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Dynamics of solitons in Bose–Einstein condensate with time-dependent atomic scattering length*

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The evolution of solitons in Bose–Einstein condensates (BECs) with time-dependent atomic scattering length in an expulsive parabolic potential is studied. Based on the extended hyperbolic function method, we successfully obtain the bright and dark soliton solutions. In addition, some new soliton solutions in this model are found. The results in this paper include some in the literature (*Phys. Rev. Lett.* **94** (2005) 050402 and *Chin. Phys. Lett.* **22** (2005) 1855).

Keywords: Gross–Pitaevskii equation, soliton solution, time-dependent atomic scattering length, extended hyperbolic function method

PACC: 0340K, 0290, 0230

1. Introduction

Research on the nonlinear property of matter wave has received much attention due to the experimental realization of Bose–Einstein condensates (BECs) in weakly interacting atomic gases.[1] The generation as well as the dynamics of solitary wave pulses in BECs is one of the most important related problems. Experimental observations of dark and bright solitons have recently been reported.[2,3] Theoretically, several methods of creating solitary waves have been proposed.[4] In the case of nonlinear matter waves, bright solitons are expected only for attractive interaction ($a > 0$), whereas dark solitons are expected to exist for repulsive interaction ($a < 0$). Recently, experiments have demonstrated that variation of the effective scattering length (SL), even including its sign, can be achieved by utilizing the so-called Feshbach resonance.[5–7] In Ref.[8], it has been demonstrated that the variation of nonlinearity of the Gross–Pitaevskii (GP) equation via Feshbach resonance provides a powerful tool for controlling the generation of bright and dark soliton trains starting from periodic waves. Sinusoidal variation of the scattering length also has been used to form patterns, such as Faraday waves.

As is known, the condensate wavefunction is described by the GP equation. In the physically important case of cigar-shaped BECs, it is reasonable to reduce the GP equation to a one-dimensional (1D) nonlinear Schrödinger equation (NLSE),[9–12]

$$\mathrm{i}\frac{\partial u(x,t)}{\partial t} + \frac{\partial^2 u(x,t)}{\partial x^2} + 2a(t)|u(x,t)|^2 u(x,t) + \frac{1}{4}\lambda^2 x^2 u(x,t) = 0. \quad (1)$$

In Eq.(1), time t and coordinate x are measured in units $2/\omega_\perp$ and $a_\perp$, where $a_\perp = (\hbar/m\omega_\perp)^{1/2}$ and $a_0 = (\hbar/m\omega_0)^{1/2}$ are linear oscillator lengths in the transverse and cigar-axis directions, respectively. $\omega_\perp$ and ω_0 are corresponding harmonic oscillator frequencies, m is the atomic mass, and $\lambda = 2|\omega_0|/\omega_\perp \ll 1$. The Feshbach-managed nonlinear coefficient reads $a(t) = |a_s(t)|/a_\mathrm{B}$, where a_B is the Bohr radius. In Ref.[13], Liang *et al* presented a thorough analysis of the dynamics of a bright soliton of BECs with time-varying atomic scattering length in an expulsive parabolic potential by using the Darboux transformation.[14,15] Their results showed that in a safe range of parameters, the bright soliton can be compressed into very high local matter densities by increasing the absolute value of the atomic scattering length. In Ref.[16], based on the extended tanh-function method, Zhang *et al* obtained both bright and dark solitons of Eq.(1).

However, the construction of the exact solutions

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of nonlinear partial differential equations (NLPDEs) is one of the most important and essential tasks in nonlinear science. With the help of exact solutions, when they exist, the phenomena modelled by these NLPDEs can be better understood.[17−21] They can also help to select the experimental parameters, to analyse the stability of the solutions and to check numerical analysis for these NLPDEs. In this paper, by using the extended hyperbolic function method, [22−28] we not only obtain bright and dark solitons solutions, but also find two new families of soliton solutions. The results in this paper can include those in Refs.[13] and [16].

The rest of this paper is organized as follows. In Section 2, we simply introduce the extended hyperbolic function method. In Section 3, applying the method, with the aid of Maple, we obtain four families of exact solutions of Eq.(1). Then we further investigate the main features of various analytical solutions obtained by using direct computer simulations. The last section is a short summary and discussion.

