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Electron acceleration by two crossed Bessel–Gaussian beams in vacuum*

Zhao Zhi-Guo(赵志国)[a)b)†] and Lü Bai-Da(吕百达)[a)]

[a)] *Institute of Laser Physics and Chemistry, Sichuan University, Chengdu* 610064, *China*

[b)] *Department of Physics, Luoyang Normal College, Luoyang* 471022, *China*

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The direct acceleration of electrons by using two linearly polarized crossed Bessel–Gaussian (BG) beams with equal frequency and amplitude in vacuum is proposed and studied. It is shown that two linearly polarized BG beams of the same order (0 or 1) with a π-rad phase difference have a resultant non-zero longitudinal electric field on the z-axis and can be used, in principle, to accelerate electrons.

Keywords: laser electron acceleration, crossed Bessel–Gaussian (BG) beams, phase difference
PACC: 4260, 4262

1. Introduction

The basic idea of laser electron acceleration is that in accordance with the Maxwell equations there exists a finite longitudinal electric-field component for a high-power focused laser that can be used to directly accelerate electrons in vacuum, where the difficulties associated with the plasma are eliminated.[1,2] Recently, the rapid development of ultrashort high-power laser technology[3,4] has given rise to a renewed interest in the possibility of laser acceleration of electrons.[5−13] Several methods to accelerate electrons by using a single Hermite–Gaussian beam, a Bessel beam and two crossed plane waves, two crossed Gaussian beams, etc., have been proposed, and the main features of laser-accelerated electrons have been reviewed.[8] In Refs.[1] and [13] it is shown that a 'nondiffracting' Bessel beam is, in principle, able to provide a longitudinal electric-field component on axis that can be used to accelerate electrons in vacuum. However, an ideal Bessel beam cannot be realized physically because it carries infinite energy.[14] Another possible way is to use the Bessel–Gaussian (BG) beam,[15] which carries finite energy and exhibits the 'nondiffracting' property upon propagation over a large range. Such a type of beam can be produced by several methods and has great potential for applications.[16]

The aim of the present paper is to study the possibility of directly accelerating electrons by two crossed BG beams in vacuum. In Section 2 the theoretical model of accelerating electrons by two linearly polarized crossed BG beams with equal frequency and amplitude is presented. The general characteristics by using a resultant longitudinal electric field of two crossed BG beams to accelerate electrons are analysed in Section 3. Finally, the main results obtained in this paper are summarized in Section 4.

2. Theoretical model

Assume that there are two linearly polarized BG beams with equal frequency and amplitude in vacuum. The two BG beams propagate along the z_1- and z_2-axes which as shown in Fig.1 rotate through angles θ and $-\theta$ about the z-axis, respectively. The transverse electric-field components in the cylindrical system (r, φ, z) are expressed as[16]

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†E-mail: zhiguo.zhao@tom.com

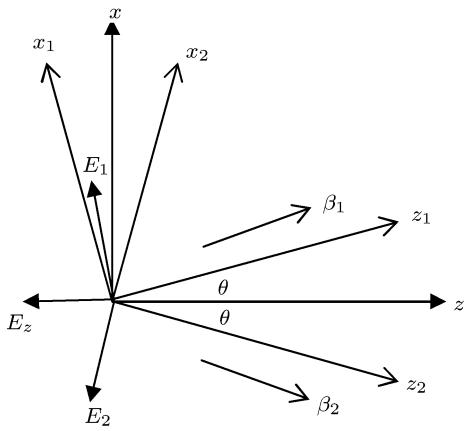


Fig.1. Electric fields for two crossed BG beams in rectangular coordinate system.

