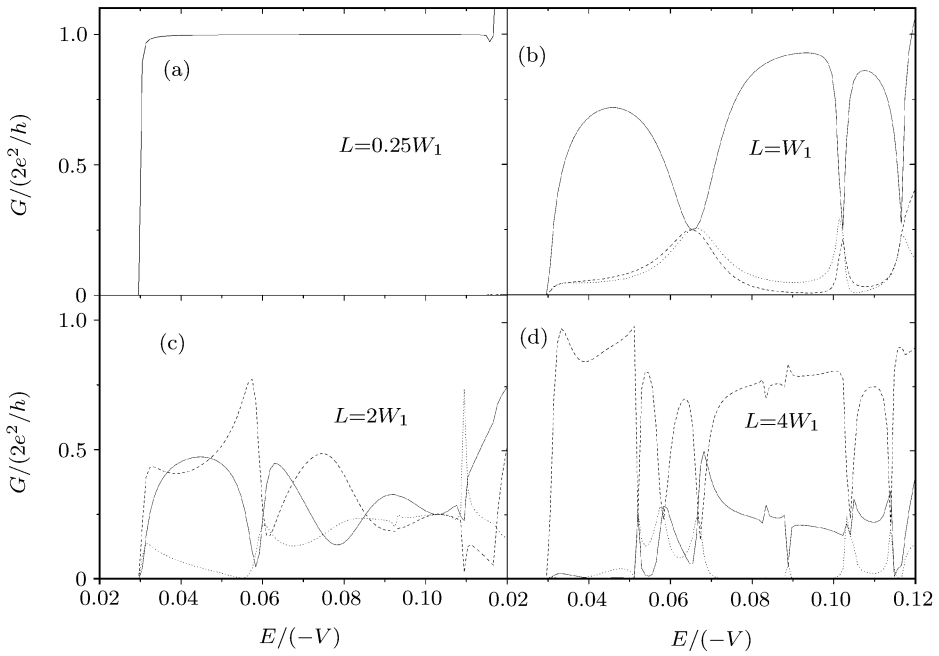


Fig.2. Conductances as a function of the electron energy E for a QFED in Fig.1 with different barrier lengths L ($W_1 = W_2 = 2W = 16$, $P = 0$). (a) $L = 0.25W_1$, (b) $L = W_1$, (c) $L = 2W_1$, (d) $L = 4W_1$. The solid, dashed and dotted curves represent respectively conductance G_{21}, G_{41} and G_{31}. P and E are in units of hopping energy $-V$, the length and width are in units of lattice constant a.

To investigate the dependence of the electron transfer on length L of the potential barrier, we calculate the conductances as a function of L for three different electron energies E as shown in Fig.3. It is seen that the conductances for the three cases are quite different. The conductances for the electron energy E=0.0449 in Fig.3(b) show perfect periodic patterns which are analogous to sine- or cosine-shaped curves. The complete electron transfer between two QWRs periodically occurs, such as L=66, 132 etc., where G_{21} and G_{41} reach their extrema 0 or 1($2e^2/h$). Each of the periods of the conductances G_{21} and G_{41} is twice as long as the transfer length (or coupling length) determined by the length scale of complete transfer from one wire to another. The phases of G_{21} and G_{41} are completely opposite to each other. While for the electron with a lower energy in Fig.3(a) or higher energy in Fig.3(c), the complete periodic transfer does not take place any more. In Fig.3(a) the curves of conductance for E=0.0314 exhibit quasi-periodic patterns, but the maximum transfer efficiency is only about 85% (for example at L=64). This results from the fact that in the scattering region of the structure there exists a high reflection to the lower-energy electrons. For$E$=0.0538 in Fig.3(c), the complete transfer appears at some potential length such as L=68, but the periodicity of conductance curve is destroyed due to the interference among more modes in the coupling region. Moreover, for the later two cases, there are strong oscillations superposed on the conductance curves, and the side reflection conductance G_{31} cannot be omitted. Our calculations indicate that, for the electrons with ener-

gies in the region E=0.0380–0.0506, the complete periodic transfer can take place with the increase of coupling region length L, and the thinner barrier (smaller W) can extend this energy region (not shown here). These results demonstrate that the complete periodic transfer in a QEFD depends on the electron energy distribution.