2. Summary of the extended method

In Ref.[22], we present an extended hyperbolic function method to construct new soliton solutions of NLPDEs. The key idea in this method is to take full advantage of the solutions of an auxiliary ordinary differential equation. The auxiliary equation reads

$$\phi'^2 = b_0 + b_1\phi + b_2\phi^2 + b_3\phi^3 + b_4\phi^4, \tag{2}$$

where $\phi' := \mathrm{d}\phi/\mathrm{d}\xi, \xi = \xi(x, y, z, t)$ and b_0, b_1, b_2, b_3, b_4 are constants. Equation (2) has the following solutions:

$$\begin{aligned}
&\phi_1(\xi) = \pm\sqrt{-\frac{b_2}{b_4}}\mathrm{sech}(\sqrt{b_2}\xi),\ b_1 = b_3 = 0,\ b_0 = \frac{b_2^2}{4b_4},\ b_2 > 0, b_4 < 0;\\
&\phi_2(\xi) = \pm\sqrt{-\frac{b_2}{2b_4}}\tanh\left(\sqrt{-\frac{b_2}{2}}\xi\right),\ b_1 = b_3 = 0,\ b_0 = \frac{b_2^2}{4b_4},\ b_2 < 0, b_4 > 0;\\
&\phi_3(\xi) = \pm\frac{1}{2}\sqrt{\frac{b_2}{b_4}}\left[1 + \tanh\left(\frac{\sqrt{b_2}}{2}\xi\right)\right],\ b_0 = b_1 = 0,\ b_3^2 = 4b_2b_4,\ b_2 > 0,\ b_4 > 0;\\
&\phi_4(\xi) = \pm\sqrt{\frac{b_2}{b_4}}\frac{2\mathrm{sech}^2\left(\frac{\sqrt{b_2}}{2}\xi\right)}{4 - \left(1 - \tanh\left(\frac{\sqrt{b_2}}{2}\xi\right)\right)^2},\ b_0 = b_1 = 0,\ b_3 = 4b_2b_4,\ b_2 > 0,\ b_4 > 0;\\
&\phi_5(\xi) = \frac{\sqrt{-\frac{b_2}{2b_4}}\tanh(\sqrt{-2b_2}\xi)}{1 \pm \mathrm{sech}(\sqrt{-2b_2}\xi)},\ b_1 = b_3 = 0,\ b_0 = \frac{b_2^2}{4b_4},\ b_2 < 0,\ b_4 > 0.
\end{aligned} \tag{3}$$

A nonlinear NLPDE with two variables x and t is given as follows:

$$P(u_x, u_t, u_{xx}, u_{tt}, u_{xt}, \ldots) = 0. \tag{4}$$

Step 1 We assume Eq.(4) has the following solutions:

$$u(x,t) = \sum_{i=0}^{n} b_i\phi^n(\xi), \tag{5}$$

where $b_i = b_i(x,t), (i = 0, 1, \ldots, n), \xi = \xi(x,t)$ are all unknown functions of $x, t, \phi(\xi)$ satisfying Eq.(2).

The parameter n can be found by balancing the highest order derivative term and the nonlinear term in Eq.(4) (n is a positive integer).

Step 2 Substituting Eq.(5) along with Eq.(2) into Eq.(4), and extracting the numerator of the resulting system, we can obtain a set of algebraic polynomials for $\phi^i(\xi)(i = 0, 1, \ldots)$. Setting the coefficients of these terms $\phi^i(\xi)$to zero, we have a system of overdetermined PDEs with respect to unknown functions $\{b_i(i = 0, 1, \ldots), \xi\}$.

Step 3 Solving the above system by use of symbolic computation system-Maple, we would end up with the explicit expressions for $b_i, (i = 0, 1, \ldots, n)$ and ξ or the constraints among them.

Thus according to Eqs.(3), (5) and the conclusions in Step 3, we can obtain many families of exact

solutions for Eq.(4).

3. Results and discussion

In order to obtain some exact solutions of Eq.(1), first we make the transformation

$$u(x,t)=\psi(x,t)\exp(\mathrm{i}\theta(x,t)). \tag{6}$$

Then substituting Eq.(6) into Eq.(1) and setting the real and imaginary parts of the resulting equation equal to zero, we obtain the following sets of partial differential equations:

$$-\psi\theta_t+\psi_{xx}-\psi\theta_x^2+2a(t)\psi^3+\frac{1}{4}\lambda^2x^2\psi=0, \tag{7}$$