$$E_{xj}(r_j,\varphi_j,z_j,t)=\frac{\mathrm{i}^n E_0 w_0}{w_j}\exp\mathrm{i}n\varphi_j \times\exp\left[-\frac{1}{w_j^2}\left(r_j^2+\frac{\alpha^2 z_j^2}{k^2}\right)\right] \times J_n\left[\frac{2\alpha r_j}{kw_j^2}\left(Z_R+\mathrm{i}z_j\right)\right]\exp\left(-\mathrm{i}\psi_j\right), \tag{1a}$$

$$E_{yj}(r_j,\varphi_j,z_j,t)=0, \tag{1b}$$

where

$$\psi_j=\beta z-\omega t+\frac{k}{2R_j(z_j)}\left(r_j^2+\frac{\alpha^2 z^2}{k^2}\right)-\arctan\left(\frac{z_j}{Z_R}\right)+\phi_{0j}, \tag{2}$$

j=1, 2 (unless otherwise stated), $J_n(\cdot)$ denotes the Bessel function of order n, k is the wave number related to the wave length λ, frequency ω and light velocity c in vacuum by $k=2\pi/\lambda=\omega/c$, and $\alpha^2+\beta^2=k^2$ ($\alpha<<k$), $w_j=w_0\,(1+z_j^2/Z_{\mathrm{R}}^2)^{1/2}$ is the beam width at the z_j plane with w_0 being the waist width, $Z_{\mathrm{R}}=\pi w_0^2/\lambda$ is the Rayleigh range, $R_j(z)=z_j\,(1+Z_{\mathrm{R}}^2/z_j^2)$ denotes the radius of curvature, E_0 and ϕ_{0j} denote the initial constant amplitude and phase, respectively.

Because of the relations $E_{zj}=-(\mathrm{i}/\beta)\partial E_{xj}/\partial_{xj}$,[1,17] and $\partial/\partial x=\cos\varphi\,(\partial/\partial r)-(\sin\varphi/r)\partial/\partial\varphi$, the longitudinal electric field E_{zj} of the jth BG beam in the rectangular coordinate system (x, y, z) is given by

$$E_{zj}(x_j,y_j,z_j,t)=-\frac{\mathrm{i}^{n+1}E_0 w_0}{\beta w_j}\left\{\frac{\alpha w_0 x_j}{w_j r_j}\exp\left(\mathrm{i}\arctan\frac{z_j}{Z_{\mathrm{R}}}\right)J_{n-1}\left[\frac{2\alpha r_j}{kw^2}\left(Z_{\mathrm{R}}+\mathrm{i}z_j\right)\right] -\left[\left(\frac{2}{w_j^2}+\frac{n}{r_j^2}+\frac{\mathrm{i}k}{R_j}\right)x_j+\frac{\mathrm{i}ny_j}{r_j^2}\right]J_n\left[\frac{2\alpha r_j}{kw_j^2}\left(Z_{\mathrm{R}}+\mathrm{i}z_j\right)\right]\right\}\exp\left(-\mathrm{i}\psi_j\right), \tag{3}$$

where $r_j=(x_j^2+y_j^2)^{1/2}$.

The resultant transverse and longitudinal electric-field components are expressed as

$$E_x(x,y,z,t)=\left[E_{x1}(x_1,y_1,z_1,t)+E_{x2}(x_2,y_2,z_2,t)\right]\cos\theta +\left[E_{z1}(x_1,y_1,z_1,t)-E_{z2}(x_2,y_2,z_2,t)\right]\sin\theta, \tag{4a}$$

$$E_z(x,y,z,t)=-\left[E_{x1}(x_1,y_1,z_1,t)-E_{x2}(x_2,y_2,z_2,t)\right]\sin\theta +\left[E_{z1}(x_1,y_1,z_1,t)+E_{z2}(x_2,y_2,z_2,t)\right]\cos\theta, \tag{4b}$$

where $x_1=x\cos\theta-z\sin\theta$, $y_1=y$, $z_1=x\sin\theta+z\cos\theta$, $x_2=x\cos\theta+z\sin\theta$, $y_2=y$ and $z_2=-x\sin\theta+z\cos\theta$.

