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Implementing the Deutsch–Jozsa algorithm by using Schrödinger cat states in cavity QED*

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We propose a scheme to implement the Deutsch–Jozsa algorithm by using Schrödinger cat states in cavity quantum electron-dynamics (QED). The scheme is based on the Raman interaction of a degenerate three-level Λ-type atom with a coherent state in a cavity. By using Schrödinger cat states, the atomic spontaneous emission can be minimized and the Hadamard transformation in our scheme is not needed.

Keywords: cavity QED, Schrödinger cat states, Deutsch–Jozsa algorithm
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1. Introduction

Compared with the classical computer, the quantum computer has a particular advantage: higher computing rapidity, especially in performing some complex algorithms[1−4] such as quantum factorization, Grover searching and the Deutsch–Jozsa algorithm. The Deutsch–Jozsa algorithm is a simpler quantum algorithm. It distinguishes whether a Boolean function $f(x)$ with 2^n inputs is constant or balanced. If $f(x)$ is equal for all values of x, it is constant; otherwise, $f(x)$ is balanced, that is, equal to 1 for exactly half of all the possible x, and 0 for the other half. To identify the function to be constant or balanced, a classical computer needs at least $2^{n-1}+1$ queries as there may be $2^{n-1}0$'s before finally 1 appears, showing that the function is balanced. In contrast, the Deutsch–Jozsa algorithm determines the function using only one query.

At present, there have been various schemes for implementing the Deutsch–Jozsa algorithm. For example, in experiment, implementing this algorithm by using the nuclear magnetic resonance technologies and trapped ions, etc., are reported;[5−7] in theory, the schemes for implementing the algorithm are proposed by using atomic ensembles or adiabatic evolution.[8,9] On the other hand, the cavity QED system is a qualified candidate for implementing quantum information processing.[10−14] Recently, Zheng[15] proposed a scheme to implement the algorithm in a cavity, which is based on the resonant interaction of a two-level atom with a cavity mode. In this paper, we propose a scheme to implement the Deutsch–Jozsa algorithm by using Schrödinger cat states[16,17] in cavity QED. In our scheme, the atomic spontaneous emission can be minimized for the atomic state remaining in the lower levels of the Λ-type three-level atom, and the Hadamard transformation is not needed.

2. Basic circuit of the Deutsch–Jozsa algorithm

We consider here the two-qubit Deutsch–Jozsa algorithm. The basic circuit of the algorithm is shown in Fig.1, which shows how the Deutsch–Jozsa algorithm evaluates the Boolean function $f(x)$ and gives the result. The input state (i) and the auxiliary qubit (j) are initially prepared in the state

$$|\psi_0\rangle = |0\rangle_i |1\rangle_j . \tag{1}$$

Then a Hadamard transformation is performed on each qubit

$$|0\rangle \to \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \quad |1\rangle \to \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle), \tag{2}$$

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leading the state $|\psi_0\rangle$ to

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)_i \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)_j . \tag{3}$$

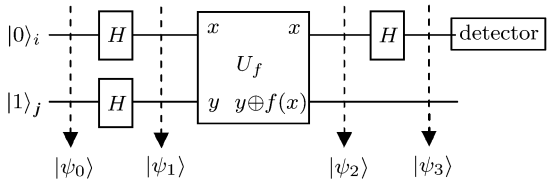


Fig.1. Basic circuit to perform the Deutsch–Jozsa algorithm. The detector measures the state $|0\rangle_i$ or $|1\rangle_i$.

Next, the function $f(x)$ is characterized by the unitary mapping transformation U_f

$$U_f : |x, y\rangle \to |x, y \oplus f(x)\rangle \tag{4}$$

where $\oplus$ represents addition modulo 2. The system state $|\psi_1\rangle$ under the unitary operation U_f becomes

$$\begin{aligned}|\psi_2\rangle =& \frac{1}{\sqrt{2}}\left[(-1)^{f(0)}|0\rangle_i + (-1)^{f(1)}|1\rangle_i\right] \\ & \otimes \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)_j .\end{aligned} \tag{5}$$

There are four possible transformations to U_f. The relation of $U_{f,n}$ (n=1, 2, 3, 4) and the function values is given as follows:

Constant function $\begin{cases} U_{f,1} \to f(0) = f(1) = 0, \\ U_{f,2} \to f(0) = f(1) = 1; \end{cases}$

Balanced function

$$\begin{cases} U_{f,3} \to f(0) = 0,\ f(1) = 1, \\ U_{f,4} \to f(0) = 1,\ f(1) = 0. \end{cases} \tag{6}$$

After that, a Hadamard transformation depicted by Eq.(2) is performed on the qubit i, which transforms the state $|\psi_2\rangle$ to the state

$$|\psi_3\rangle = \pm\,|f(0) \oplus f(1)\rangle_i \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)_j . \tag{7}$$

According to Eq.(7), if $f(0) = f(1)$, $|f(0)\oplus f(1)\rangle_i$ is $|0\rangle_i$, we can see that if the detector shows the output remains $|0\rangle_i$, the function is constant; otherwise it is balanced.