$$\psi_t+\psi\theta_{xx}+2\psi_x\theta_x=0. \tag{8}$$

Balancing the highest order derivative term ψ_{xx} with the nonlinear term ψ^3 in Eq.(7) yields $n=1$. Therefore by the use of the extended hyperbolic function method we assume the solutions of Eqs.(7) and (8) to be in the following special forms:

$$\psi(x,t)=f(t)+h(t)\phi(\xi),\quad \xi=k(t)x+\eta(t), \tag{9}$$

$$\theta(x,t)=\Delta(t)x^2+\Gamma(t)x+\Omega(t), \tag{10}$$

where $f(t)$, $h(t)$, $k(t)$, $\eta(t)$, $\Delta(t)$, $\Gamma(t)$ and $\Omega(t)$ are functions of t to be determined, with $\phi(\xi)$ satisfying Eq.(2). Substituting Eqs.(9), (10) and (2) into Eqs.(7), (8) and collecting the coefficients of power of $x^k\phi^l$, and $\sqrt{b_0+b_1\phi+b_2\phi^2+b_3\phi^3+b_4\phi^4}$ ($k=0,1,2$; $l=0,1,2,3$) with the aid of Maple, then setting each of the coefficients to zero, we can obtain a set of over-determined ordinary differential equations (ODEs) with respect to the unknown derivative functions $\{a(t),\ f(t),\ h(t),\ k(t),\ \eta(t),\ \Delta(t),\ \Gamma(t),\ \Omega(t)\}$ as follows:

$$h'+2h\Delta=0, \tag{11}$$

$$h(k'+4k\Delta)=0, \tag{12}$$

$$f'+2f\Delta=0, \tag{13}$$

$$h(\eta'+2k\Gamma)=0, \tag{14}$$

$$2h(ah^2+b_4k^2)=0, \tag{15}$$

$$\frac{3}{2}h(4afh+b_3k^2)=0, \tag{16}$$

$$-\frac{1}{4}h(4\Delta'+16\Delta^2-\lambda^2)=0, \tag{17}$$

$$-h(\Gamma'+4\Gamma\Delta))=0, \tag{18}$$

$$-h(\Omega'+\Gamma^2-b_2k^2-6af^2)=0, \tag{19}$$

$$f\Omega'+f\Gamma^2-2af^3-\frac{1}{2}b_1hk^2=0. \tag{20}$$

Solving Eqs.(11)–(20) by means of Maple, we obtain the following results.

Case 1

$$\begin{aligned}
&f(t)=b_1=b_3=0,\quad b_0=\frac{b_2^2}{4b_4},\quad k(t)=C_1a(t),\\
&h(t)=\pm C_1\sqrt{-b_4a(t)},\\
&\Gamma(t)=C_2a(t),\quad \Delta=-\frac{a'(t)}{4a(t)},\\
&\Omega(t)=(b_2C_1^2-C_2^2)\int a(t)^2\mathrm{d}t+C_3,\\
&\eta(t)=-2C_1C_2\int a(t)^2\mathrm{d}t+C_4,\\
&a(t)=\frac{2\lambda\mathrm{e}^{\lambda t}}{C_5\mathrm{e}^{2\lambda t}-C_6},\quad b_4<0,\quad a(t)>0,
\end{aligned} \tag{21}$$

where C_1, C_2, C_3, C_4, C_5, C_6 are arbitrary constants, $a'(t)=\dfrac{\mathrm{d}a(t)}{\mathrm{d}t}$.

Case 2

$$\begin{aligned}
&f(t)=b_1=b_3=0,\quad b_0=\frac{b_2^2}{4b_4},\quad k(t)=C_1a(t),\\
&h(t)=\pm C_1\sqrt{-b_4a(t)},\\
&\Gamma(t)=C_2a(t),\quad \Delta=-\frac{a'(t)}{4a(t)},\\
&\Omega(t)=(b_2C_1^2-C_2^2)\int a(t)^2\mathrm{d}t+C_3,\\
&\eta(t)=-2C_1C_2\int a(t)^2\mathrm{d}t+C_4,\\
&a(t)=\frac{2\lambda\mathrm{e}^{\lambda t}}{C_5\mathrm{e}^{2\lambda t}-C_6},\quad b_4>0,\quad a(t)<0.
\end{aligned} \tag{22}$$

