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Passive control of chaotic system with multiple strange attractors*

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In this paper we present a new simple controller for a chaotic system, that is, the Newton–Leipnik equation with two strange attractors: the upper attractor (UA) and the lower attractor (LA). The controller design is based on the passive technique. The final structure of this controller for original stabilization has a simple nonlinear feedback form. Using a passive method, we prove the stability of a closed-loop system. Based on the controller derived from the passive principle, we investigate three different kinds of chaotic control of the system, separately: the original control forcing the chaotic motion to settle down to the origin from an arbitrary position of the phase space; the chaotic intra-attractor control for stabilizing the equilibrium points only belonging to the upper chaotic attractor or the lower chaotic one, and the inter-attractor control for compelling the chaotic oscillation from one basin to another one. Both theoretical analysis and simulation results verify the validity of the suggested method.

Keywords: chaos, passive control, the Newton–Leipnik equation attractor
PACC: 0545, 0547

1. Introduction

Chaotic control is one of the topics acquiring great important position and much attention in physics and engineering publications. Many researchers have used modern elegant theories to control chaotic systems. Linear state feedback is easily implemented for the nonlinear chaotic systems.[1] The Lyapunov-type method is a more general synthesis approach to nonlinear controller design.[2] The feedback linearization technique is an effective nonlinear geometric theory for nonlinear chaos controls.[3] Adaptive control methods were proposed in Ref.[4], where some parameters of chaotic systems were unknown. The bang-bang controller was also possible to drive some of chaotic oscillators such as a Lorenz system being driven to equilibrium within a short time.[5] Prediction feedback control outperformed delay control in driving a chaotic system to the anticipated orbits on account of its relevance to the historical information of chaotic motion.[6] The inverse system method is found to be more suitable for chaotic control as long as the inverse model of the controlled chaotic system is available.[7]

Recently, the concept of passivity of nonlinear systems has aroused the new interest in nonlinear controls.[8] Consider a nonlinear affine system

$$\begin{aligned} \dot{x} &= f(x) + g(x)u, \\ y &= h(x), \end{aligned} \tag{1}$$

where the state $x \in \Re^n$, the input $u \in \Re^m$, and output $y \in \Re^m$. Both f and the m column of g are smooth vector fields, h is the smooth mapping. We suppose that the vector field f has at least one equilibrium point, and without loss of generality we can assume that the equilibrium point is $x = 0$. If the equilibrium point is not at $x = 0$, we can shift the equilibrium point to $x = 0$ by coordinate transform.

The system in the form of Eq.(1) is said to be passive if there exists a real-valued constant β such that for any $\forall t \geq 0$ the following inequality holds:

$$\int_0^t u^{\mathrm{T}}(\tau)y(\tau)\mathrm{d}\tau \geq \beta,$$

or, there exist a constant $\rho > 0$ and a real-valued constant β such that

$$\int_0^t u^{\mathrm{T}}(\tau)y(\tau)\mathrm{d}\tau + \beta \geq \rho \int_0^t y^{\mathrm{T}}(\tau)y(\tau), \tag{2}$$

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whose physical meaning is that the energy of the nonlinear system can be increased only through the supply from an external source. The stability properties of passive systems are well known.[9,10] Once the system has been rendered passive, the controller law such as $u = -\varphi(y)$ can asymptotically stabilize the equilibrium point $x = 0$ of the nonlinear system (1).

In this paper we use the passive technique to control a chaotic system: the Newton–Leipnik system. The controller proposed here is in a very simple nonlinear feedback form. Using the passive method it is also very easy to prove the stability of a closed-loop system.

The rest of the paper is organized as follows. In Section 2 the passive control of the Newton–Leipnik system is described in detail. Namely, a procedure to design a feedback passivation controller is presented. In Section 3, the numerical results are given. Finally, conclusions are drawn from the present study in Section 4.

