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The statistical fluctuation of a single-mode laser system driven by coloured pump noise with signal modulation and the quantum noise with cross-correlation between its real and imaginary parts*

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Using the linear approximation method, this paper studies the statistical property of a single-mode laser driven by both coloured pump noise with signal modulation and the quantum noise with cross-correlation between its real and imaginary parts, and calculates the steady-state mean normalized intensity fluctuation and intensity correlation time. It analyses the influences of the modulation signal, the net gain coefficient, the noise and its correlation form on the statistical fluctuation of the laser system respectively. It is found that the coloured pump noise modulated by the signal has a great suppressing action on the statistical fluctuation of the laser system; the pump noise self-correlation time and the specific frequency of modulation signal have the result that the statistical fluctuation tends to zero. Furthermore, the 'colour' correlation of pump noise has much influences on the statistical fluctuation of the laser system. Increasing the intensity of pump noise will augment the statistical fluctuation of the laser system, but the intensity of quantum noise and the coefficient of cross-correlation between its real and imaginary parts have less influence on the statistical fluctuation of the laser system. Therefore, from the conclusions of this paper the statistical property can be known and a theoretical basis for steady operation and output of the laser system can be provided.

Keywords: signal, noise, single-mode laser, steady-state mean normalized intensity fluctuation, intensity correlation time

PACC: 4250, 0540, 4260K

1. Introduction

Noise and its cross-correlation form has great influence on the dynamic behaviour of nonlinear stochastic system. As a result, many researchers recently study its action on the dynamic behaviour of a nonlinear stochastic system: they have introduced cross-correlation noise into the laser system and studied the transient state property, the steady state property and the statistical fluctuation of the laser system driven by different forms of cross-correlation noise respectively.[1−12] We have studied the dynamic property and the statistical fluctuation of a single-mode laser system driven by both pump noise and quantum noise with cross-correlation between its real and imaginary parts.[13−16] However, in the above-mentioned studies, one emphasized the influence of cross-correlation noise on the dynamic behaviour of a nonlinear stochastic system, but gave little consideration to the influence of noise modulated by the signal on the dynamic behaviour of the laser system. Dykman has introduced into the laser system the noise with signal modulation.[17] Signal modulated noise can better describe the laser system than ordinary noise, because when a laser system is modulated by a signal, the noise of the system will simultaneously be modulated by the signal, and the modulation noise will highly affect the dynamic behaviour of the laser system. The study of the single-mode laser system driven by noise with signal modulation can comprehensively reveal how modulation signal and noise affect the statistical fluctuation of the laser system, and can provide

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a theoretical basis for steady operation and output of the laser system. Therefore, our study has the background of certain practical application.

In this paper, by adopting the linear approximation, we have calculated the steady-state mean normalized intensity fluctuation (SSMNIF) $C(0)$ and intensity correlation time (ICT) T of a single-mode laser driven by both coloured pump noise with signal modulation and the quantum noise with cross-correlation between its real and imaginary parts. On the basis of the research about the influence of the modulation signal, the net gain of the system, the noise and its correlation form on the statistical fluctuation of the laser system, we can provide a theoretics foundation and optimum scheme for steady output of the laser system.

2. The SSMNIF $C(0)$ and the ICT T

The Langevin equation of amplitude for a loss-noise model of a single-mode laser is given by[18]

$$\frac{\mathrm{d}r}{\mathrm{d}t'} = a_0 r - Ar^3 + \frac{Q}{2r}(1-|\lambda_q|) + rp_{\mathrm{R}}(t') + \varepsilon_r(t'). \quad (1)$$

If we think that the noise is modulated by a periodic signal $B\cos\Omega t'$ in a single-mode laser system,[17] the intensity Langevin equation for a single-mode laser loss-noise model is given by

$$\frac{\mathrm{d}I}{\mathrm{d}t'} = 2a_0 I - 2AI^2 + Q(1-|\lambda_q|) + 2Ip_{\mathrm{R}}(t')B\cos\Omega t' + 2\sqrt{I}\varepsilon_r(t'). \quad (2)$$

