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Probabilistic teleportation of a non-symmetric three-particle state*

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This paper proposes a scheme for teleporting a kind of essential three-particle non-symmetric entangled state, which is much more valuable than a GHZ and W state for some applications in quantum information processing. In comparison with previous proposal of teleportation, the resources of entangled states as quantum channel and the number of classical messages required by our scheme can be cut down. Moreover, it is shown that there exists a class of transformations which ensure the success of this scheme, because the two-particle transformation performed by the receiver in the course of teleportation may be a generic two-particle operation instead of a control-NOT (CNOT) operation. In addition, all kinds of transformations performed by sender and receiver are given in detail.

Keywords: probabilistic teleportation, three-particle entangled state, non-maximally entangled state, unitary transformation

PACC: 0365, 0367

1. Introduction

Quantum entanglement sits at the heart of the quantum communication, and serves as one of the most crucial resources in many applications of quantum information processing. There are three important quantum communication schemes: quantum cryptography, quantum dense coding and quantum teleportation; all of them have utilized entanglement. Quantum cryptography can ensure the security of the key distribution and thus realize, together with the one-time pad scheme, absolutely secret communication.[1] Quantum dense coding[2] can make one send classical information more efficiently. And, with quantum teleportation which is perhaps the most striking example of the use of entanglement in assisting communication, one can transfer quantum information, namely, the quantum state itself, from one place to another.[3] Due attention has been paid to generate entanglement; nevertheless, quantum entanglement has to be transported among spatially separated spots in some applications, such as in quantum computation.[4] So, it is most advantageous to transmit the entanglement initially imposed on an unknown multipartite state to a multipartite state at a remote place in quantum information. Quantum teleportation is a nice way to transfer a quantum state from sender to receiver via a quantum channel with the help of local operations and classical communications.

Quantum teleportation has been considered to be one of the singular fruits of the burgeoning field of quantum information theory, since the seminal work of Bennett *et al* in 1993.[3] At present, various kinds of quantum teleportation have been proposed.[5−22] As far as pure three-qubit states is concerned, it has been proved that[23] three entangled qubits can be classified into two equivalent classes, such as GHZ-type and W-type which are all symmetric three-particle states, under stochastic local operations and classical communication transformations. It has been shown that, when the three-particle GHZ state is shared as quantum channel,[5] the fidelity of teleportation is $\langle F_{\rm GHZ}\rangle = 2/3 + (\sin(2\theta))/3$ and when the three-particle W state is utilized,[6] the fidelity is $\langle F_{\rm W}\rangle = 7/9$. At the

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same time, three-qubit states can be divided into five types [24], where non-symmetric entangled states appear. Recently, based on the three-qubit states classification by unitary transformation, Bae *et al* [25] considered three-party quantum teleportation with non-symmetric states, and found that when the three parties take the state $|\varphi\rangle = (|000\rangle + |101\rangle + |111\rangle)/\sqrt{3}$ as quantum channel, there was an optimal scenario which produces the fidelity $\langle F_\varphi\rangle = 8/9$. The fidelity between the state the first party wish to send and the state the final party received not only quantifies how successfully the parties accomplish quantum teleportation, but also implies the robustness of the quantum state. In other words, $|\varphi\rangle$ is an extremely useful and important state owing to $\langle F_\varphi\rangle > \langle F_{\rm W}\rangle > \langle F_{\rm GHZ}\rangle$. Teleportation about a general three-particle GHZ state,[17−20] that is $|\varphi_{\rm GHZ}\rangle = x\,|000\rangle + y\,|111\rangle$, or a general three-particle W state,[21] namely $|\varphi_{\rm W}\rangle = x\,|001\rangle + y\,|010\rangle + z\,|100\rangle$, have been investigated. So, it is very significant to directly transmit the following state which is a general state corresponding to $|\varphi\rangle$ state:

$$|\varphi\rangle_{123} = x_1\,|000\rangle_{123} + x_2\,|101\rangle_{123} + x_3\,|111\rangle_{123}\,,$$
$$|x_1|^2 + |x_2|^2 + |x_3|^3 = 1. \qquad (1)$$

Generally, any state of N-partite system can be teleported by using N pairs of entangled particles as quantum channel and the transmission classical information.[8] Particularly, the teleportation of an arbitrary three-particle entangled state has been investigated, when three entangled pairs are used as the quantum channel.[22] The important three-particle state $|\varphi\rangle_{123}$, which will be transmitted between two parties in our scheme, can be successfully teleported by using the protocol in Ref.[22] which is applicable to teleportation of an arbitrary three-particle entangled state. However, in comparison with the scheme in Ref.[22], the goal of our scheme is to weaken the requirement for the quantum channel, i.e. to use a reduced amount of entanglement shared between the two parties, and to decrease the transmission of classical messages in the course of completing the same task.