2. Passive control of the Newton–Leipnik system

If the system (1) has relative degree [1,...,1] at x=0 ($L_g h(0)$ is nonsingular) and the distribution spanned by the vector field $g_1(x), ..., g_m(x)$ is involutive, then it can be represented in the so-called normal form

$$\begin{aligned} \dot{z} &= f_0(z) + p(z, y)y, \\ \dot{y} &= b(z, y) + a(z, y)u. \end{aligned} \tag{3}$$

The nonlinear system (3) may be rendered passive by using a state feedback in the form[11]

$$u = \alpha(x) + \beta(x)v. \tag{4}$$

Now we use this idea to control the chaos in the Newton–Leipnik system. The Newton–Leipnik system model was obtained by modifying Euler's rigid body equations with the addition of a linear feedback in 1981.[12] The controlled differential equations of the Newton–Leipnik chaotic system are as follows:[12]

$$\begin{aligned} \dot{x}_1 &= -\gamma x_1 + x_2 + 10x_2x_3, \\ \dot{x}_2 &= -x_1 - 0.4x_2 + 5x_1x_3, \\ \dot{x}_3 &= \sigma x_3 - 5x_1x_2 + u, \end{aligned} \tag{5}$$

where x_1, x_2 and x_3 represent angular velocities about principal axes, which are fixed in the body, γ and σ are positive parameters that are related to the eigenvalues of inertia moments of the body. The Newton–Leipnik system model shows a double strange attractor for the nominal parameters $\gamma = 0.4$ and $\sigma = 0.175$ when control action is not applied. For the initial states $x_0^{(1)} = (0.349, 0, -0.160)^{\mathrm{T}}$ and $x_0^{(2)} = (0.349, 0, -0.180)^{\mathrm{T}}$ we obtain two strange attractors, see Fig.1. As discussed in the previous work on the Newton–Leipnik system,[12,13] we refer to the attractor A1 in the upper space as the upper attractor (UA) and the attractor A2 in the lower space as the lower attractor (LA). The equilibrium points of the system (5) with the nominal parameters $\gamma = 0.4$ and $\sigma = 0.175$ are

$$\begin{aligned} x_{\mathrm{e}}^{(1)} &= (0, 0, 0)^{\mathrm{T}}, \\ x_{\mathrm{e}}^{(2)} &= (0.2390, 0.0308, 0.2103)^{\mathrm{T}}, \\ x_{\mathrm{e}}^{(3)} &= (-0.2390, -0.0308, 0.2103)^{\mathrm{T}}, \\ x_{\mathrm{e}}^{(4)} &= (0.0315, -0.1224, -0.1103)^{\mathrm{T}}, \\ x_{\mathrm{e}}^{(5)} &= (-0.0315, 0.1224, -0.1103)^{\mathrm{T}}. \end{aligned}$$

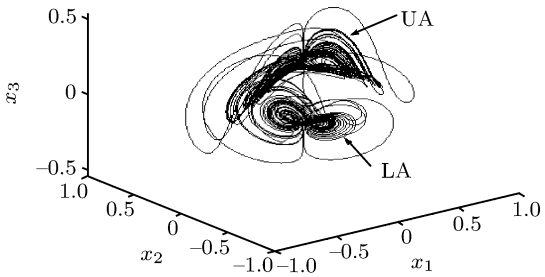


Fig.1. The double strange attractors of the Newton–Leipnik system.

All these equilibrium points are unstable. Note that the points $x_{\mathrm{e}}^{(2)}$ and $x_{\mathrm{e}}^{(3)}$ lie in the eye-centres of UA, while the points $x_{\mathrm{e}}^{(4)}$ and $x_{\mathrm{e}}^{(5)}$ are situated in the eye-centres of LA.

Theorem 1 The chaotic system (5) is a minimum phase system.