The statistical properties of noises $p_{\mathrm{R}}(t')$ and $\varepsilon_r(t')$ are as follows

$$\begin{aligned} &\langle p_{\mathrm{R}}(t')\rangle = \langle \varepsilon_r(t')\rangle = 0, \quad \langle p_{\mathrm{R}}(t')p_{\mathrm{R}}(s)\rangle = \frac{P}{2\tau}\mathrm{e}^{-\dfrac{|s-t'|}{\tau}}, \quad -1 \le \lambda_q \le 1, \\ &\langle \varepsilon_r(t')\varepsilon_{\mathrm{r}}(s)\rangle = Q(1+|\lambda_q|)\delta(t'-s), \quad \langle p_{\mathrm{R}}(t')\varepsilon_r(s)\rangle = 0. \end{aligned} \quad (3)$$

In Eqs.(1), (2) and (3), a_0 and A represent the net gain coefficient and self-saturation coefficient respectively, I is the laser intensity, B is the amplitude of a modulation signal, Ω is the signal frequency, $p_{\mathrm{R}}(t')$ is the real part of the pump noise, and $\varepsilon_r(t')$ is the quantum noise of phase lock, P and Q are the intensities of the pump noise and the quantum noise respectively, τ is the pump noise self-correlation time, λ_{q} is the cross-correlation coefficient between the real part and the imaginary part of the quantum noise. Let $I = I_0 + \delta(t')$, where $I_0 = \dfrac{a_0}{A}$ is the deterministic steady-state intensity, and $\delta(t')$ is the perturbational term. If we linearize Eq.(2) around the deterministic steady-state intensity I_0, we will get

$$\frac{\mathrm{d}\delta(t')}{\mathrm{d}t'} = -\gamma\delta(t') + 2I_0 p_{\mathrm{R}}(t')B\cos\Omega t' + 2\sqrt{I_0}\varepsilon_r(t') + Q(1-|\lambda_q|), \quad (4)$$

where $\gamma = 2a_0$.

According to the steady-state mean intensity correlation function (SSMICF) defined by

$$\begin{aligned} C(t) &= \lim_{t'\to\infty} \frac{\overline{\langle I(t')I(t'+t)\rangle} - \overline{\langle I(t')\rangle^2}}{\overline{\langle I(t')\rangle^2}} \\ &= \lim_{t'\to\infty} \left(\frac{\dfrac{\Omega}{2\pi}\displaystyle\int_{t'}^{t'+2\pi/\Omega} \langle I(t')I(t'+t)\rangle \mathrm{d}t' - \frac{\Omega}{2\pi}\int_{t'}^{t'+2\pi/\Omega} \langle I(t')\rangle^2 \mathrm{d}t'}{\dfrac{\Omega}{2\pi}\displaystyle\int_{t'}^{t'+2\pi/\Omega} \langle I(t')\rangle^2 \mathrm{d}t'} \right), \end{aligned}$$

we can obtain SSMICF by solving Eq.(4)

$$C(t) = \left(\frac{2Q(1+|\lambda_q|)}{\gamma I_0} + \frac{B^2P(2\pi\gamma^2+\Omega^3)(\Omega^2-\gamma^2+\tau^{-2})}{\tau^2\Omega\gamma(k_1^2+\Omega^2)(K_2^2+\Omega^2)(\gamma^2+\Omega^2)}\right)\mathrm{e}^{-\gamma|t|} + \frac{B^2P(\Omega^2+\gamma^2-\tau^{-2})}{\tau(k_1^2+\Omega^2)(k_2^2+\Omega^2)}\mathrm{e}^{-|t|/\tau}\cos\Omega t + \frac{2\Omega B^2P}{\tau^2(k_1^2+\Omega^2)(k_2^2+\Omega^2)}\mathrm{e}^{-|t|/\tau}\sin\Omega|t|, \tag{5}$$

