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Experimental observation of partial amplitude death in coupled chaotic oscillators*

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The dynamics of coupled Lorenz circuits is investigated experimentally. The partial amplitude death reported in *Phys. Rev.* E **72**, 057201 (2005) is verified by physical experiments with electronic circuits. With the increase of coupling constant, the coupled circuits undergo the transition from the breakdown of both the reflection symmetry and the translational symmetry to the partial amplitude death. Its stability is also confirmed by analysing the effects of noise.

Keywords: Partial amplitude death, chaotic oscillators, Lorenz circuit
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1. Introduction

Coupled oscillators constitute an effective and popular paradigm for the study of interacting oscillatory processes in the physical and biological sciences,[1,2] such as the coupled laser systems, Josephson junction arrays, electrical circuits, secure communications,[3−5] etc. Due to its importance and perspective of potential application, investigations on the coupled periodic oscillators have long been on the upsurge. However, efforts are turned to the study of the chaotic oscillators recently. Many researchers are attracted by the rich dynamics arising from the interaction between simple units, such as different types of synchronization (complete synchronization, phase synchronization, generalized synchronization, lag synchronization) in coupled periodical and chaotic oscillators, riddled basin, kinds of bifurcations of the synchronous chaos,[6−11] among which the most widely studied phenomenon is synchronization, which is realized when the originally independent oscillators have established functional relations due to their mutual interaction.[12] The synchronization may be observed even under weak coupling.

However, some coupled systems cannot realize chaos synchronization even when the interactions among the elements are strong enough. They will exhibit interesting dynamics such as amplitude death, which means that units of the coupled system evolve to a stable equilibrium.[13] Such behaviours have been widely observed in coupled systems such as limited-cycle oscillators, Lorenz oscillators, Rossler oscillators,[14] and chemical oscillator systems, e.g. coupled B–Z reactions carried out in coupled tank reactors experimentally.[15] Moreover, various mechanisms of amplitude death have been reported as the mismatch of the coupled oscillators' nature frequencies,[16] the time delay interaction caused by the finite speed of information transformation.[17] Recently, the amplitude death has been found even in the coupled identical oscillators due to the wide time scales of the subsystems.[18] The partial amplitude death, which is quite different in characteristics and regime from the amplitude death mentioned above, was reported also in an identical coupled Lorenz system in our previous work.[19] The strong interaction between two identical coupled Lorenz oscillators tends to break down both the reflection symmetry and the translational symmetry of coupled system, which makes one element oscillate with a large amplitude and the other element oscillates with small amplitude or even stays at rest-state, and as a result, the partial amplitude death occurs. In this paper, we experimentally investigate the dynamics in coupled Lorenz circuits in which the partial amplitude death can occur when the coupling is strong. The stability of partial amplitude death is explored by introducing a noise into the circuits.

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The rest of this paper is organized as follows. In Section 2, we present the experimental setup. In Section 3, we exhibit the experimental results and discuss the effects of noise on the state of partial amplitude death system. Finally, a brief conclusion drawn from the present study is given in Section 4.

2. Experimental setup

The analogue circuit representing the Lorenz oscillator as shown in Fig.1(a) is the same as that reported in Ref.[20]. It is composed of three integrators (TL082PC), each with a variable, denoted respectively by V_{C_1}, V_{C_2} and V_{C_3}. The nonlinear terms in the Lorenz equation are created by analogue multipliers (AD633/AD with a gain coefficient of 0.1). The tolerances of the capacity and resistance are 5% and 1% respectively. According to Kirchhoff's laws and the 'golden rules'[21] for the operational amplifiers (OPAMPS), the circuit can be described by the following equations:

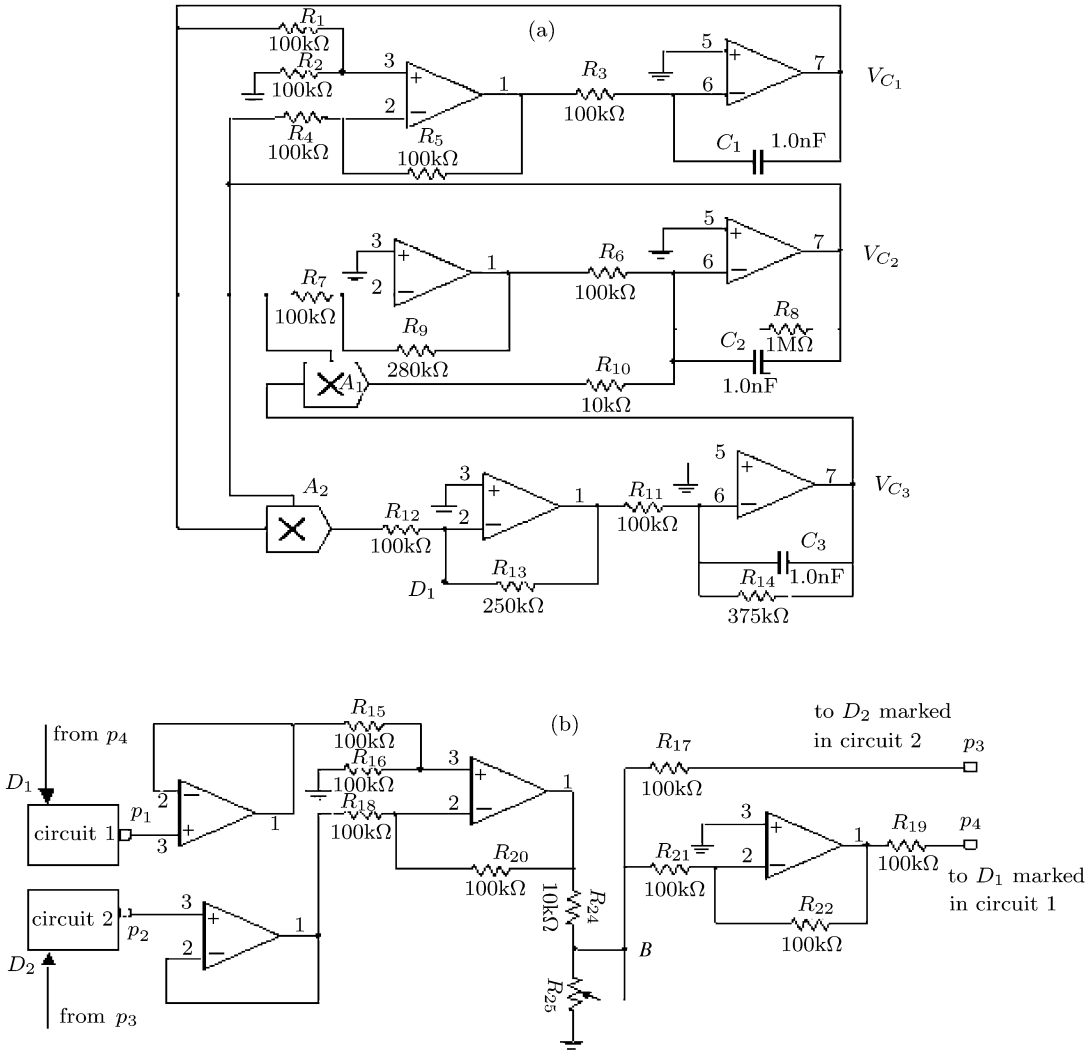


Fig.1. (a) The circuit (circuit 1) representing Lorenz equation, V_{C_1}, V_{C_2}, V_{C_3} are the output voltages representing the three variables in the rescaled Lorenz equation. (b) The coupling circuit.