Proof Apply the control action u to the third equation of Eq.(5). Suppose that the state component x_3 is the system output, and $z_1 = x_1$, $z_2 = x_2$, $y = x_3$, then the Newton–Leipnik equation (5) can be rewritten in the normal form of Eq.(3)

$$\begin{aligned} \dot{z}_1 &= -\gamma z_1 + z_2 + 10z_2y, \\ \dot{z}_2 &= -z_1 - 0.4z_2 + 5z_1y, \\ \dot{y} &= \sigma y - 5z_1z_2 + u. \end{aligned}$$

We have

$$\begin{aligned} \dot{z} &= f_0(z) + p(z, y)y, \\ \dot{y} &= b(z, y) + a(z, y)u, \end{aligned}$$

where

$$\begin{aligned} &f_0(z) = [-\gamma z_1 + z_2, -z_1 - 0.4z_2]^{\mathrm{T}}, \\ &p(z,y) = [10z_2, 5z_1]^{\mathrm{T}}, \\ &a(z,y) = 1, \\ &b(z,y) = \sigma y - 5z_1 z_2. \end{aligned} \tag{6}$$

Our goal is to design a smooth control (4) for the Newton–Leipnik equation to make a closed-loop system passive. Choose a storage function candidate

$$\boldsymbol{V}(z,y) = \boldsymbol{W}(z) + \frac{1}{2}y^2, \tag{7}$$

where

$$\boldsymbol{W}(z) = \frac{1}{2}(z_1^2 + z_2^2)$$

is the Lyapunov function with $\boldsymbol{W}(0) = 0$.

The zero dynamics of the Newton–Leipnik system (3) describes the internal dynamics, which is consistent with external constraint y=0, i.e.

$$\dot{z} = f_0(z). \tag{8}$$

Considering Eq.(8) and both γ and σ being positive constants, we have

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{W}(z) =& \frac{\partial}{\partial z}\boldsymbol{W}(z) f_0(z) \\ =& [z_1, z_2][-\gamma z_1 + z_2, -z_1 - 0.4z_2]^{\mathrm{T}}, \\ =& -\gamma z_1^2 - 0.4z_2^2 \leq 0. \end{aligned} \tag{9}$$

Because $\boldsymbol{W}(z) \leq 0$, it can be concluded that $\boldsymbol{W}(z)$ is the Lyapunov function of $f_0(z)$ and that $f_0(z)$ is globally asymptotically stable. The zero dynamics of the Newton-Leipnik system is Lyapunov stable, i.e. the Newton-Leipnik system is in a minimum phase.[9] Meanwhile, $L_g h(0) = 1 \neq 0$, i.e. $L_g h(0)$ is nonsingular, and the system has relative degree [1, ..., 1], so the system can be equivalent to a passive system.

Theorem 2 The chaotic system (5) can be equivalent to a passive system using state feedback, therefore it can be globally asymptotically stabilized.

Proof According to Theorem 1, the chaotic dynamical system (5) is a minimum phase system. The derivative of $\boldsymbol{V}(z,y)$ along the trajectory of the Newton–Leipnik system (5) is

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{V}(z,y) \leq& \frac{\partial}{\partial z}\boldsymbol{W}(z)\dot{z} + y\dot{y}, \\ =& \frac{\partial}{\partial z}\boldsymbol{W}(z) f_0(z) + \frac{\partial}{\partial z}\boldsymbol{W}(z) p(z,y) y \\ &+ [b(z,y) + a(z,y)u]y. \end{aligned} \tag{10}$$

As the Newton–Leipnik system is in a minimum phase

$$\frac{\partial}{\partial z}\boldsymbol{W}(z) f_0(z) \leq 0,$$

the inequality (10) becomes

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{V}(z,y) \leq& \frac{\partial \boldsymbol{W}(z)}{\partial z} p(z,y) y \\ &+ [b(z,y) + a(z,y)u]y. \end{aligned}$$

If we select the feedback control (4) to be in the following form and consider the expressions (6),[9,10] we have

$$\begin{aligned} u =& a^{-1}(z,y)\left[-b(z,y) - \frac{\partial \boldsymbol{W}}{\partial z} p(z,y) - \lambda y + v \right] \\ =& -\sigma y - 10 z_1 z_2 - \lambda y + v, \end{aligned} \tag{11}$$

where λ is a positive constant and v is an external input signal that is connected to the reference input. The above inequality can be rewritten as