By letting x=0 and y=0 in Eqs.(4), the resultant transverse and longitudinal electric-field components on the z-axis are given by

$$E_x(0,0,z,t)=\frac{2\mathrm{i}^n w_0 E_0}{w}\exp\left[-\frac{1}{w^2}\left(z^2\sin^2\theta+\frac{\alpha^2 z^2\cos^2\theta}{k^2}\right)\right] \times\left\{\left[\cos\theta-\frac{\mathrm{i}z\sin^2\theta}{\beta}\left(\frac{2}{w^2}+\frac{n}{z^2\sin^2\theta}+\frac{\mathrm{i}k}{R}\right)\right]J_n(d)+\frac{\mathrm{i}\alpha w_0\sin\theta}{\beta w} \times\exp\left(\mathrm{i}\arctan\frac{z\cos\theta}{Z_{\mathrm{R}}}\right)J_{n-1}(d)\right\}\exp\left(-\mathrm{i}\psi_M\right)\cos\left(\frac{\phi_{01}-\phi_{02}}{2}\right), \tag{5a}$$

$$E_z(0,0,z,t) = \frac{2\mathrm{i}^{n+1}E_0 w_0}{w}\exp\left[-\frac{1}{w^2}\left(z^2\sin^2\theta + \frac{\alpha^2 z^2\cos^2\theta}{k^2}\right)\right] \times\left\{\left[\sin\theta + \frac{\mathrm{i}z\sin\theta\cos\theta}{\beta}\left(\frac{2}{w^2}+\frac{n}{z^2\sin^2\theta}+\frac{\mathrm{i}k}{R}\right)\right]J_n(d) - \frac{\mathrm{i}\alpha w_0\cos\theta}{\beta w} \times\exp\left(\mathrm{i}\arctan\frac{z\cos\theta}{Z_\mathrm{R}}\right)J_{n-1}(d)\right\}\exp\left(-\mathrm{i}\psi_M\right)\sin\left(\frac{\phi_{01}-\phi_{02}}{2}\right), \tag{5b}$$

where $d = \alpha z\sin\theta\ Z_\mathrm{R}\ (Z_\mathrm{R}+\mathrm{i}z\cos\theta)/(z^2+Z_\mathrm{R}^2)$, $w = w_0\ (1+z^2\cos^2\theta/Z_\mathrm{R}^2)^{1/2}$, $R = z\cos\theta + Z_\mathrm{R}^2/z\cos\theta$, and

$$\psi_\mathrm{M} = \beta z\cos\theta - \omega t + \frac{k}{2R}\left[z^2\sin^2\theta + \left(\frac{\alpha z\cos\theta}{k}\right)^2\right] - \arctan\left(\frac{z\cos\theta}{Z_\mathrm{R}}\right) + \frac{\phi_{01}+\phi_{02}}{2}. \tag{6}$$

It turns out from Eqs.(5) and (6) that the resultant transverse and longitudinal electric-field components on the z-axis are mainly dependent on the initial amplitude E_0, phase ϕ_{oj}, beam order n and angle θ.

3. Acceleration of electrons by crossed BG beams

To effectively accelerate electrons by lasers, it is required that there exists a non-zero resultant longitudinal electric field on the z-axis, which can be maximized. From Eqs.(5) we see that, if $\phi_{01} - \phi_{02} = \pi$, E_z (0, 0, z) takes its maximum value and E_x (0, 0, z)=0, i.e. the two crossed BG beams with a π-rad phase difference are favourable for the use in electron acceleration.

Assuming that $\phi_{01}-\phi_{02}=\pi$, $\theta << 1$, and recalling Bessel function expansions

$$J_0(\xi) = 1 - \frac{\xi^2}{4} + O(\xi^4),$$
$$J_1(\xi) = \frac{\xi}{2} - \frac{\xi^3}{16} + O(\xi^5), \tag{7}$$

which are valid on or near the axis i.e. $\xi << 1$,[1] it is readily seen that for $n > 1$ the longitudinal electric-field on the z-axis has E_z (0, 0, z)=0, and makes no sense to accelerate electrons.