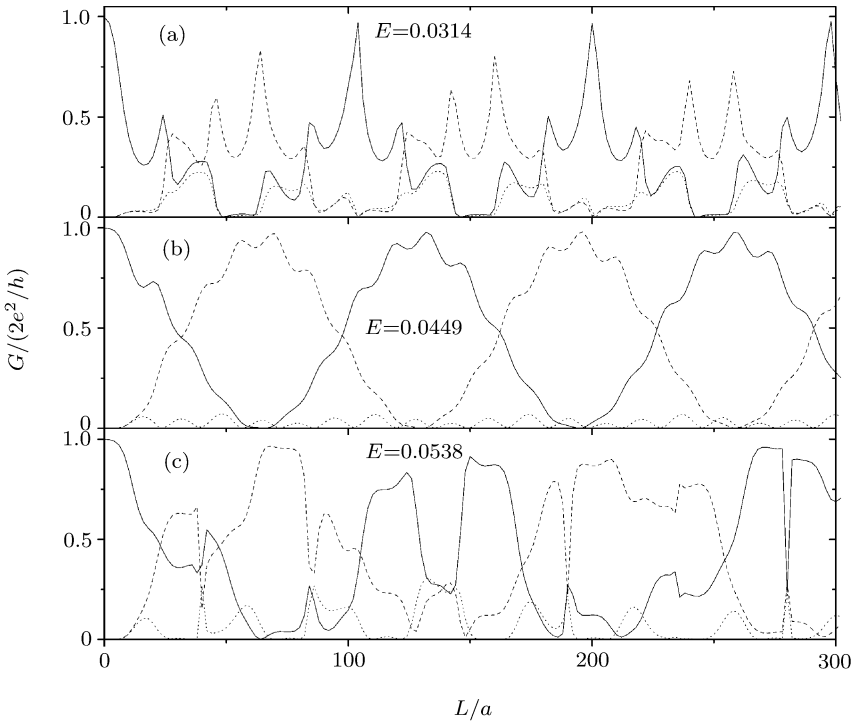


Fig.3. Conductances versus barrier length L for a QFED with different electron energies $E(W_1 = W_2{=}2W = 16,\ P = 0)$. (a) $E = 0.0314$, (b) $E = 0.0449$, (c) $E = 0.0538$. The solid, dashed and dotted curves represent conductance G_{21}, G_{41} and G_{31}, respectively.

In Figs.4(a) and 4(b), we show the conductances G_{21} and G_{41} as a function of the potential height P and barrier length L (E=0.0449), respectively. Two features are deserving of special attention. One is that, for a given potential height P, the conductances G_{21} and G_{41} show fine periodic structures and inverse phase with the increase of length L. The period of conductance (or transfer length) increases with the potential height P. The other is that, for a fixed L, the conductance exhibits a quasi-periodic pattern and the period of curves is inverse to the length L. Therefore complete periodic transfer can also be achieved by varying the potential height, which is easy to realize experimentally by modifying voltage applied to the barrier.[2] To clearly show the relation between the electron transfer and the potential height, Fig.5(a) presents the conductances as a function of the potential height P with E=0.0449. It is found that the conductance curves exhibit a fine quasi-periodic pattern without any additional oscillation. The complete transfer between upper and bottom QWRs takes place in turn with the increase of P. The diagram suggests a good practical way to transfer electrons between two QWRs. However, for electrons with energies beyond the energy region discussed above, the complete periodic transfer cannot be achieved by adjusting the potential height. This can be illustrated from the conductance as a function of the potential height for the case of the lower-energy electron in Fig.5(b) and for the case of the higher-energy electron in Fig.5(c). It is clearly seen that the irregular oscillations are superposed on the conductance profiles and most extrema of the curves are not $1(2e^2/h)$ or 0. In the two cases, the complete transfer cannot be obtained by varying the potential height. Further, the complete electron transfer is also dependent on the electron energy distribution, which is demonstrated above.