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{V}(z,y) \leq vy - \lambda y^2. \tag{12}$$

Then, taking integration over both sides of the inequality (12) gives

$$\begin{aligned} &\boldsymbol{V}(z,y) - \boldsymbol{V}(z_0,y_0) \\ &\leq \int_0^t v(\tau) y(\tau) \mathrm{d}\tau - \int_0^t \lambda y(\tau)^2 \mathrm{d}\tau. \end{aligned} \tag{13}$$

As $\boldsymbol{V}(z,y) \geq 0$ let $\mu = \boldsymbol{V}(z_0,y_0)$, then we have

$$\begin{aligned} \int_0^t v(\tau) y(\tau) \mathrm{d}\tau + \mu \geq& \int_0^t \lambda y(\tau)^2 \mathrm{d}\tau + V(z,y) \\ \geq& \int_0^t \lambda y(\tau)^2 \mathrm{d}\tau, \end{aligned}$$

which satisfies the passive definition (2). Therefore, the Newton–Leipnik system (5) is rendered passive under the feedback control (11).

If an external signal v=0 (zero-input state response), from the inequality (13) we have

$$\boldsymbol{V}(z,y) \leq \boldsymbol{V}(z_0,y_0). \tag{14}$$

Because $\boldsymbol{W}(z)$ is radially unbounded, it follows from the function (7) that $\boldsymbol{V}(z,y)$ is also radially unbounded, so that the closed-loop system is bounded-state stable for $[z^{\mathrm{T}}, y]^{\mathrm{T}}$. This means that we can use the very simple passive control (11) to regulate the Newton–Leipnik system to the origin. If an external signal $v \neq 0$, then we want to steer the state of Newton–Leipnik system to a set-point, which is of one of the nontrivial equilibrium points $x_{\mathrm{e}}^{(i)} =$

$[x_{e1}^{(i)}, x_{e2}^{(i)}, x_{e3}^{(i)}]^{\mathrm{T}}$, ($i$=2, 3, 4, 5). Substituting Eq.(11) to Eq.(5), we have

$$\begin{aligned} & v = 15x_{e1}^{(i)}x_{e2}^{(i)} + \lambda x_{e3}^{(i)}, \\ & i \in I, \\ & I = \{2,3,4,5\}. \end{aligned} \tag{15}$$

In order to distinguish the stabilization of $x_e^{(2)}$ from $x_e^{(3)}$; $x_e^{(4)}$ from $x_e^{(5)}$, we need to add the constrained condition to Eq.(15), so it is converted into the following forms:

The added external signal to stabilize $x_e^{(2)}$ is in the following form:

$$\begin{aligned} & v = 15x_{e1}^{(i)}x_{e2}^{(i)} + \lambda x_{e3}^{(i)}, \quad x_1 > 0, \quad i = 2; \\ & v = 0, \quad x_1 \le 0. \end{aligned} \tag{16}$$

The added external signal to stabilize $x_e^{(3)}$ is in the following form:

$$\begin{aligned} & v = 15x_{e1}^{(i)}x_{e2}^{(i)} + \lambda x_{e3}^{(i)}, \quad x_1 < 0, \quad i = 3; \\ & v = 0, \quad x_1 \ge 0. \end{aligned} \tag{17}$$

The added external signal to stabilize $x_e^{(4)}$ is in the following form:

$$\begin{aligned} & v = 15x_{e1}^{(i)}x_{e2}^{(i)} + \lambda x_{e3}^{(i)}, \quad x_2 < 0, \quad i = 4; \\ & v = 0, \quad x_2 \ge 0. \end{aligned} \tag{18}$$

The added external signal to stabilize $x_e^{(5)}$ is in the following form:

$$\begin{aligned} & v = 15x_{e1}^{(i)}x_{e2}^{(i)} + \lambda x_{e3}^{(i)}, \quad x_2 > 0, \quad i = 5. \\ & v = 0, \quad x_2 \le 0. \end{aligned} \tag{19}$$