For two crossed BG beams of order n=1 the resultant longitudinal electric-field component on the z axis reads

$$E_z(0,0,z) = -\frac{\alpha E_0 Z_\mathrm{R}^2}{\beta}\frac{1}{Z_\mathrm{R}^2+z^2}\exp\left[-\frac{\left(k^2\theta^2+\alpha^2\right)}{k^2\theta_0^2}\frac{z^2}{Z_\mathrm{R}^2+z^2}\right]\exp\left(-\mathrm{i}\psi_t\right), \tag{8}$$

where $\theta_0 = w_0/Z_\mathrm{R}$ (diffraction angle), and

$$\psi_t = -\int\frac{k}{2\gamma_z^2}\mathrm{d}z - \frac{\alpha^2}{2k}z - \frac{\left(\theta^2+\alpha^2/k^2\right)}{\theta_0^2}\frac{Z_\mathrm{R}z}{Z_\mathrm{R}^2+z^2} - 2\arctan\frac{z}{Z_\mathrm{R}} + \phi_{01}, \tag{9}$$

$\gamma_z = (1-\upsilon_z^2/c^2)^{-1/2}$, υ_z is the electron velocity along the z-axis and $\upsilon_z/c \approx 1\text{–}1/2\gamma_z^2$, $t = \int\mathrm{d}z/\upsilon_z$.

The real part of Eq.(8) is written as

$$E_z(0,0,z) = -\frac{E_0\alpha Z_\mathrm{R}^2}{\beta\left(Z_\mathrm{R}^2+z^2\right)}\exp\left[-\frac{\left(\theta^2+\alpha^2/k^2\right)}{\theta_0^2}\frac{z^2}{Z_\mathrm{R}^2+z^2}\right]\cos\psi_t. \tag{10}$$

From Eq.(9) the phase velocity of the accelerating field is given by[18]

$$\upsilon_\mathrm{ph} = \omega/\left|\nabla\psi_t\right| = c\left[1 - \frac{\alpha^2}{2k^2} - \frac{\left(\theta^2+\alpha^2/k^2\right)\left(1-\left(z/Z_\mathrm{R}\right)^2\right)+2\theta_0^2\left(1+\left(z/Z_\mathrm{R}\right)^2\right)}{2\left(1+\left(z/Z_\mathrm{R}\right)^2\right)}\right]^{-1}. \tag{11}$$

Near the waist $|z/Z_\mathrm{R}| << 1$, if $\left[\frac{\alpha^2}{2k^2} + \frac{\left(\theta^2+\alpha^2/k^2\right)\left(1-\left(z/Z_\mathrm{R}\right)^2\right)+2\theta_0^2\left(1+\left(z/Z_\mathrm{R}\right)^2\right)}{2\left(1+\left(z/Z_\mathrm{R}\right)^2\right)}\right] << 1$, we obtain

$$\frac{v_{ph}}{c} \approx 1 + \frac{k^2\theta^2 + 2(\alpha^2 + k^2\theta_0^2)}{2k^2} = 1 + \frac{1}{2\gamma_c^2} > 1, \quad (12)$$

where $\gamma_c = k/[k^2\theta^2 + 2(\alpha^2 + k^2\theta_0^2)]^{1/2}$ is the critical energy.[6] Equation (12) indicates that the phase velocity $v_{\rm ph}$ is greater than the light velocity c in vacuum, and the axial field slips ahead of electrons. The slippage distance $z_{\rm s}$, defined as the distance required for electrons to phase slip by π, is

$$z_{\rm s} \approx \pi \left(\omega \left| v_{ph}^{-1} - v_z^{-1} \right| \right)^{-1} = \frac{\gamma_c^2 \lambda}{1 + \gamma_c^2/\gamma_z^2} = \frac{\lambda}{\theta^2 + 2(\alpha^2/k^2 + \theta_0^2)}. \quad (13)$$

In the high-energy regime $\gamma_z >> \gamma_{\rm c}$, the slippage is dominated by diffraction, i.e. the third and fourth terms on the right-hand side of Eq.(9), and electrons can be accelerated. However, in the low-energy regime $\gamma_z << \gamma_{\rm c}$, there are oscillations of $-E_z(z)/E_0$ with $z/Z_{\rm R}$, which cannot be used to effectively accelerate electrons. The normalized longitudinal electric field on the z-axis $-E_z(z)/E_0$ versus normalized distance