Case 3

$$\begin{aligned}
&k(t)=C_1a(t),\quad h(t)=\pm C_1\sqrt{-b_4a(t)},\\
&f(t)=\pm\frac{b_3C_1}{4}\sqrt{\frac{-a(t)}{b_4}},\\
&\Gamma(t)=C_2a(t),\quad \Delta=-\frac{a'(t)}{4a(t)},\\
&\Omega(t)=-\frac{-8b_2b_4C_1^2+3b_3^2C_1^2+8b_4C_2^2}{8b_4}\int a(t)^2\mathrm{d}t+C_3,\\
&\eta(t)=-2C_1C_2\int a(t)^2\mathrm{d}t+C_4,\\
&a(t)=\frac{2\lambda\mathrm{e}^{\lambda t}}{C_5\mathrm{e}^{2\lambda t}-C_6},\quad b_3^3-4b_2b_3b_4+8b_1b_4^2=0,\\
&b_4>0,\quad a(t)<0,
\end{aligned} \tag{23}$$

Case 4

$$k(t)=C_1a(t),\quad h(t)=\pm C_1\sqrt{-b_4a(t)},$$
$$f(t)=\mp\frac{b_3C_1}{4}\sqrt{\frac{-a(t)}{b_4}},$$
$$\Gamma(t)=C_2a(t),\quad \Delta=-\frac{a'(t)}{4a(t)},$$
$$\Omega(t)=-\frac{-8b_2b_4C_1^2+3b_3^2C_1^2+8b_4C_2^2}{8b_4}\int a(t)^2\mathrm{d}t+C_3,$$
$$\eta(t)=-2C_1C_2\int a(t)^2\mathrm{d}t+C_4,$$
$$a(t)=\frac{2\lambda\mathrm{e}^{\lambda t}}{C_5\mathrm{e}^{2\lambda t}-C_6},\quad b_3^3-4b_2b_3b_4+8b_1b_4^2=0,$$
$$b_4<0,\quad a(t)>0, \tag{24}$$

Therefore from Eqs.(3), (6), (9), (10), (21)–(23),we obtain following soliton solutions of Eq.(1):

Family 1 Bright soliton solution ($b_2>0, b_4<0, a(t)>0$):

$$\begin{aligned}u_1=&C_1\sqrt{b_2a(t)}\mathrm{sech}\left[\sqrt{b_2}(C_1a(t)x-2C_1C_2\int a(t)^2\mathrm{d}t+C_4)\right]\\&\cdot\exp\left[\mathrm{i}(-\frac{a'(t)}{4a(t)}x^2+C_2a(t)x+(b_2C_1^2-C_2^2)\int a(t)^2\mathrm{d}t+C_3)\right].\end{aligned} \tag{25}$$

Family 2 Dark soliton solution ($b_2<0, b_4>0, a(t)<0$):

$$\begin{aligned}u_2=&C_1\sqrt{\frac{b_2}{2}a(t)}\tanh\left[\sqrt{-\frac{b_2}{2}}(C_1a(t)x-2C_1C_2\int a(t)^2\mathrm{d}t+C_4)\right]\\&\cdot\exp\left[\mathrm{i}\left(-\frac{a'(t)}{4a(t)}x^2+C_2a(t)x+(b_2C_1^2-C_2^2)\int a(t)^2\mathrm{d}t+C_3\right)\right].\end{aligned} \tag{26}$$

Family 3 New soliton solutions ($b_2<0, b_4>0, a(t)<0$):

$$\begin{aligned}u_3=&C_1\sqrt{\frac{b_2}{2}a(t)}\frac{\tanh[\sqrt{-2b_2}(C_1a(t)x-2C_1C_2\int a(t)^2\mathrm{d}t+C_4)]}{1+\mathrm{sech}[\sqrt{-2b_2}(C_1a(t)x-2C_1C_2\int a(t)^2\mathrm{d}t+C_4)]}\\&\cdot\exp\left[\mathrm{i}\left(-\frac{a'(t)}{4a(t)}x^2+C_2a(t)x+(b_2C_1^2-C_2^2)\int a(t)^2\mathrm{d}t+C_3\right)\right],\end{aligned} \tag{27}$$

$$\begin{aligned}u_4=&C_1\sqrt{\frac{b_2}{2}a(t)}\frac{\tanh[\sqrt{-2b_2}(C_1a(t)x-2C_1C_2\int a(t)^2\mathrm{d}t+C_4)]}{1-\mathrm{sech}[\sqrt{-2b_2}(C_1a(t)x-2C_1C_2\int a(t)^2\mathrm{d}t+C_4)]}\\&\cdot\exp\left[\mathrm{i}\left(-\frac{a'(t)}{4a(t)}x^2+C_2a(t)x+(b_2C_1^2-C_2^2)\int a(t)^2\mathrm{d}t+C_3\right)\right].\end{aligned} \tag{28}$$