3. Simulation results

Usually, the parameters of the Newton–Leipnik system are selected as $\gamma = 0.4$, and $\sigma = 0.175$. First, let us check the zero-input state response control and the intra-attractor control. The zero-input state response control is to drive the complete dynamics to the origin. The Newton–Leipnik system starts from $x_0 = [0.5, -1, 2]^{\mathrm{T}}$. We select $\lambda = 2$ and $\lambda = 5$, and use the controller as described in the expression (11) with v=0. According to the inequality (14), the closed-loop system converges to origin $[0, 0, 0]^{\mathrm{T}}$; the results are shown in Fig.2.

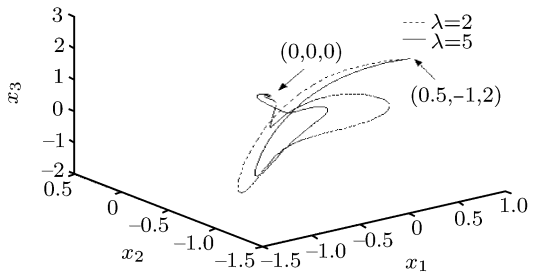


Fig.2. The Newton–Leipnik system is driven from (0.5, –1, 2) to the origin.

We note that the system is free of chaotic transients. Each of the states subject to passive control reaches a constant value in a short time and stays there in a long period. The positive constant λ only influences the transient dynamics. The bigger the value of λ, the faster the response is. The intra-attractor control is to stabilize the equilibrium points, which are affiliated with the upper attractor (UA) and the lower attractor (LA), respectively. Now we manage to force the Newton-Leipnik system into the set-points like $x_e^{(2)}$ and $x_e^{(3)}$, v=0.5310 when λ=2, the system states from (0.1, –0.1, 0.1) to $x_e^{(2)}$ and $x_e^{(3)}$ are drawn as shown in Fig.3 and Fig.4 respectively.

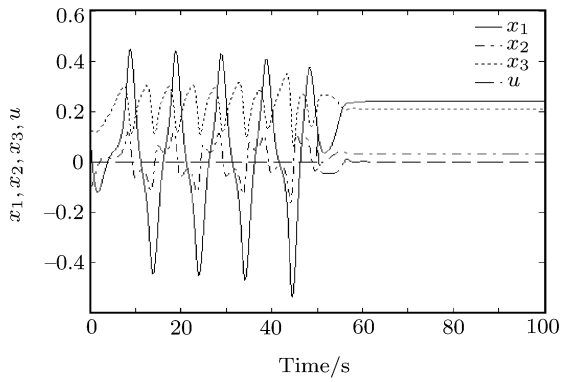


Fig.3. The Newton–Leipnik system is driven from (0.1, −0.1, 0.1) to $x_e^{(2)}$, control action is applied from t=50s on.

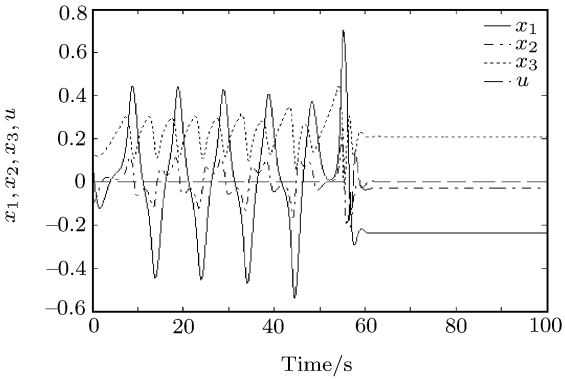


Fig.4. The Newton–Leipnik system is driven from (0.1, −0.1, 0.1) to $x_e^{(3)}$, control action is applied from t=50s on.