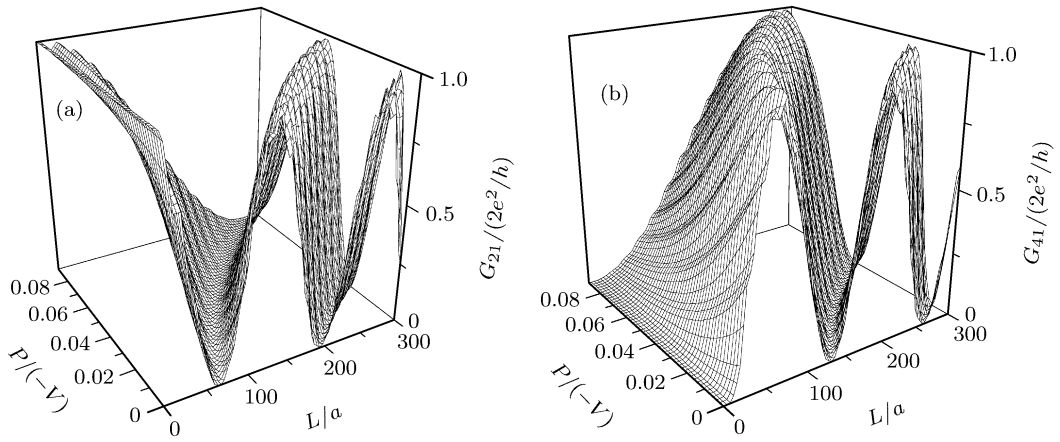


Fig.4. Conductances as a function of the potential barrier height P and length L for a QFED ($W_1 = W_2 = 2W = 16$, $E = 0.0449$). (a) G_{21}, (b) G_{41}.

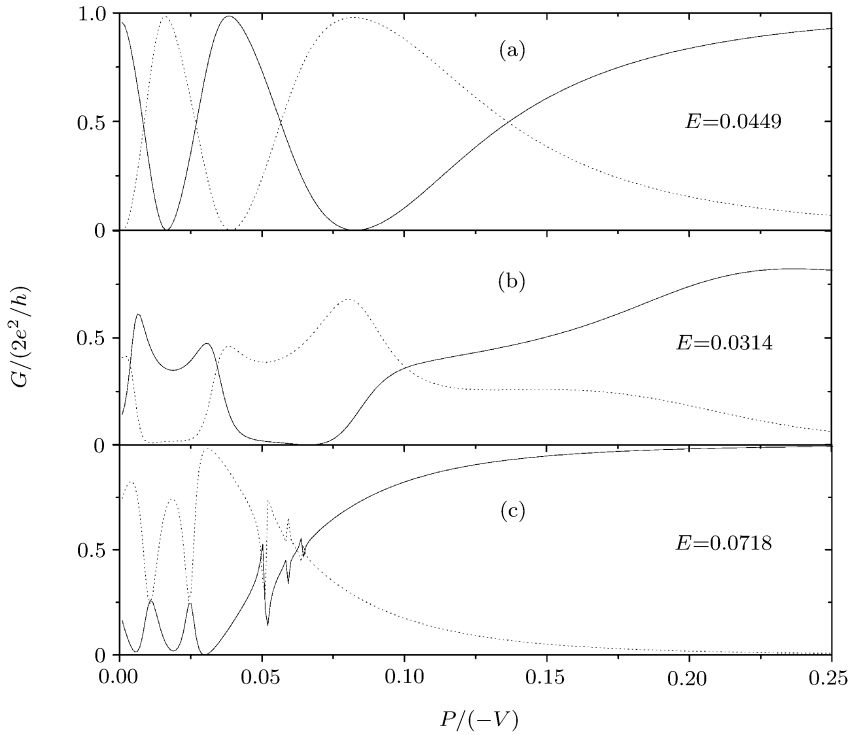


Fig.5. Conductances versus the potential height P for a QFED with different electron energies ($L = 280$, $W_1 = W_2 = 2W = 16$). (a) E=0.0449, (b) E=0.0314, (c) E=0.0718. The solid and dotted curves respectively represent G_{21} and G_{41}.

In Fig.6, the conductances for the QFED including two asymmetric QWRs are calculated. One can see that, on one hand the asymmetric geometry changes the transfer length, on the other hand the transfer efficiency is decreased by the asymmetry. This coincides with the prediction of Yang *et al*[15] that a complete transfer is forbidden unless the two wells are either identical or have equal energy levels.