where $k_1 = \gamma - \tau^{-1}$, $k_2 = \gamma + \tau^{-1}$ and $\tau \neq \dfrac{1}{2a_0}$. Then let $t = 0$ the SSMNIF $C(0)$ is obtained

$$C(0) = \frac{2Q(1+|\lambda_q|)}{\gamma I_0} + \frac{B^2P(2\pi\gamma^2+\Omega^3)(\Omega^2-\gamma^2+\tau^{-2})}{\tau^2\Omega\gamma(k_1^2+\Omega^2)(k_2^2+\Omega^2)(\gamma^2+\Omega^2)} + \frac{B^2P(\Omega^2+\gamma^2-\tau^{-2})}{\tau(k_1^2+\Omega^2)(k_2^2+\Omega^2)}. \tag{6}$$

According to the intensity correlation time defined by

$$T = \int_0^\infty \phi(t)\mathrm{d}t,$$

where $\phi(t) = \dfrac{C(t)}{C(0)}$, inserting $C(t)$ and $C(0)$, we have

$$T = \frac{2Q(1+|\lambda_q|)}{\gamma^2 I_0 C(0)} + \frac{B^2P(2\pi\gamma^2+\Omega^3)(\Omega^2-\gamma^2+\tau^{-2})}{\tau^2\Omega\gamma^2(k_1^2+\Omega^2)(k_2^2+\Omega^2)(\gamma^2+\Omega^2)C(0)} + \frac{B^2P(3\Omega^2+\gamma^2-\tau^{-2})}{(k_1^2+\Omega^2)(k_2^2+\Omega^2)(1+\tau^2\Omega^2)C(0)}. \tag{7}$$

In the following, we will discuss the influences of the different parameters of the modulation signal and the noise on SSMNIF $C(0)$ above the threshold ($a_0 > 0$) and in the case of pump noise self-correlation time $\tau \ll 1$.

3. The changes of SSMNIF $C(0)$ with pump noise self-correlation time τ

Based on the Eq.(6), $C(0)$ vs τ curve is plotted as shown in Fig.1. It is seen from Fig.1, that $C(0)$

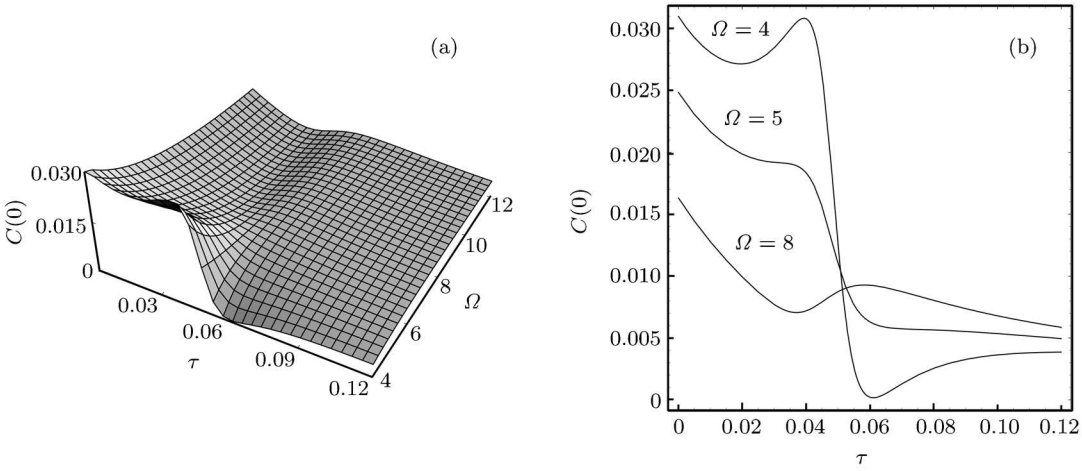


Fig.1. $C(0)$ versus τ for different values of Ω with the parameters $A = 1$, $Q = 0.001$, $P = 0.001$, $B = 20$, $a_0 = 10$, $\lambda_q = 0.5$.

changes with τ and exhibits a complicated process for different values of Ω. That is, the $C(0)$ vs τ curve has two minima and one maximum. It is indicated that the 'coloured' pump noise has great influence on the statistical fluctuation of the laser system.