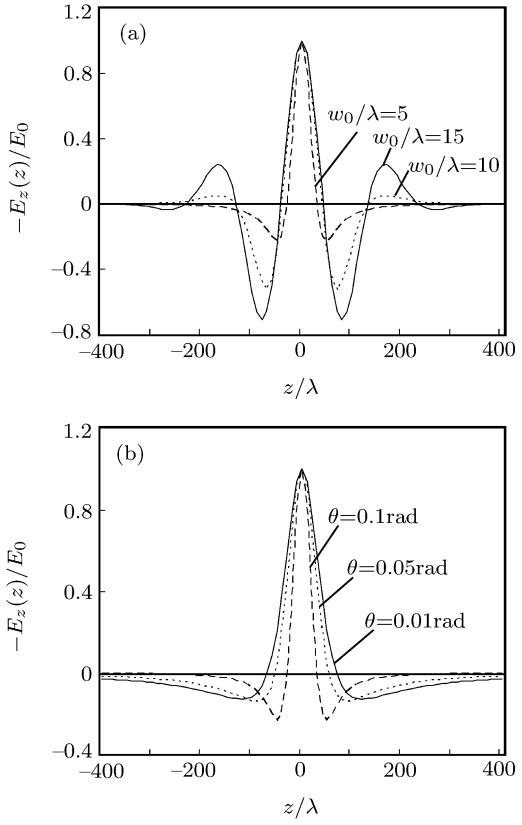


Fig.2. Normalized accelerating longitudinal field $-E_z(z)/E_0$ versus normalized distance z/λ. (a) w_0/λ=5, 10, 15. (b)θ=0.01, 0.05, 0.1rad.

z/λ in the high-energy regime is depicted in Fig.2 for different values of (a) w_0/λ=15, 10, 5, θ=0.1rad, and (b) θ=0.01, 0.05, 0.1rad, w_0/λ=5, respectively. As can be seen, the accelerating field depends on w_0 and θ, and the slippage distance $z_{\rm s}$ increases with the increase of w_0 and decrease of θ, which is consistent with Eq.(13).

By using the relation between the accelerating on-axis electric field and gradient of a potential $E_z = -\partial {\rm U}(z)/\partial z$, the potential is expressed as

$$U(z) = \frac{k^2\alpha\theta_0^2 E_0}{\beta\left(\alpha^2 + k^2\theta^2\right)} \times \exp\left[-\frac{\left(\alpha^2 + k^2\theta^2\right) z^2}{k^2\theta_0^2(Z_{\rm R}^2 + z^2)}\right] \times \sin\left[\frac{\left(\alpha^2 + k^2\theta^2\right) Z_{\rm R} z}{k^2\theta_0^2(Z_{\rm R}^2 + z^2)} - \phi_{01}\right]. \quad (14)$$

The energy gain ΔW from the injection point $z_{\rm I}$ to the extraction point $z_{\rm F}$ is given by $\Delta W = e[U(z_{\rm F}) - U(z_{\rm I})]$ with e being the electron charge. For the case of $z_{\rm F} = -z_{\rm I} = z_0$ ΔW is written as

$$\Delta W = \frac{4ek\alpha E_0}{\beta\left(\alpha^2 + k^2\theta^2\right)} \cos\phi_{01} \times \exp\left[-\frac{\left(\alpha^2 + k^2\theta^2\right) z_0^2}{k^2\theta_0^2(Z_{\rm R}^2 + z_0^2)}\right] \times \sin\left[\frac{\left(\alpha^2 + k^2\theta^2\right) Z_{\rm R} z_0}{k^2\theta_0^2(Z_{\rm R}^2 + z_0^2)}\right], \quad (15)$$

which indicates that the energy gain depends on the waist width w_0, angle θ, initial phase ϕ_{01}, distance z_0, wave number k and its transverse part α. If $z_0/\lambda \to \infty$, we have $\Delta W \to 0$, which is consistent with the Lawson-Woodward theorem.[19,20] But, a finite energy gain is achieved over a finite interaction range $2z_0$ and is maximized if z_0 satisfies

$$\tan\left[\frac{\left(\alpha^2 + k^2\theta^2\right)}{k^2\theta_0^2}\frac{Z_{\rm R} z_0}{Z_{\rm R}^2 + z_0^2}\right] = \frac{Z_{\rm R}^2 - z_0^2}{2Z_{\rm R} z_0}. \quad (16)$$