In this paper, we come up with a scheme for teleporting a non-symmetric three-particle entangled state which is of the utmost importance for some applications[25] in the quantum information. In contrast to the previous protocol,[22] which used three pairs of entangled particles as quantum channels, our scheme need a small number of entanglement, two pairs of non-maximally entangled states, to achieve the same task. Furthermore, only five classical bits recording the outcome of sender's measurement should be sent to the receiver in this scheme, while six classical bits are required by the protocol in Ref.[22]. Therefore, the present scheme may be more easily implemented than the previous scheme[22] due to the fact that the resources of entangled states as quantum channel and the number of classical information transmitted can be cut down. In addition, it will be shown that there exists a class of transformations which ensure the success of our scheme, because Bob can perform different two-particle transformation involved in the process of teleportation. Finally, all kinds of transformations that Alice and Bob need to do are given in detail.

2. Teleportation of a three-particle entangled state

Suppose that an unknown three-particle state represented by Eq.(1) needs to be transmitted between two parties, traditionally called Alice (sender) and Bob (receiver), who are spatially separated.

Of course, use of a maximally entangled state as quantum channel makes the teleportation procedure perfect. However, because of the impossibility of blocking the quantum state from interacting with the environment, the initially maximally entangled states will evolve into a non-maximally entangled state or a more general case, mixed state. So, in our scheme the two partially entangled states

$$|\phi\rangle_{45} = a\,|00\rangle_{45} + b\,|11\rangle_{45}\,, \qquad (2)$$

$$|\phi\rangle_{67} = c\,|00\rangle_{67} + d\,|11\rangle_{67}\,, \qquad (3)$$

are utilized as quantum channels, where a and b satisfy $|a| \geq |b|$, $|a|^2 + |b|^2 = 1$, c and d satisfy $|c| \geq |d|$, $|c|^2 + |d|^2 = 1$. Alice is in possession of particles 1, 2, 3, 4 and 6, while particles 5 and 7 are in Bob's possession. Therefore, the state of the whole system composed of particles 1, 2, 3, and the quantum channel at the beginning of the protocol is simply

$$|\Phi\rangle_{1234567} = |\varphi\rangle_{123} \otimes |\phi\rangle_{45} \otimes |\phi\rangle_{67}\,. \qquad (4)$$

In order to teleport the three-particle state $|\varphi\rangle_{123}$, firstly, Alice performs joint measurements on particles 26 and 34 in the Bell basis. As a result, the state of particles 1, 5 and 7 collapses into one of the following unnormalized states, at random (16 kinds in all):

$$\left\langle \Phi^{+}\right|_{34}\left\langle \Phi^{\pm}\right|_{26}\Phi\rangle_{1234567}=\frac{1}{2}(x_1ac\left|000\right\rangle_{175}+x_2bc\left|101\right\rangle_{175}\pm x_3bd\left|111\right\rangle_{175}), \tag{5}$$

$$\left\langle \Phi^{-}\right|_{34}\left\langle \Phi^{\pm}\right|_{26}\Phi\rangle_{1234567}=\frac{1}{2}(x_1ac\left|000\right\rangle_{175}-x_2bc\left|101\right\rangle_{175}\mp x_3bd\left|111\right\rangle_{175}), \tag{6}$$

$$\left\langle \Psi^{+}\right|_{34}\left\langle \Phi^{\pm}\right|_{26}\Phi\rangle_{1234567}=\frac{1}{2}(x_1bc\left|001\right\rangle_{175}+x_2ac\left|100\right\rangle_{175}\pm x_3ad\left|110\right\rangle_{175}), \tag{7}$$

$$\left\langle \Psi^{-}\right|_{34}\left\langle \Phi^{\pm}\right|_{26}\Phi\rangle_{1234567}=\frac{1}{2}(x_1bc\left|001\right\rangle_{175}-x_2ac\left|100\right\rangle_{175}\mp x_3ad\left|110\right\rangle_{175}), \tag{8}$$