3.1. The influence of frequency Ω of modulation signal on the SSMNIF $C(0)$ vs τ curve

The three-dimensional plot in Fig.1(a) exhibits how the frequency Ω of modulation signal influence the change of the $C(0)$ with τ. The Fig.1(b) is its sectional drawing. By virtue of Fig.1, we can see that, with increasing Ω, the $C(0)$ vs τ curve experiences a changing process from having two minima and one maximum to having one minimum and one maximum only, and there are changes of the size and the position of the minimum and maximum. This shows that the frequency Ω has a great influence on the statistical fluctuation of the laser system. It is noted that, as $\Omega = 4$, the value of $C(0)$ will tend to zero near the $\tau = 0.06$. It is shown that the specific frequency of modulation signal has the result that SSMNIF $C(0)$ tends to zero when pump noise is coloured correlation.

3.2. The influence of amplitude B of modulation signal on the SSMNIF $C(0)$ vs τ curve

The three-dimensional plot in Fig.2(a) exhibits how the amplitude B of the modulation signal influences the change of $C(0)$ with τ. Figures 2(b) and (c) are its sectional drawing. It is seen from Eq.(6) that, when the laser system has no input signal, $C(0)$ is determined by Q, λ_q, a_0 and A, not relevant to τ, thus the curve of $C(0)$ vs τ exhibits a straight line, as shown in Fig.2(c). After inputting the modulation