3. The model

We now consider a degenerate Λ-type three-level atom interacting with a single-mode cavity. The relevant levels and the transitions of the atom are shown in Fig.2. In the atom, the energies of two lower states $|e\rangle$ and $|g\rangle$ are equal. Assume that the atomic transition frequency ω_0 is highly detuned from the cavity field frequency ω_c, so that the upper-level state $|i\rangle$ can be eliminated adiabatically, and the atomic spontaneous emission can be minimized since the atomic states are only in the lower levels. Thus, in the interaction picture, the effective Hamiltonian for this system is given by[10,18,19]

$$\begin{aligned}H_{\text{eff}} =& -\lambda a^{+}a\left(|e\rangle\langle g| + |g\rangle\langle e|\right) \\ & - a^{+}a\left(\beta_1|e\rangle\langle e| + \beta_2|g\rangle\langle g|\right),\end{aligned} \tag{8}$$

where $\lambda = \dfrac{g_1 g_2}{\Delta}$, $\beta_1 = \dfrac{g_1^2}{\Delta}$, $\beta_2 = \dfrac{g_2^2}{\Delta}$, $\Delta = \omega_0 - \omega_c$, $a\,(a^{+})$ is an annihilation (creation) operator of the cavity field, $g_1\,(g_2)$ is the coupling constant between the atomic transition $|i\rangle \to |e\rangle$ $(|g\rangle)$ and the cavity mode. Suppose $g_1 = g_2 = g$, then $\lambda = \beta_1 = \beta_2 = \dfrac{g^2}{\Delta}$.

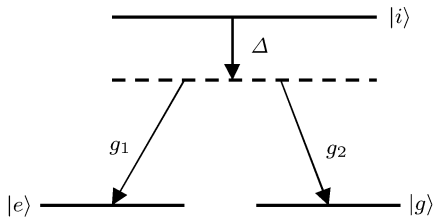


Fig.2. Schematic circuit of a degenerate Λ-type three-level atom interacting with a single-mode cavity.

Then the evolution of an atom–cavity system for different initial states is given by

$$\begin{aligned}|e\rangle|\alpha\rangle \to & \frac{1}{2}[(|\alpha\rangle + |\alpha e^{2i\lambda t}\rangle)|e\rangle \\ & - (|\alpha\rangle - |\alpha e^{-2i\lambda t}\rangle)|g\rangle];\end{aligned} \tag{9a}$$

$$\begin{aligned}|e\rangle|-\alpha\rangle \to & \frac{1}{2}[(|\alpha\rangle + |\alpha e^{2i\lambda t}\rangle)|e\rangle \\ & + (|\alpha\rangle - |\alpha e^{-2i\lambda t}\rangle)|g\rangle];\end{aligned} \tag{9b}$$

$$\begin{aligned}|g\rangle|\alpha\rangle \to & \frac{1}{2}[(|\alpha\rangle + |\alpha e^{2i\lambda t}\rangle)|g\rangle \\ & - (|\alpha\rangle - |\alpha e^{-2i\lambda t}\rangle)|e\rangle];\end{aligned} \tag{9c}$$

$$\begin{aligned}|g\rangle|-\alpha\rangle \to & \frac{1}{2}[(|\alpha\rangle + |\alpha e^{2i\lambda t}\rangle)|g\rangle \\ & + (|\alpha\rangle - |\alpha e^{-2i\lambda t}\rangle)|e\rangle],\end{aligned} \tag{9d}$$

where $|\alpha\rangle\,(|-\alpha\rangle)$ is a coherent state.

Thus, if a Λ-type atom in a superposition state

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|e\rangle \pm |g\rangle) \tag{10}$$

is projected into $|\alpha\rangle(|-\alpha\rangle)$ with the interaction time $t = \dfrac{\pi}{2\lambda}$, a CNOT operation will be implemented,

namely

$$|+\rangle|\alpha\rangle \rightarrow |+\rangle|-\alpha\rangle; \tag{11a}$$

$$|+\rangle|-\alpha\rangle \rightarrow |+\rangle|\alpha\rangle; \tag{11b}$$

$$|-\rangle|\alpha\rangle \rightarrow |-\rangle|\alpha\rangle; \tag{11c}$$

$$|-\rangle|-\alpha\rangle \rightarrow |-\rangle|-\alpha\rangle. \tag{11d}$$

showing that if and only if the atomic state is $|+\rangle$, the coherent state will change.