For a fixed laser power P=20TW the normalized energy gain $\Delta W(z_0)/(eE_0\lambda)$ versus normalized acceleration distance z_0/λ is plotted in Fig.3 for different values of (a) w_0/λ=5, 10, 15, θ=0.1rad, and (b) θ=0.01, 0.05, 0.1rad, w_0/λ=5, and ϕ_{01}=0, α=0.03k. It is seen that the maximum energy gain $\Delta W_{\rm max}/(eE_0\lambda)$ increases as the waist width w_0 and the angle θ decrease if the laser power is fixed. For example, $\Delta W_{\rm max}/(eE_0\lambda)$=1, 0.74, and 0.55 at z_0/λ=29, 39, and 43 for w_0/λ=5, 10, 15 in Fig.3(a), and $\Delta W_{\rm max}/(eE_0\lambda)$=1, 0.78, and 0.46 for θ=0.01, 0.05, 0.1rad in Fig.3(b), respectively. The results are in agreement with those from Eq.(16).

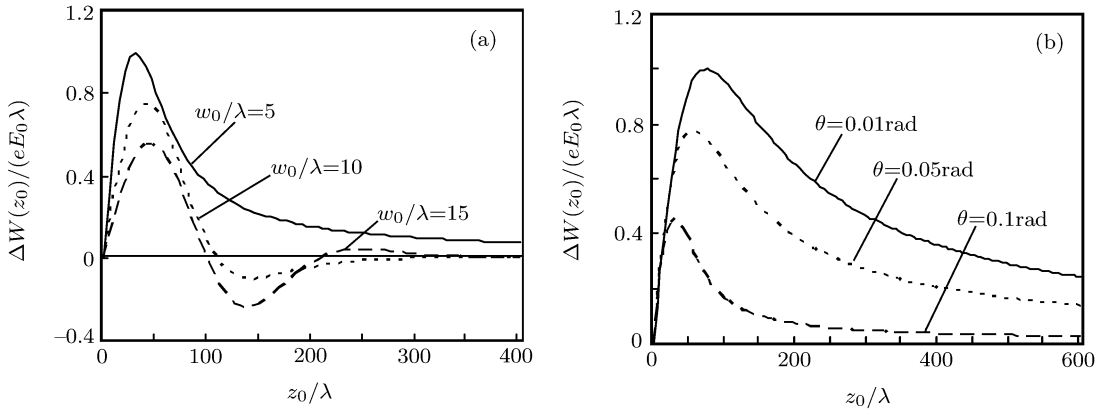


Fig.3. Normalized energy gain $\Delta W(z_0)/(\ eE_0\lambda)$ versus normalized acceleration distance z_0/λ. (a) w_0/λ=5, 10, 15. (b)θ=0.01, 0.05, 0.1rad.

For two crossed BG beams of order n=0 the resultant longitudinal electric-field component on the z-axis is

$$E_z(0,0,z) = -\frac{2k\theta E_0 Z_{\rm R}^2}{\beta}\frac{1}{Z_{\rm R}^2+z^2}\exp\left(-\frac{\left(k^2\theta^2+\alpha^2\right)}{k^2\theta_0^2}\frac{z^2}{Z_{\rm R}^2+z^2}\right)\exp\left(-{\rm i}\psi_t\right), \tag{17}$$

$$\psi_t = -\int\frac{k}{2\gamma_z^2}{\rm d}z - \frac{\alpha^2}{2k}z - \frac{\left(\theta^2+\alpha^2/k^2\right)}{\theta_0^2}\frac{Z_{\rm R}z}{Z_{\rm R}^2+z^2} - 2\arctan\frac{z}{Z_{\rm R}} + \phi_{01}. \tag{18}$$