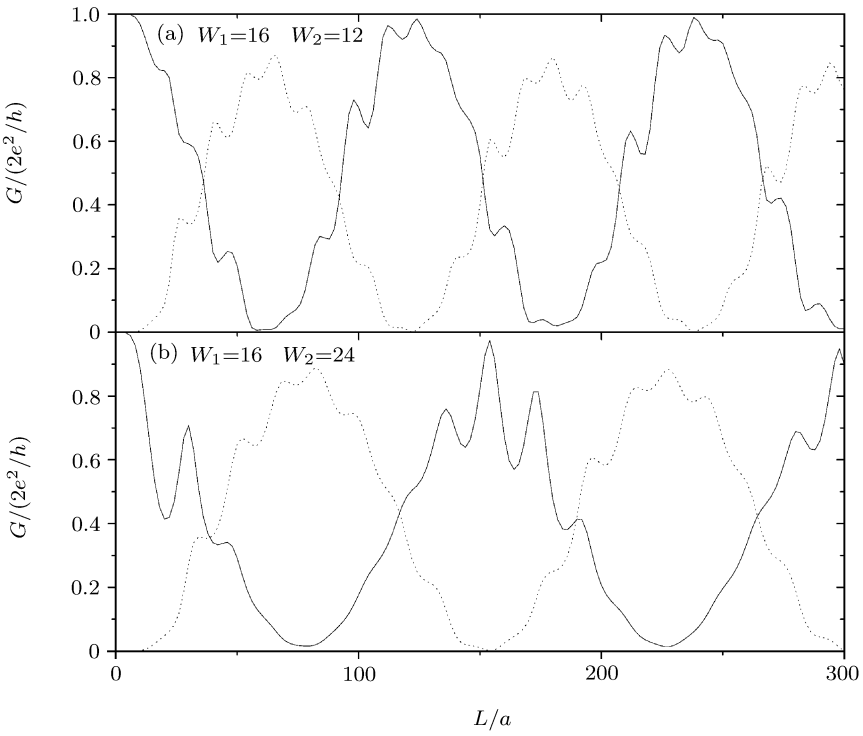


Fig.6. Conductances as a function of the barrier length L for the asymmetric QWRs in a QFED (W=8, E=0.0449, P=0). (a) W_1=16, W_2=12; (b) W_1=16, W_2=24. The solid and dotted curves represent G_{21} and G_{41}, respectively.

In addition, we also discuss the influence of temperature on transfer efficiency. Figure 7 presents the conductances as a function of the potential height for three different temperatures. It is found that the transfer efficiency distinctly decreases at higher temperatures, which can be understood by considering that the thermal effect destroys the quantum interference in the QFED.

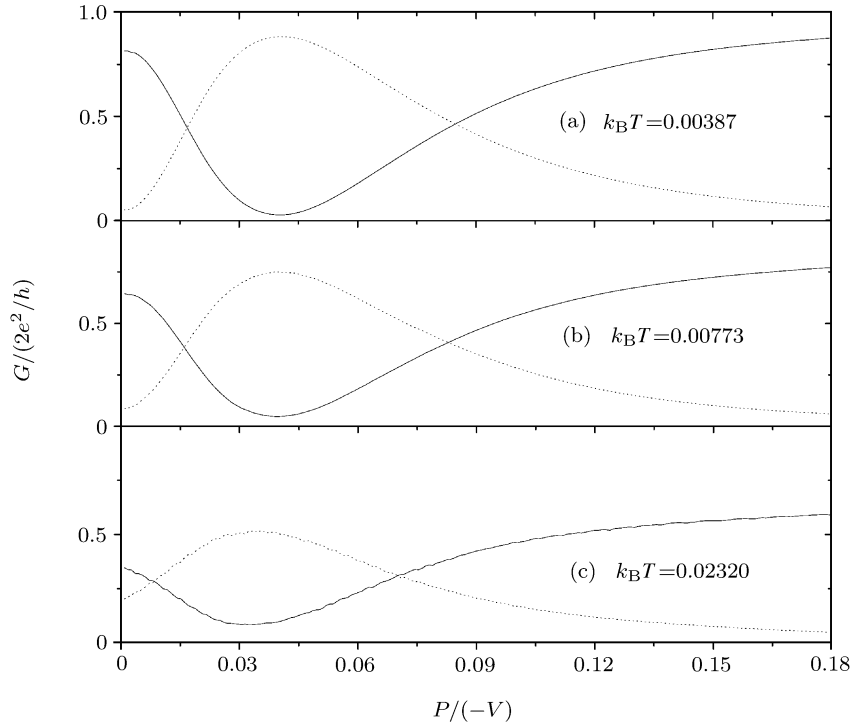


Fig.7. Finite-temperature conductances versus the potential height P for a QFED at different temperatures T (L=132, E=0.0449, $W_1 = W_2 = 2W = 16$). (a) k_BT=0.00387, (b) k_BT=0.00773, (c) k_BT=0.02320. The solid and dotted curves represent G_{21} and G_{41}, respectively. k_BT is in units of hopping energy $-V$.

3. Conclusions

Some nonideal effects of a QFED including electron energy distribution, asymmetric geometry and finite temperature have been studied by adopting a lattice Green function method. It is found that the complete periodic transfer depends on the electron energy distribution. Only for the electrons with energies in a certain region can a complete transfer between two QWRs be obtained by changing the potential height and/or barrier length. However, the conductance for the electrons with energies beyond the region exhibits many irregular oscillations, and thus the periodicity of conductance and complete electron transfer does not hold any more. Other nonideal factors such as finite temperature and asymmetric geometry can reduce the transfer efficiency and change the transfer length.

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