$$\left\langle \Phi^{+}\right|_{34}\left\langle \Psi^{\pm}\right|_{26}\Phi\rangle_{1234567}=\frac{1}{2}(x_1ad\left|010\right\rangle_{175}+x_2bd\left|111\right\rangle_{175}\pm x_3bc\left|101\right\rangle_{175}), \tag{9}$$

$$\left\langle \Phi^{-}\right|_{34}\left\langle \Psi^{\pm}\right|_{26}\Phi\rangle_{1234567}=\frac{1}{2}(x_1ad\left|010\right\rangle_{175}-x_2bd\left|111\right\rangle_{175}\mp x_3bc\left|101\right\rangle_{175}), \tag{10}$$

$$\left\langle \Psi^{+}\right|_{34}\left\langle \Psi^{\pm}\right|_{26}\Phi\rangle_{1234567}=\frac{1}{2}(x_1bd\left|011\right\rangle_{175}+x_2ad\left|110\right\rangle_{175}\pm x_3ac\left|100\right\rangle_{175}), \tag{11}$$

$$\left\langle \Psi^{-}\right|_{34}\left\langle \Psi^{\pm}\right|_{26}\Phi\rangle_{1234567}=\frac{1}{2}(x_1bd\left|011\right\rangle_{175}-x_2ad\left|110\right\rangle_{175}\mp x_3ac\left|100\right\rangle_{175}). \tag{12}$$

The four Bell states of particles 26, 34 can be expressed as $\left|\Phi^{\pm}\right\rangle_{ij}=(\left|00\right\rangle_{ij}\pm\left|11\right\rangle_{ij})/\sqrt{2}$, $\left|\Psi^{\pm}\right\rangle_{ij}=(\left|01\right\rangle_{ij}\pm\left|10\right\rangle_{ij})/\sqrt{2}$, where ij=26 and 34.

Without loss of generality, suppose that the state of particles 1, 5 and 7 conditioned on Alice's measurement result $\left|\Phi^{+}\right\rangle_{34},\left|\Psi^{+}\right\rangle_{26}$ is

$$\left\langle \Psi^{+}\right|_{34}\left\langle \Psi^{+}\right|_{26}\Phi\rangle_{1234567}=\frac{1}{2}(x_1bd\left|011\right\rangle_{175}+x_2ad\left|110\right\rangle_{175}+x_3ac\left|100\right\rangle_{175}). \tag{13}$$

In order to separate particle 1 from the entanglement of particles 1, 5 and 7, Alice performs a Hadamard operation on particle 1, that is $H\left|0\right\rangle_1=(\left|0\right\rangle_1+\left|1\right\rangle_1)/\sqrt{2}$, $H\left|1\right\rangle_1=(\left|0\right\rangle_1-\left|1\right\rangle_1)/\sqrt{2}$. Afterwards, the state expressed in Eq.(13) is transformed into

$$\begin{aligned}\left|\varphi\right\rangle_{157}=&\frac{1}{2\sqrt{2}}[(x_1bd\left|11\right\rangle_{75}+x_2ad\left|10\right\rangle_{75}\\&+x_3ac\left|00\right\rangle_{75})\otimes\left|0\right\rangle_1\\&+(x_1bd\left|11\right\rangle_{75}-x_2ad\left|10\right\rangle_{75}\\&-x_3ac\left|00\right\rangle_{75})\otimes\left|1\right\rangle_1].\end{aligned} \tag{14}$$

Then, Alice measures particle 1 in the basis$\{\left|0\right\rangle,\left|1\right\rangle\}$. The state of particles 5 and 7 conditioned on Alice's measurement result $\left|0\right\rangle_1$ collapses into

$$\frac{1}{2\sqrt{2}}(x_1bd\left|11\right\rangle_{75}+x_2ad\left|10\right\rangle_{75}+x_3ac\left|00\right\rangle_{75}). \tag{15}$$

Otherwise, the state will be

$$\frac{1}{2\sqrt{2}}(x_1bd\left|11\right\rangle_{75}-x_2ad\left|10\right\rangle_{75}-x_3ac\left|00\right\rangle_{75}). \tag{16}$$

After that, in order to make Bob know that he acquires the state with certainty, via classical channel Alice sends Bob a message instructing him the outcomes of her measurement, which include five classical bits, i.e. the results of two Bell-state measurements of particles 16 and 34, and the measurement of particle 1.