$$
\begin{aligned}
V_{C_1} =& -\frac{1}{R_3C_1}\left[V_{C_1}\cdot\frac{R_2}{R_1+R_2}\left(1+\frac{R_5}{R_4}\right)-\frac{R_5}{R_4}V_{C_2}\right],\\
V_{C_2} =& -\frac{1}{R_8C_2}V_{C_2}+\frac{R_9/R_7}{R_6C_2}V_{C_1}-\frac{0.1}{R_{10}C_2}V_{C_1}V_{C_3},\\
V_{C_3} =& -\frac{1}{R_{14}C_3}V_{C_3}+\frac{R_{13}/R_{12}}{R_{11}C_3}\cdot 0.1\cdot V_{C_1}V_{C_2}. \qquad (1)
\end{aligned}
$$

To explore the behaviours of two coupled oscillators, we make a copy (named as circuit 2) of the circuit in Fig.1(a) and denote the points by $V_{C_1}^{(2)}$, $V_{C_2}^{(2)}$, $V_{C_3}^{(2)}$ and D_2 (figure of circuit 2 is not shown here) corresponding to points of V_{C_1}, V_{C_2}, V_{C_3} and D_1 marked in Fig.1(a). The circuit in Fig.1(b) is the connection circuit determining the coupling mode and coupling coefficient, which consists of two obstructers, one subtractor amplifier and one amplifier with a gain of -1. Signals from V_{C_2} and $V_{C_2}^{(1)}$ in the two Lorenz circuits are input by p_1 and p_2 and transmitted to places marked by D_2 (in circuit 2 which is not shown here) and D_1 in Fig.1(a) through p_3 and p_4 respectively. Thus, the coupled system can be described by the following equations:

$$
\begin{aligned}
V_{C_1} =& \sigma(V_{C_2}-V_{C_1}),\\
V_{C_2} =& RV_{C_1}-V_{C_2}-10V_{C_1}V_{C_3},\\
V_{C_3} =& 2.5V_{C_1}V_{C_2}-bV_{C_3}+\varepsilon(V_{C_2}^{(1)}-V_{C_2}),\\
V_{C_1}^{(1)} =& \sigma(V_{C_2}^{(1)}-V_{C_1}^{(1)}),\\
V_{C_2}^{(1)} =& RV_{C_1}^{(1)}-V_{C_2}^{(1)}-10V_{C_1}^{(1)}V_{C_3}^{(1)},\\
V_{C_3}^{(1)} =& 2.5V_{C_1}^{(1)}V_{C_2}^{(1)}-bV_{C_3}^{(1)}+\varepsilon(V_{C_2}-V_{C_2}^{(1)}). \qquad (2)
\end{aligned}
$$

According to the values of electronic elements shown in Fig.1, the parameters in Eq.(2) can be given as σ=10$(\Omega\cdot\mathrm{F})^{-1}$, R=28$(\Omega\cdot\mathrm{F})^{-1}$, and b=2.664$(\Omega\cdot\mathrm{F})^{-1}$, ε is couple coefficient and can be expressed as

$$
\varepsilon = 10\cdot\frac{R_{13}}{R_{19}}\cdot\frac{R_{25}}{R_{25}+R_{24}}=\frac{25R_{25}}{R_{25}+R_{24}}, \qquad (3)
$$

where R_{24}=10kΩ, R_{25} is a box-type rheostat with maximum value of 10kΩ by which we change the coupling coefficient. Owing to the voltage limitation of OP-AMPS, ε is not allowed to be larger than $\varepsilon_{\max}$ (it is less than 10 experimentally). Otherwise, the OP-AMPS will be saturated to distort the output wave. In the experiment, data acquisition is processed with 61110E DAQ card of National Instruments connected to a computer with a sampling rate of 60,000p/s and input limitation of $\pm$10V. Software in Labview is used to analyse the experimental data.