The real part of Eq.(17) is

$$E_z(0,0,z) = -\frac{2k\theta E_0 Z_{\rm R}^2}{\beta\left(Z_{\rm R}^2+z^2\right)}\exp\left[-\frac{\left(\theta^2+\alpha^2/k^2\right)}{\theta_0^2}\frac{z^2}{Z_{\rm R}^2+z^2}\right]\cos\psi_t. \tag{19}$$

The phase velocity of the accelerating field $v_{\rm ph}$ and slippage distance $z_{\rm s}$ are the same as the expressions (12) and (13), respectively. The potential is given by

$$U(z) = \frac{4k^2\theta E_0}{\beta\left(\alpha^2+k^2\theta^2\right)}\exp\left[-\frac{\left(\alpha^2+k^2\theta^2\right)z^2}{k^2\theta_0^2(Z_{\rm R}^2+z^2)}\right]\sin\left[\frac{\left(\alpha^2+k^2\theta^2\right)Z_{\rm R}z}{k^2\theta_0^2(Z_{\rm R}^2+z^2)} - \phi_{01}\right]. \tag{20}$$

The energy gain ΔW is expressed as

$$\Delta W = \frac{8ek^2\theta E_0}{\beta\left(\alpha^2+k^2\theta^2\right)}\cos\phi_{01}\exp\left[-\frac{\left(\alpha^2+k^2\theta^2\right)z_0^2}{k^2\theta_0^2(Z_{\rm R}^2+z_0^2)}\right]\sin\left[\frac{\left(\alpha^2+k^2\theta^2\right)Z_{\rm R}z_0}{k^2\theta_0^2(Z_{\rm R}^2+z_0^2)}\right]. \tag{21}$$

ΔW reaches its maximum if z_0 satisfies

$$\tan\left[\frac{\left(\alpha^2+k^2\theta^2\right)}{k^2\theta_0^2}\frac{Z_{\rm R}z_0}{Z_{\rm R}^2+z_0^2}\right] = \frac{Z_{\rm R}^2-z_0^2}{2Z_{\rm R}z_0}. \tag{22}$$

It is readily seen that inserting α=0 into Eqs.(19), (20), (21) and (22), the results reduce to those for two crossed Gaussian beams and agree with those in Ref.[6].

For two crossed BG beams with order n=0, and P=20TW, λ=1μm, θ=0.1rad, w_0/λ=5, 10, 15, we have the slippage distance $z_{\rm s}$=50.24μm, 72.33μm, 78.74μm, the diffraction angles θ_0=64mrad, 32mrad, 21 mrad, respectively. Within the interaction distance $z_{\rm s}=2z_0$, we obtain the energy gain

$$\Delta W({\rm MeV}) = 22.1[P({\rm TW})]^{1/2} = 98.9{\rm MeV},$$
$$\Delta W({\rm MeV}) = 10.62[P({\rm TW})]^{1/2} = 47.5{\rm MeV},$$
$$\Delta W({\rm MeV}) = 5.87[P({\rm TW})]^{1/2} = 26.23{\rm MeV},$$

and the average accelerating gradient $< E_z >= \Delta W/ez_{\rm s}$=1.97TV/m, 0.66TV/m, and 0.33TV/m for w_0/λ=5, 10, and 15, respectively. The results show that for a fixed laser power P, W and $< E_z >$ increase but $z_{\rm s}$ decreases as w_0/λ decreases.

4. Concluding remarks

In conclusion, the general characteristics of accelerating electrons by using two linearly polarized crossed BG beams with equal frequency and amplitude has been studied. It is found that two crossed BG beams of the same order (n=0 or n=1) have a non-zero resultant longitudinal electric- field on the z-axis and can be used, in principle, to accelerate electrons. The initial phase difference plays a crucial role. For two linearly polarized crossed BG beams with a π-rad phase difference, the resultant transverse electric field vanishes, and there exists the non-zero resultant longitudinal field on the z-axis, which can be maximized and used to accelerate electrons.

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