With these classical bits come from Alice in hand, Bob introduces an additional qubit 8 with the state $\left|0\right\rangle_8$ because entanglement teleportation transfers not only the amount of entanglement but also the entanglement structure. At the final stage of the teleportation, the original state of qubits 1, 2 and 3 initially possessed by Alice will be restored on qubits 5, 7 and 8 owned by Bob. At the moment, the total state of Bob's system is

$$\frac{1}{2\sqrt{2}}(x_1bd\left|11\right\rangle_{75}+x_2ad\left|10\right\rangle_{75}+x_3ac\left|00\right\rangle_{75})\otimes\left|0\right\rangle_8, \tag{17}$$

or

$$\frac{1}{2\sqrt{2}}(x_1bd\left|11\right\rangle_{75}-x_2ad\left|10\right\rangle_{75}-x_3ac\left|00\right\rangle_{75})\otimes\left|0\right\rangle_8. \tag{18}$$

To begin with, Bob performs two-particle transformation F_{58} on particles 5 and 8, which should have a nonzero entangling power. Here, it is required that F_{58} is an invertible transformation, i.e. full rank. Thus, we can find an appropriate and fully determined unitary transformation V_{58}, once the transformation

F_{58} is known. We make F_{58} and V_{58} fulfil the following requirements:

$$V_{58}F_{58}\left|00\right\rangle = \left|00\right\rangle ; \quad V_{58}F_{58}\left|10\right\rangle = \left|11\right\rangle . \tag{19}$$

Eq.(19) can be rewritten in the more convenient form of

$$V_{58}F_{58} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ z_{21} & z_{22} & z_{23} & z_{24} \\ z_{31} & z_{32} & z_{33} & z_{34} \\ 0 & 0 & 1 & 0 \end{pmatrix}, \tag{20}$$

where z_{ij} are complex parameters. Indeed, it is shown that there exists a class of transformation which satisfies Eq.(19) or Eq.(20), when F_{58} is given. In particular, we can take identity operation I_{58} as V_{58} if transformation F_{58} is CNOT operation. In the teleportation of a general three-particle entangled W state[21] or a two-particle entangled state,[14] CNOT operation has been utilized. Furthermore, quantum CNOT gate has been demonstrated successfully in Ref.[26]. In this paper, we also take this technique, that is, CNOT operation, to realize our scheme of teleportation instead of generic two-particle transformation in order to implement easily.

Bob performs CNOT operation with the particle 5 as the control qubit and the particle 8 as the target qubit. Equation (17) is rewritten in the following way:

$$\frac{1}{2\sqrt{2}}(x_1bd\left|111\right\rangle_{578} + x_2ad\left|010\right\rangle_{578} + x_3ac\left|000\right\rangle_{578}). \tag{21}$$

Then, Bob needs to establish a correspondence such that the coefficients x_1, x_2, x_3 can correspond to $\left|000\right\rangle, \left|101\right\rangle, \left|111\right\rangle$ respectively. This can be realized by performing unitary transformation $U_1 = (\sigma_x)_5 \otimes (\sigma_x)_7 \otimes (\sigma_x)_8$ on particles 5, 7 and 8 shown in Eq.(21), that is

$$\frac{1}{2\sqrt{2}}(x_1bd\left|000\right\rangle_{578} + x_2ad\left|101\right\rangle_{578} + x_3ac\left|111\right\rangle_{578}). \tag{22}$$

Note that if Bob performs generic transformation F_{58} on particles 5 and 8, he may take the transformation $U_1 = (\sigma_x)_5 \otimes (\sigma_x)_7 \otimes (\sigma_x)_8 \otimes V_{58}$ on particles 5, 7 and 8 to establish correspondence.

Similarly, after Bob performs CNOT operation on particles 5 and 8, Eq.(18) is rewritten as

$$\frac{1}{2\sqrt{2}}(x_1bd\left|111\right\rangle_{578} - x_2ad\left|010\right\rangle_{578} - x_3ac\left|000\right\rangle_{578}). \tag{23}$$

Bob performs unitary transformation $U_1 = (\sigma_x)_5 \otimes (\sigma_x)_7 \otimes (\mathrm{i}\sigma_y)_8$ to obtain the correspondence.