3. Experimental results

In this section we investigate the partial amplitude death in the two coupled Lorenz circuits. The coupling constant ε is determined by R_{25}, see Eq.(3). We change ε by varying R_{25} between zero and $R_{\max}$=5kΩ. With the increase of R_{25}, the dynamics of the two coupled circuits exhibits various phenomena such as the sequent breakdown of reflection symmetry and translational symmetry, crisis-induced intermittency and partial amplitude death. The phase portraits projected in the V_{C_1}-V_{C_3} plane for different R_{25} are shown in Fig.2, and well describe the transition to partial amplitude death. As the coupling is turned on, the reflection symmetry in any single Lorenz oscillator is broken down. One wing of the attractor expands and the trajectory spends more time on it as shown in Figs.2(a) and (b). At a critical value of R_{25} (R_{25}=2.7kΩ), a two-wing structure of the attractor disappears: one wing collapses and only the expanding wing survives as shown in Fig.2(c). However the translational symmetry between the two oscillators is still maintained. Further increasing the value of R_{25}, the translational symmetry is broken down also and two circuits behave as different attractors with different sizes. The process of the transition can be seen in Figs.2(c) and (d). When the R_{25} is adjusted to 3.3kΩ, an interesting state appears in which one of the attractors shrinks into a straight line parallel to the $V_{C_3}^{(1)}$ axis in Fig.2(e). It indicates that partial amplitude death occurs with $V_{C_1}^{(2)}$ being independent of time while $V_{C_3}^{(2)}$ still being time-dependent.

More knowledge about the behaviours described in Fig.2 can be gained by recording the time sequences of $V_{C_1}(V_{C_1}^{(2)})$ and $V_{C_2}(V_{C_2}^{(2)})$. $V_{C_2}(V_{C_2}^{(2)})$ behaviours are similar to $V_{C_1}(V_{C_1}^{(2)})$, and the results are not shown here. Before the translational symmetry is broken down, two oscillators jump between two wings of the attractor but they are not in synchronization with each other as seen in Figs.3(a) and (b). Moreover, two oscillators with the same attractor are in an antiphase state for a range of R_{25} (2–2.7kΩ) where one will oscillate with a large amplitude if the other oscillates with a small amplitude as shown in Fig.3(c). Each circuit jumps intermittently between the oscillations with a small and a large amplitude.

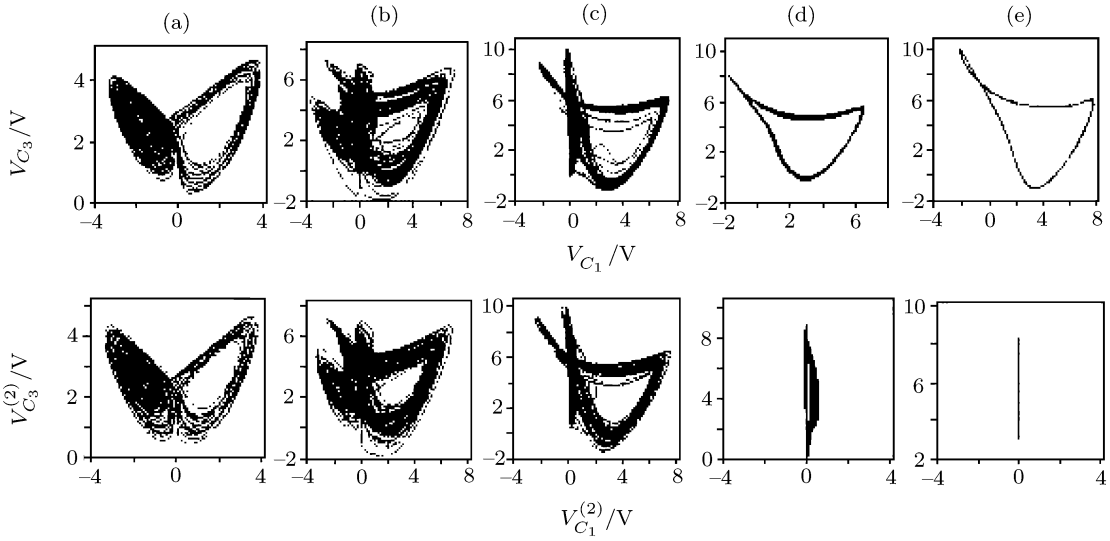


Fig.2. The phase portraits in V_{C_1}-V_{C_3} plane with different R_{25} (R_{25}=300Ω, 1.4kΩ, 2.7kΩ, 2.74kΩ, 3.3kΩ for (a)–(e) respectively). Top figures are for circuit 1 and the bottom ones for circuit 2.