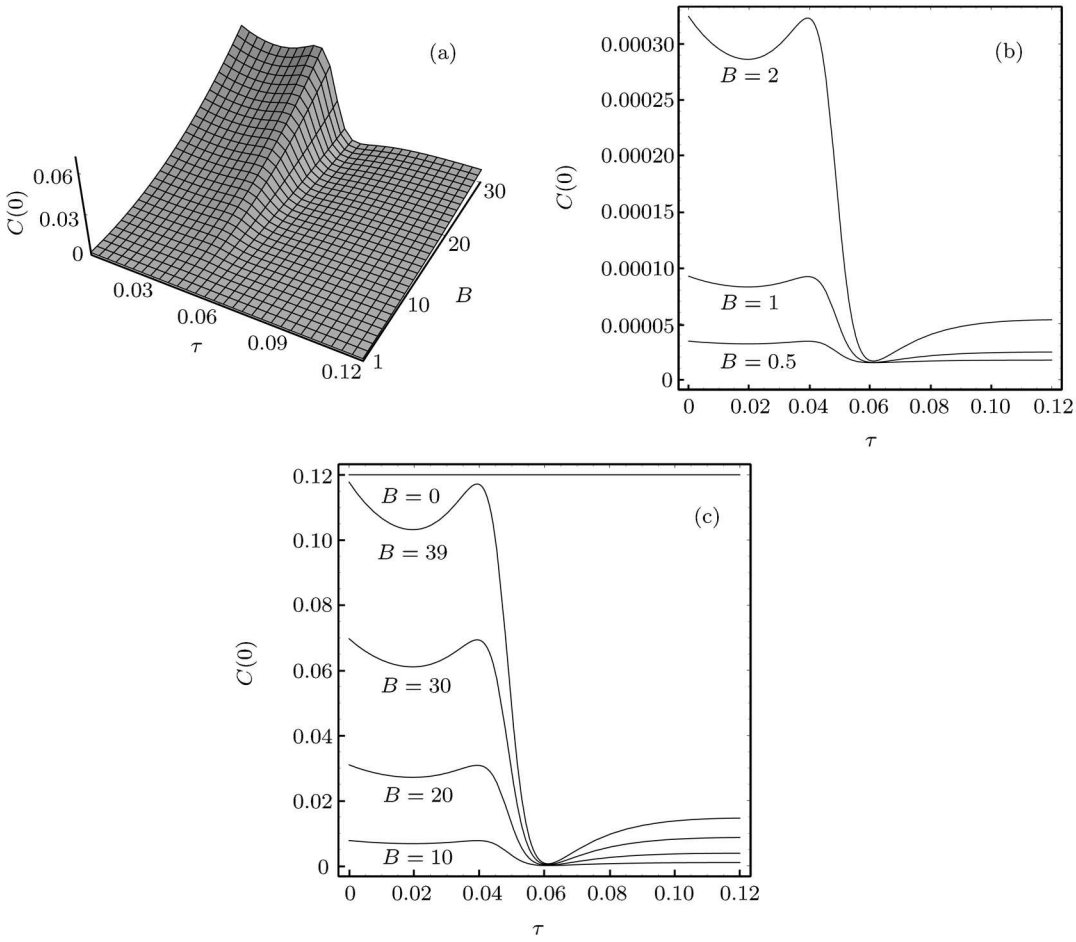


Fig.2. $C(0)$ versus τ for different values of B with the parameters $A = 1$, $Q = 0.001$, $P = 0.001$, $\Omega = 4$, $a_0 = 10$, $\lambda_q = 0.5$.

signal into the laser system, the curve exhibits a complicated changing process. That is, the curve has two minima and one maximum, as shown in Figs.2(b), (c). It is noted that, 1) as B is smaller the value of $C(0)$ is smaller too, i.e. the small amplitude of signal has a great suppressive action on the statistical fluctuation; 2) as the amplitude of signal $B < 40$, compared with the laser system which has no input signal, the $C(0)$ vs τ curve is all below the straight line and, as B increases, the values of $C(0)$ all tend to zero near $\tau = 0.06$. This shows that the interaction between modulation signal and coloured pump noise can suppress the statistical fluctuation of the laser system, and the specific frequency of modulation signal may have the result that SSMNIF $C(0)$ tends to zero as changing range of the amplitude of modulation signal is very small.

3.3. The influence of intensity of noise and its cross-correlation forms on the $C(0)$ vs τ curve

By choosing λ_q, Q and P as parameters respectively, the $C(0)$ vs τ curves are plotted, as shown in Fig.3. It is seen from Figs,3(a), (b) that, as the intensity of the quantum noise changes, the cross-correlation forms between the real part and imaginary part of the quantum noise are zero correlation, positive correlation and negative correlation respectively, the $C(0)$ vs τ curve all superpose and remain unchanged. It is shown that the quantum noise and its the cross-correlation between the real part and imaginary parts have less influences on the statistical fluctuation of the laser system, but when pump noise intensity P increases, the statistical fluctuation of the laser system will augment, as shown in Fig.3(c).