In a similar manner for other 30 kinds of cases, all possible transformations are given in Table 1, where we observe that Bob's necessary unitary operation U_1 depends only on Alice's measurement result.

Table 1. All transformations performed by Alice and Bob. For convenience, we denote Alice's Bell-state measurements, result of particle 1, and Bob's unitary transformation as BM_{26}, BM_{34}, H_1, Bob's U_1 respectively. Here σ_x, σ_y, σ_z are Pauli matrices and I is identity matrix.

BM_{34}, BM_{26}	H_1	Bob's U_1	BM_{34}, BM_{26}	H_1	Bob's U_1
$\left\|\Phi^+\right\rangle_{34}\left\|\Phi^+\right\rangle_{26}$	0	$I_5 \otimes I_7 \otimes I_8$	$\left\|\Phi^+\right\rangle_{34}\left\|\Psi^+\right\rangle_{26}$	0	$I_5 \otimes (\sigma_x)_7 \otimes I_8$
$\left\|\Phi^+\right\rangle_{34}\left\|\Phi^+\right\rangle_{26}$	1	$I_5 \otimes I_7 \otimes (\sigma_z)_8$	$\left\|\Phi^+\right\rangle_{34}\left\|\Psi^+\right\rangle_{26}$	1	$I_5 \otimes (\sigma_x)_7 \otimes (\sigma_z)_8$
$\left\|\Phi^+\right\rangle_{34}\left\|\Phi^-\right\rangle_{26}$	0	$I_5 \otimes (\sigma_z)_7 \otimes I_8$	$\left\|\Phi^+\right\rangle_{34}\left\|\Psi^-\right\rangle_{26}$	0	$I_5 \otimes (\mathrm{i}\sigma_y)_7 \otimes I_8$
$\left\|\Phi^+\right\rangle_{34}\left\|\Phi^-\right\rangle_{26}$	1	$(\sigma_z)_5 \otimes (\sigma_z)_7 \otimes I_8$	$\left\|\Phi^+\right\rangle_{34}\left\|\Psi^-\right\rangle_{26}$	1	$(\sigma_z)_5 \otimes (\mathrm{i}\sigma_y)_7 \otimes I_8$
$\left\|\Phi^-\right\rangle_{34}\left\|\Phi^+\right\rangle_{26}$	0	$I_5 \otimes I_7 \otimes (\sigma_z)_8$	$\left\|\Phi^-\right\rangle_{34}\left\|\Psi^+\right\rangle_{26}$	0	$I_5 \otimes (\sigma_x)_7 \otimes (\sigma_z)_8$
$\left\|\Phi^-\right\rangle_{34}\left\|\Phi^+\right\rangle_{26}$	1	$I_5 \otimes I_7 \otimes I_8$	$\left\|\Phi^-\right\rangle_{34}\left\|\Psi^+\right\rangle_{26}$	1	$I_5 \otimes (\sigma_x)_7 \otimes I_8$
$\left\|\Phi^-\right\rangle_{34}\left\|\Phi^-\right\rangle_{26}$	0	$(\sigma_z)_5 \otimes (\sigma_z)_7 \otimes I_8$	$\left\|\Phi^-\right\rangle_{34}\left\|\Psi^-\right\rangle_{26}$	0	$(\sigma_z)_5 \otimes (\mathrm{i}\sigma_y)_7 \otimes I_8$
$\left\|\Phi^-\right\rangle_{34}\left\|\Phi^-\right\rangle_{26}$	1	$I_5 \otimes (\sigma_z)_7 \otimes I_8$	$\left\|\Phi^-\right\rangle_{34}\left\|\Psi^-\right\rangle_{26}$	1	$I_5 \otimes (\mathrm{i}\sigma_y)_7 \otimes I_8$
$\left\|\Psi^+\right\rangle_{34}\left\|\Phi^+\right\rangle_{26}$	0	$(\sigma_x)_5 \otimes I_7 \otimes (\sigma_x)_8$	$\left\|\Psi^+\right\rangle_{34}\left\|\Psi^+\right\rangle_{26}$	0	$(\sigma_x)_5 \otimes (\sigma_x)_7 \otimes (\sigma_x)_8$
$\left\|\Psi^+\right\rangle_{34}\left\|\Phi^+\right\rangle_{26}$	1	$(\mathrm{i}\sigma_y)_5 \otimes I_7 \otimes (\sigma_x)_8$	$\left\|\Psi^+\right\rangle_{34}\left\|\Psi^+\right\rangle_{26}$	1	$(\sigma_x)_5 \otimes (\sigma_x)_7 \otimes (\mathrm{i}\sigma_y)_8$
$\left\|\Psi^+\right\rangle_{34}\left\|\Phi^-\right\rangle_{26}$	0	$(\sigma_x)_5 \otimes (\mathrm{i}\sigma_y)_7 \otimes (\sigma_x)_8$	$\left\|\Psi^+\right\rangle_{34}\left\|\Psi^-\right\rangle_{26}$	0	$(\sigma_x)_5 \otimes (\mathrm{i}\sigma_y)_7 \otimes (\sigma_x)_8$
$\left\|\Psi^+\right\rangle_{34}\left\|\Phi^-\right\rangle_{26}$	1	$(\mathrm{i}\sigma_y)_5 \otimes (\sigma_z)_7 \otimes (\sigma_x)_8$	$\left\|\Psi^+\right\rangle_{34}\left\|\Psi^-\right\rangle_{26}$	1	$(\mathrm{i}\sigma_y)_5 \otimes (\mathrm{i}\sigma_y)_7 \otimes (\sigma_x)_8$
$\left\|\Psi^-\right\rangle_{34}\left\|\Phi^+\right\rangle_{26}$	0	$(\mathrm{i}\sigma_y)_5 \otimes I_7 \otimes (\sigma_x)_8$	$\left\|\Psi^-\right\rangle_{34}\left\|\Psi^+\right\rangle_{26}$	0	$(\sigma_x)_5 \otimes (\sigma_x)_7 \otimes (\mathrm{i}\sigma_y)_8$
$\left\|\Psi^-\right\rangle_{34}\left\|\Phi^+\right\rangle_{26}$	1	$(\sigma_x)_5 \otimes I_7 \otimes (\sigma_x)_8$	$\left\|\Psi^-\right\rangle_{34}\left\|\Psi^+\right\rangle_{26}$	1	$(\sigma_x)_5 \otimes (\sigma_x)_7 \otimes (\sigma_x)_8$
$\left\|\Psi^-\right\rangle_{34}\left\|\Phi^-\right\rangle_{26}$	0	$(\mathrm{i}\sigma_y)_5 \otimes (\sigma_z)_7 \otimes (\sigma_x)_8$	$\left\|\Psi^-\right\rangle_{34}\left\|\Psi^-\right\rangle_{26}$	0	$(\mathrm{i}\sigma_y)_5 \otimes (\mathrm{i}\sigma_y)_7 \otimes (\sigma_x)_8$
$\left\|\Psi^-\right\rangle_{34}\left\|\Phi^-\right\rangle_{26}$	1	$(\sigma_x)_5 \otimes (\mathrm{i}\sigma_y)_7 \otimes (\sigma_x)_8$	$\left\|\Psi^-\right\rangle_{34}\left\|\Psi^-\right\rangle_{26}$	1	$(\sigma_x)_5 \otimes (\mathrm{i}\sigma_y)_7 \otimes (\sigma_x)_8$