4. Implementation of the Deutsch–Jozsa algorithm

To implement the Deutsch–Jozsa algorithm, we encode the input qubit in the atomic state $|+\rangle$ ($|-\rangle$) and the auxiliary qubit in coherent state$|\alpha\rangle$ ($|-\alpha\rangle$). A schematic diagram of implementing the Deutsch–Jozsa algorithm is shown in Fig.3. The atom is initially prepared in the state $|e\rangle$and the cavity mode in the Schrödinger cat states[16,17] $\frac{1}{\sqrt{2}}(|\alpha\rangle - |-\alpha\rangle)$. Thus the state of the atom–cavity system is

$$|\phi_1\rangle = \frac{1}{\sqrt{2}}|e\rangle(|\alpha\rangle - |-\alpha\rangle), \tag{12}$$

which can be rewritten as[20]

$$|\phi_1\rangle = \frac{1}{2}(|+\rangle + |-\rangle)(|\alpha\rangle - |-\alpha\rangle). \tag{13}$$

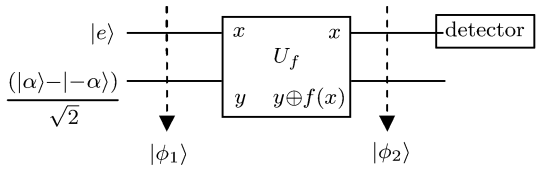


Fig.3. The circuit of implementing the Deutsch–Jozsa algorithm in our scheme. The detector measures the atomic state in the basis $\{|e\rangle, |g\rangle\}$.

It is clear that the state $|\phi_1\rangle$ has the same form as Eq.(3) only if we suppose

$$|+\rangle = |0\rangle_i, \quad |-\rangle = |1\rangle_i, \tag{14a}$$

$$|\alpha\rangle = |0\rangle_j, \quad |-\alpha\rangle = |1\rangle_j. \tag{14b}$$

The same rule is suitable for the following passage.

Then a unitary operation U_f is performed on the atom–cavity system.

(a) For the case $f(0) = f(1) = 0$, U_{f1} is an identity operation. So it does not require any interaction between the atom and the cavity at all[8] and the whole system output state $|\phi_2\rangle$ remains in the initial state $|\phi_1\rangle$. Namely

$$\begin{aligned}|\phi_2\rangle =& \frac{1}{\sqrt{2}}|e\rangle(|\alpha\rangle - |-\alpha\rangle) \\ =& \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)\frac{1}{\sqrt{2}}(|\alpha\rangle - |-\alpha\rangle).\end{aligned} \tag{15}$$

(b) For the case $f(0) = f(1) = 1$, we firstly inject the atom into the cavity with the interaction time $t = \frac{\pi}{2\lambda}$ to implement a CNOT operation (11), which leads the system state $|\phi_1\rangle$ to

$$-|g\rangle\frac{1}{\sqrt{2}}(|\alpha\rangle - |-\alpha\rangle). \tag{16}$$

Secondly, we perform a transformation

$$|g\rangle \rightarrow |e\rangle \tag{17}$$

with a classical field. The system state depicted by Eq.(16) evolves to

$$\begin{aligned}|\phi_2\rangle =& -|e\rangle\frac{1}{\sqrt{2}}(|\alpha\rangle - |-\alpha\rangle) \\ =& -\frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)\frac{1}{\sqrt{2}}(|\alpha\rangle - |-\alpha\rangle).\end{aligned} \tag{18}$$

(c) For the case $f(0) = 0,\ f(1) = 1$, we firstly implement a CNOT operation on the system state (13), and then let the atom go through a classical field to implement the transformation

$$|g\rangle \rightarrow -|g\rangle. \tag{19}$$

After that, the system state $|\phi_1\rangle$ becomes

$$\begin{aligned}|\phi_2\rangle =& |g\rangle\frac{1}{\sqrt{2}}(|\alpha\rangle - |-\alpha\rangle) \\ =& \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle)\frac{1}{\sqrt{2}}(|\alpha\rangle - |-\alpha\rangle),\end{aligned} \tag{20}$$

(d) For the case $f(0) = 1,\ f(1) = 0$, the atom is injected into the cavity to implement the CNOT operation (18), the system will evolve to the state (16), namely

$$\begin{aligned}|\phi_2\rangle =& -|g\rangle\frac{1}{\sqrt{2}}(|\alpha\rangle - |-\alpha\rangle) \\ =& -\frac{1}{\sqrt{2}}(|+\rangle - |-\rangle)\frac{1}{\sqrt{2}}(|\alpha\rangle - |-\alpha\rangle).\end{aligned} \tag{21}$$

Making a simple calculation, we can rewrite Eqs.(15), (18), (20), (21) into a formula similar to Eq.(5).

Since the atomic qubit in the state $|\phi_2\rangle$ is actually in the state $|e\rangle$ or $|g\rangle$, we need not make a Hadamard operation on it, but detect it in the basis $\{|e\rangle, |g\rangle\}$. If the measurement result is $|e\rangle$, namely$|\phi_2\rangle = \pm\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\frac{1}{\sqrt{2}}(|\alpha\rangle - |-\alpha\rangle)$, we can come to the conclusion that the function is constant; otherwise, the it is balanced.

5. Conclusion

In summary, we have proposed a scheme to implement the Deutsch–Jozsa algorithm by using Schrödinger cat states. Compared with the previous schemes, two distinct features of our scheme are that the atomic spontaneous emission is minimized because the atomic state remains in the lower levels of the Λ-type three-level atom, and the Hadamard transformation is not needed. Thus, we can implement the Deutsch–Jozsa algorithm in a simpler way.

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