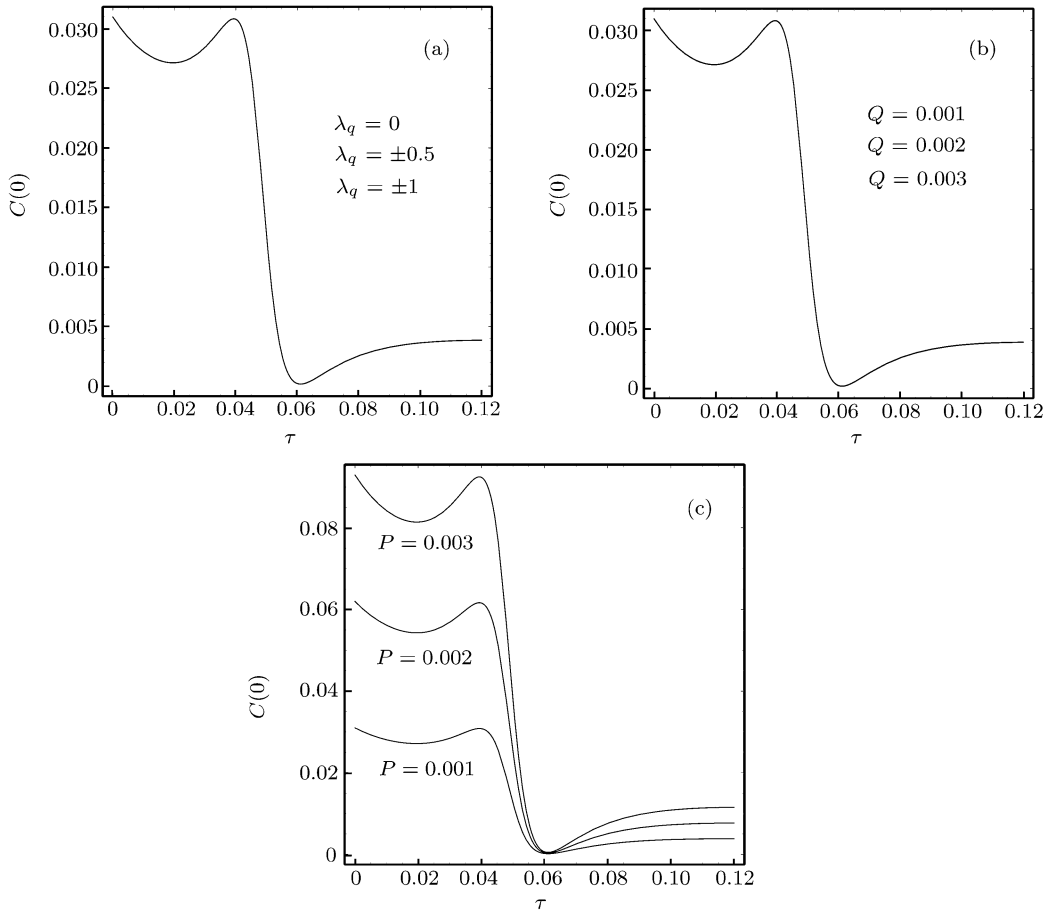


Fig.3. $C(0)$ versus τ for different values of λ_q, Q and P with the parameters $A = 1$, $B = 20$, $\Omega = 4$, $a_0 = 10$, other (a) $Q = 0.001$, $P = 0.001$; (b) $P = 0.001$, $\lambda_q = 0.5$; (c) $Q = 0.001$, $\lambda_q = 0.5$.

3.4. The influence of the net gain coefficient a_0 on the SSMNIF $C(0)$ vs τ curve

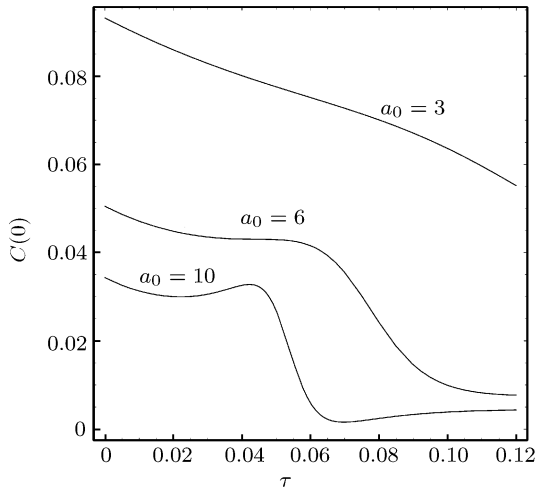


Fig.4. $C(0)$ versus τ for different values of a_0 with the parameters $A = 1$, $B = 20$, $Q = 0.001$, $P = 0.001$, $\Omega = 4$, $\lambda_q = 0.5$.

By choosing a_0 as a parameter, the $C(0)$ vs τ curve is plotted, as shown in Fig.4. It is seen from Fig.4 that, along with the parameters discussed, as a_0 increases, the whole curve falls down, and displays a changing process from monotonic descent to the appearance of two minima and one maximum. As $a_0 = 10$, the value of $C(0)$ will tend to zero near $\tau = 0.06$.