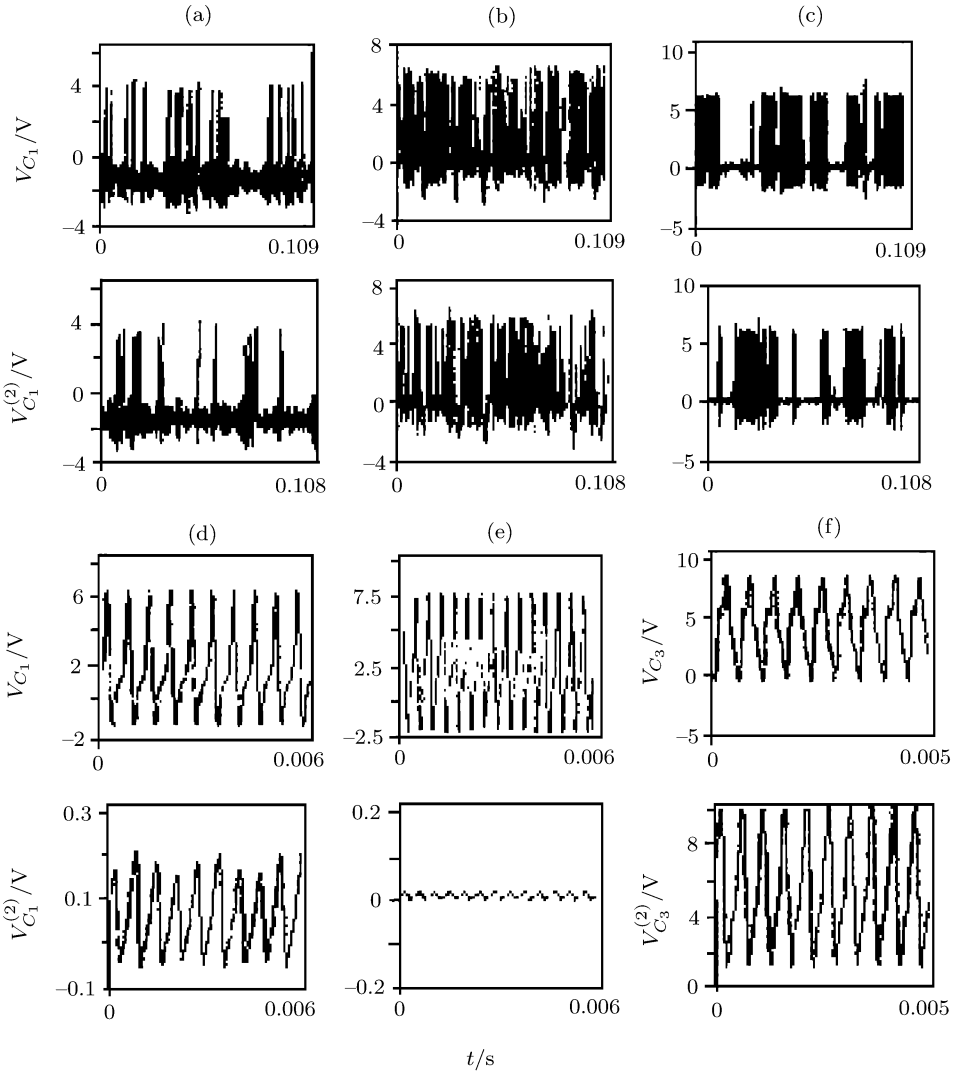


Fig.3. The time evolutions of the variable V_{C_1} are recorded with different R_{25} corresponding to (a)–(e) in Fig.2.(f) is for the time evolution of the variable V_{C_3} with the same R_{25} as that in (e). Top figures are for circuit 1 and the bottom ones for circuit 2.