Similarly, if we want to drive the Newton-Leipnik system into the set-points like $x_e^{(4)}$ and $x_e^{(5)}$, $v = -0.2784$ when λ=2, the system states from (0.01, 0.15, –0.2) to $x_e^{(4)}$ and $x_e^{(5)}$ are drawn as shown in Fig.5 and Fig.6 respectively.

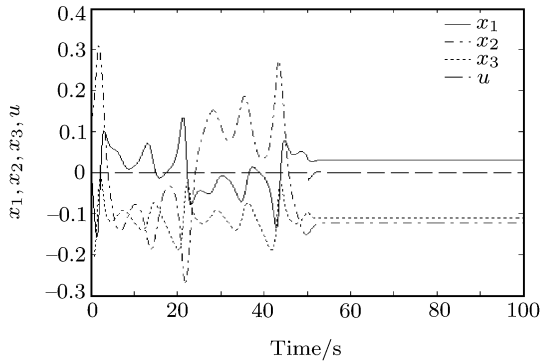


Fig.5. The Newton–Leipnik system is driven from (0.01, 0.15, –0.2) to $x_e^{(4)}$, control action is applied from t=50s on.

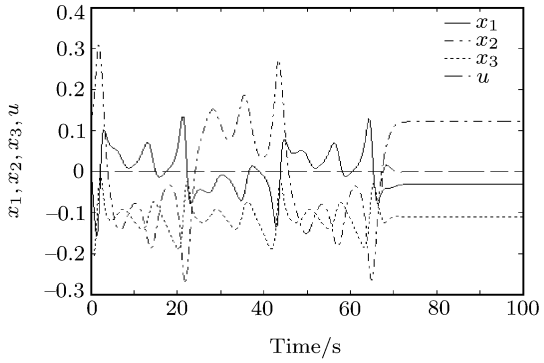


Fig.6. The Newton–Leipnik system is driven from (0.01, 0.15, –0.2) to $x_e^{(5)}$, control action is applied from t=50s on.

Note that once the Newton-Leipnik system is rendered a passive system, a very simple control modification (change the value of v) can drive the chaotic system to any set-point. Now we turn to an inter-attractor control (see Fig.7).

Starting from an initial state (0.349, 0, –0.160) belonging to the state basin of the upper attractor (UA), the trajectory settles down on this attractor after the transient. From t=50s on, the control (11) with v=0 is applied for the equilibrium of the origin, and at t=100s the control (18) is also turn on to enhance the prior control action for equilibrium point $x_e^{(4)}$ that is laying in the lower attractor (LA); while at t=300s all the control actions are set free from the system so that it returns to chaos again, this is because the accumulated error of the computation is enlarged by the Newton–Leipnik system.

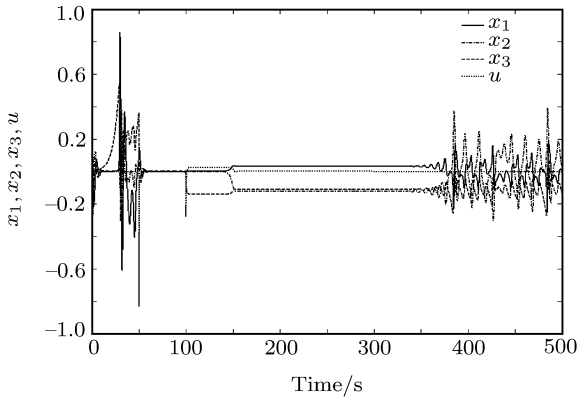


Fig.7. Inter-attractor control to take the system dynamics from UA to the origin at t=50s, and from the origin to the equilibrium $x_e^{(4)}$ of LA at t=100s; and goes back to chaos at t=300s again.

4. Conclusions

In this paper, the control of a chaotic system with multiple attractors is demonstrated. The analytical procedure is somewhat complex because we use a passive method, but the final structure is easily accepted: a simple nonlinear state feedback control. Using this special nonlinear controller we can assure the stability of a closed-loop system. Such a control can stabilize a desired motion within one attractor, as well as take the system dynamics from one attractor to another.

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