4. The ICT T changes with pump noise self-correlation time τ

The T vs τ curve is plotted based on Eq.(7), as shown in Fig.5. It is seen from Fig.1, along with the parameters discussed, the change of T with τ exhibits a complicated process. That is, the T vs τ curve has one minimum and one maximum. It is indicated that the 'coloured' pump noise has great influence on the statistical fluctuation of the laser system.

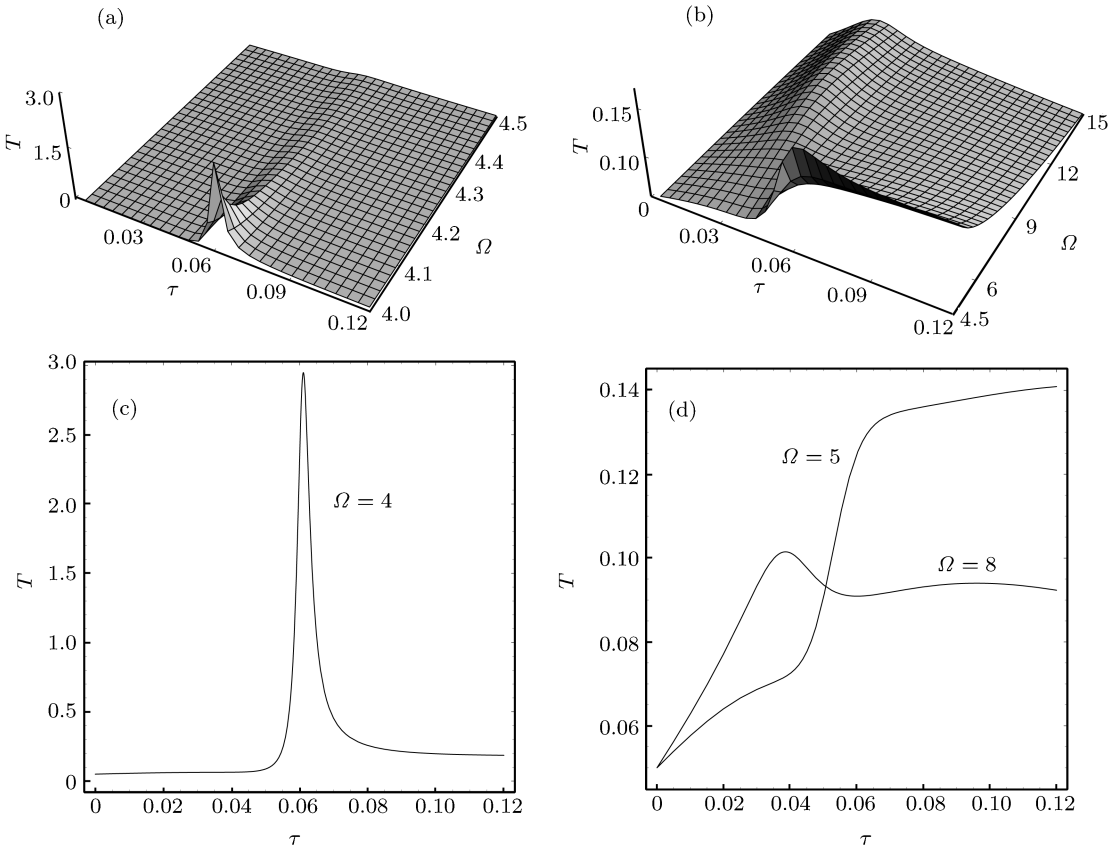


Fig.5. T versus τ for different values of Ω with the parameters $A = 1$, $Q = 0.001$, $P = 0.001$, $B = 20$, $a_0 = 10$, $\lambda_q = 0.5$.