In the present case, the state in Eq.(22) cannot directly rotated back to the desired state owing to the lack of any knowledge of the state parameters x_1, x_2, x_3. Here, the following strategy is applied. Bob introduces an auxiliary particle A with the state $|0\rangle_a$ and performs a collective unitary transformation on particles 7, 8 and A under the basis $\{|000\rangle_{78a}, |010\rangle_{78a}, |110\rangle_{78a}, |100\rangle_{78a}, |001\rangle_{78a}, |011\rangle_{78a}, |111\rangle_{78a}, |101\rangle_{78a}\}$. The unitary transformation takes the form of the following 8×8 matrices

$$U_2 = \begin{pmatrix} A_1 & A_2 \\ A_2 & -A_1 \end{pmatrix}, \tag{24}$$

where A_i (i=1, 2) is a 4×4 matrix, and may be written as

$$A_1 = \mathrm{diag}(a_1, a_2, a_3, a_4), \tag{25}$$

$$A_2 = \mathrm{diag}(\sqrt{1-a_1^2}, \sqrt{1-a_2^2}, \sqrt{1-a_3^2}, \sqrt{1-a_4^2}), \tag{26}$$

where a_i (i=1, 2, 3, 4, and $|a_i| \leq 1$) depends on the state of particles 5 and 7. For example, if we choose