With the increase of R_{25} (R_{25}=2.74kΩ), the jumps between the two kinds of oscillations become less frequent and eventually one oscillator maintains a large oscillation while the other retains a small oscillation as seen in Fig.3(d). The disappearance of the intermittency as $R_{25} > 3.2$kΩ indicates the breakdown of the translational symmetry. The temporal evolution of the trajectory in Fig.2(e) is shown in Fig.3(e) where the small oscillation of variable $V_{C_1}^{(2)}(V_{C_2}^{(2)})$ in one unit dies off (the weak oscillation in the bottom figure is caused by the small offset of the analogue multipliers in the experiment) while $V_{C_3}^{(2)}$ continues the oscillation with a high amplitude as shown in Fig.3(f). It is noticeable that the oscillator only stays at rest for the variables $V_{C_1}^{(2)}$ and $V_{C_2}^{(2)}$, while for the variable $V_{C_3}^{(2)}$ it still oscillates with a large amplitude comparable with that of the other oscillator. The phenomenon of the amplitude death in some dynamical variables is termed the partial amplitude death compared with the amplitude death found in coupled limit cycles[14,22] where the amplitude death refers to the state in which all variables of the element stay in a quenched state. As a result, the phenomenon named as the partial amplitude death in our previous work [19] is realized experimentally in the coupled identical Lorenz circuits.

The partial amplitude death is stable but sensitive to the noise. As a white noise with a zero mean value is applied to the coupled system, the system originally staying in the state of partial amplitude death will transit to oscillation, and then return to partial amplitude death when the noise is turned off. In Fig.4, we present the time series of $V_{C_1}^{(2)}$ with R_{25}=3.3kΩ. A white noise with an amplitude of 20mV generated by a functional generator (Agilent 33220A) is applied to both circuits for variables V_{C_3} and $V_{C_3}^{(2)}$ (The noise is applied at the point marked by B in Fig.1(b)) at the time t=17.676s and turned off at the time t=24.312s. The transition between the partial amplitude death and the oscillation is prominent, which means that the partial amplitude death is sensitive to noise and will damp back to the partial amplitude death rapidly as the noise disappears. The experimental results have verified the stability of the partial amplitude death in the two coupled Lorenz circuits, which is well consistent with the theoretical analysis presented in our previous work.[19]

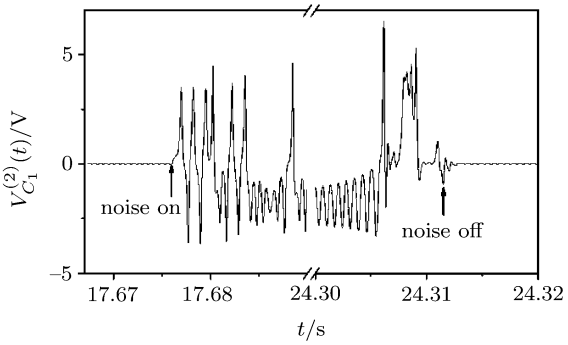


Fig.4. Time series of $V_{C_1}^{(2)}$ with R_{25}=3.3kΩ. White noise with an amplitude of 20mV is applied to the two coupled circuits at t=17.676s and turned off at t=24.312s.

4. Conclusion

We have investigated the dynamics of the coupled Lorenz circuits experimentally. The experimental result completely agrees with what is presented in our previous work.[19] We find that the translational symmetry of the coupled Lorenz circuits is broken down when the interaction between the oscillators becomes strong, which results in the partial amplitude death of the coupled Lorenz system. What is more, the partial amplitude death is completely determined by the dynamics of the coupled Lorenz system but not caused by the frequency response bandwidth limitation of the Lorenz circuit. Since the frequency response bandwidth of the circuit is mainly determined by the analogue multipliers (AD633/AD), whose frequency response bandwidth is 1MHz, while the main frequency of the Lorenz system is about 3kHz. The frequency response bandwidth of the Lorenz circuit is sufficient for the dynamics of the Lorenz system and will not influence the dynamics of the Lorenz system. Moreover, by applying a noise to the coupled circuits, we verify that the partial amplitude death is stable but sensitive to noise.

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