4.1. The influence of frequency Ω of modulation signal on the T vs τ curve

The three-dimensional plots in Figs.5(a), (b) exhibit how the frequency Ω of modulation signal influences the change of T with τ. Figures 5(c), (d) are its sectional drawing. It is seen from Fig.5 that, with increasing Ω, there are changes of the size and the position of the maximum and the minimum. Moreover, as $\Omega = 4$, the T vs τ curve has an acute peak near $\tau = 0.06$, it is shown that the fluctuation of the laser system is very small and its output is relatively stable in the case of very long correlation time. It leads to the same conclusion as that in Section 3.1.

4.2. The influence of amplitude B of a modulation signal on the τ vs τ curve

Figure 6 exhibits how the amplitude B influences the change of T with τ. It is seen from Fig.6, compared with the laser system which has not input signal (the straight line in Fig.6 ($B = 0$)), it is found that

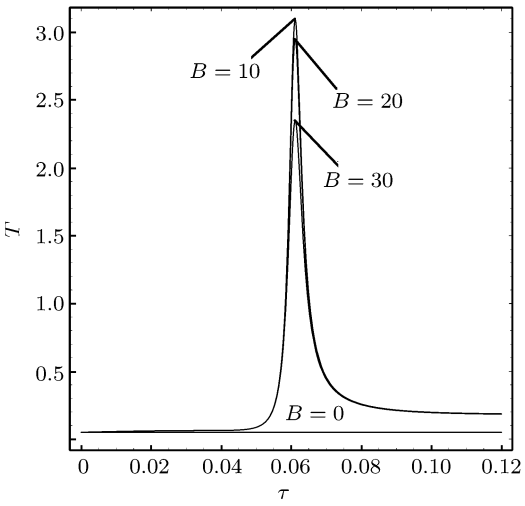


Fig.6. T versus τ for different values of B with the parameters $A = 1$, $Q = 0.001$, $P = 0.001$, $\Omega = 4$, $a_0 = 10$, $\lambda_q = 0.5$.

the value of T will increase after inputting signal into the laser system. This shows that the ICT is prolonged as a result of the interaction between the modulation signal and pump noise, and the statistical fluctuation of the laser system is suppressed. It leads to the same conclusion as that in Section 3.2.

Furthermore, when we analyse the influence of intensity of noises and net gain coefficient on the ICT we can also obtain some conclusions like those in Section 3.3 and 3.4.

5. Conclusion

The analysis and discussions above may be summarized as follows: 1) when the laser system has no input signal, the $C(0)$ vs τ curve and the T vs τ curve are all a straight line, which shows that the ' colour' of pump noise has no effect on the statistical fluctuation of the laser system; 2) after inputting signal into the laser system, $C(0)$ changes with τ and exhibits a complicated process, it shows the great influence of 'colour' of pump noise on the statistical fluctuation of the laser system; 3) when amplitude of signal $B < 40$, compared with the laser system which has no input signal, the interaction between the modulation signal and coloured pump noise may suppress the statistical fluctuation; in particular, the very small amplitude of modulation signal has a great suppressive action on the fluctuation; 4) the frequency of modulation signal has great influence on the statistical fluctuation; along with the parameters discussed, the certain frequency of the modulation signal ($\Omega = 4$) and the certain self-correlation time of pump noise ($\tau = 0.06$) have the result that the statistical fluctuation of laser system tends to zero; the laser system may maintain steady operating and output here; 5) the intensity of quantum noise and its cross-correlation between real and imaginary parts have less influence on the statistical fluctuation of the laser system, but the statistical fluctuation has been suppressed as the intensity of pump noise decreases; 6) the net gain a_0 is an important parameter in the description of the statistical property of the laser system, which reflects the operational state of the laser, when the laser system is far away the threshold, the statistical fluctuation of the laser system will decrease.

It should be noted that the discussions of this paper are based on the dimensionless self-saturation coefficient $A = 1$, which corresponds to the order of self-saturation coefficient being $10^6 \mathrm{s}^{-1}$.[19−21]

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