$$A_1 = \mathrm{diag}\left(1, \frac{b}{a}, \frac{bd}{ac}, \frac{d}{c}\right), \tag{27}$$

$$A_2 = \mathrm{diag}\left(0, \sqrt{1-\left(\frac{b}{a}\right)^2}, \sqrt{1-\left(\frac{bd}{ac}\right)^2}, \sqrt{1-\left(\frac{d}{c}\right)^2}\right), \tag{28}$$

the unitary transformation U_2 will transform the states in Eq.(22)

$$\frac{1}{2\sqrt{2}}(x_1 bd\,|000\rangle_{578} + x_2 ad\,|101\rangle_{578} + x_3 ac\,|111\rangle_{578}) \otimes |0\rangle_a \tag{29}$$

into

$$\begin{aligned} &\frac{1}{2\sqrt{2}} bd(x_1\,|000\rangle_{578} + x_2\,|101\rangle_{578} + x_3\,|111\rangle_{578})\,|0\rangle_a \\ &+ \frac{1}{2\sqrt{2}}(d\sqrt{a^2-b^2}\,|101\rangle_{578} \\ &+ \sqrt{(ac)^2-(bd)^2}\,|111\rangle_{578})\,|1\rangle_a\,. \end{aligned} \tag{30}$$

Finally, Bob measures the state of particle A in the basis $\{|0\rangle, |1\rangle\}$. From Eq.(30), it can be seen that if the result is $|1\rangle_a$, teleportation fails; if the result is $|0\rangle_a$, particles 5, 7 and 8 collapses into the following state which is just a replica of Alice's original state

$$\frac{1}{2\sqrt{2}} bd(x_1\,|000\rangle_{578} + x_2\,|101\rangle_{578} + x_3\,|111\rangle_{578}). \tag{31}$$

Meanwhile, no trace of identity of the unknown state remains in Alice's region, as is required in accordance with the no-cloning theorem.

So teleportation is successfully realized with the probability of $(1/8)|bd|^2$. Table 2 gives the values of a_1, a_2, a_3 and a_4 corresponding to all cases. It can be calculated that 32 kinds of cases have the same probability, therefore, the total probability of successful teleportation is $(1/8)$ $|bd^2\times 32=4|bd|^2$. For the maximally entangled quantum channels, namely $|a| = |b| = |c| = |d|=1/\sqrt{2}$, the total probability of successful teleportation is 1.

Table 2. Values a_i (i =1, 2, 3, 4) of unitary transformation U_2 corresponding to the states of particles 5 and 7.

States of particle 5 and 7	a_1	a_2	a_3	a_4
$\langle\Phi^{\pm}\|_{34}\langle\Phi^{\pm}\|_{26}\Phi\rangle_{1234567}$	bd/ac	d/c	1	b/c
$\langle\Psi^{\pm}\|_{34}\langle\Phi^{\pm}\|_{26}\Phi\rangle_{1234567}$	d/c	bd/ac	b/a	1
$\langle\Phi^{\pm}\|_{34}\langle\Psi^{\pm}\|_{26}\Phi\rangle_{1234567}$	b/a	1	d/c	bd/ac
$\langle\Psi^{\pm}\|_{34}\langle\Psi^{\pm}\|_{26}\Phi\rangle_{1234567}$	1	b/a	bd/ac	d/c

3. Discussions and conclusions

In this scheme, we take only two pairs of entangled particles as a quantum channel to teleport a kind of important three-particle entangled state. Comparing with previous proposals [22] which require three pairs of entangled states as quantum channel, we conclude that our scheme needs a reduced number of entangled states to accomplish the task. Therefore, the resources of entangled states as quantum channel and the number of classical messages communicated can be cut down. Moreover, in our scheme the two-particle transformation, which is performed by Bob in the process of teleportation, may be generic two-particle transformation instead of CNOT operation. Finally, we give all kinds of unitary transformations performed by Alice and Bob.

In conclusion, the central theme of this paper is to teleport a non-symmetric three-particle entangled

state which is far more useful than the GHZ and W state for some applications [25] in the quantum information. We show that, after two Bell-state measurements, a Hadamard transformation and a single qubit measurement at the sender's side, by introducing an additional particle and an auxiliary particle, an entangling transformation and a set of unitary transformation at the receiver's location, a kind of important three-particle entangled state can be teleported with certain probability.

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