Family 4 New kink soliton solutions ($b_2>0, b_4>0, a(t)<0$):

$$\begin{aligned}u_5=&\frac{C_1}{2}\sqrt{-b_2a(t)}\left[2+\tanh\left(\frac{\sqrt{b_2}}{2}(C_1a(t)x-2C_1C_2\int a(t)^2\mathrm{d}t+C_4\right)\right]\\&\cdot\exp\left[\mathrm{i}\left(-\frac{a'(t)}{4a(t)}x^2+C_2a(t)x-\frac{1}{2}(b_2C_1^2+2C_2^2)\int a(t)^2\mathrm{d}t+C_3\right)\right],\end{aligned} \tag{29}$$

$$\begin{aligned}u_6=&\frac{C_1}{2}\sqrt{-b_2a(t)}\left[5-\frac{8}{3-\tanh\left[\frac{\sqrt{b_2}}{2}(C_1a(t)x-2C_1C_2\int a(t)^2\mathrm{d}t+C_4)\right]}\right]\\&\cdot\exp\left[\mathrm{i}\left(-\frac{a'(t)}{4a(t)}x^2+C_2a(t)x-\frac{1}{2}(b_2C_1^2+2C_2^2)\int a(t)^2\mathrm{d}t+C_3\right)\right].\end{aligned} \tag{30}$$

In Families 1–4, $a'(t)=\dfrac{\mathrm{d}a(t)}{\mathrm{d}t}$, $a(t)=\dfrac{2\lambda\mathrm{e}^{\lambda t}}{C_5\mathrm{e}^{2\lambda t}-C_6}$, and C_1, C_2, C_3, C_4, C_5, C_6 are arbitrary constants. The solutions (25) and (26) have been obtained also in Ref.[16] by applying the extended tanh-function method. But to our knowledge, the

other solitons have not been reported earlier. Specifically, as an example for discussion, we select $C_5 = 0$, $C_6 = -\frac{2\lambda}{g_0}$, the scattering length is $a(t) = g_0\exp(\lambda t)$, i.e. the Feshbach-managed nonlinear coefficient. Then from Eqs.(25)–(30), we obtain

Family 1 Bright soliton solution ($b_2 > 0, b_4 < 0, (g_0 > 0)$):

$$u_1 = C_1\sqrt{b_2 g_0}\,\mathrm{e}^{\frac{\lambda t}{2}}\mathrm{sech}\left[\sqrt{b_2}\left(C_1 g_0\mathrm{e}^{\lambda t}x - \frac{2C_1C_2g_0^2}{\lambda}\mathrm{e}^{2\lambda t} + C_4\right)\right] \cdot \exp\left[\mathrm{i}\left(-\frac{\lambda}{4}x^2 + C_2 g_0\mathrm{e}^{\lambda t}x + \frac{g_0^2(b_2C_1^2 - C_2^2)}{2\lambda}\mathrm{e}^{2\lambda t} + C_3\right)\right]. \tag{31}$$

Family 2 Dark soliton solution ($b_2 < 0, b_4 > 0, g_0 < 0$):

$$u_2 = C_1\sqrt{\frac{b_2 g_0}{2}}\,\mathrm{e}^{\frac{\lambda t}{2}}\tanh\left[\sqrt{-\frac{b_2}{2}}\left(C_1 g_0\mathrm{e}^{\lambda t}x - \frac{2C_1C_2g_0^2}{\lambda}\mathrm{e}^{2\lambda t} + C_4\right)\right] \cdot \exp\left[\mathrm{i}\left(-\frac{\lambda}{4}x^2 + C_2 g_0\mathrm{e}^{\lambda t}x + \frac{g_0^2(b_2C_1^2 - C_2^2)}{2\lambda}\mathrm{e}^{2\lambda t} + C_3\right)\right]. \tag{32}$$

Family 3 New soliton solutions ($b_2 < 0, b_4 > 0, (g_0 < 0)$):

$$u_3 = C_1\sqrt{\frac{b_2 g_0}{2}}\,\mathrm{e}^{\frac{\lambda t}{2}}\frac{\tanh\left[\sqrt{-2b_2}\left(C_1 g_0\mathrm{e}^{\lambda t}x - \frac{2C_1C_2g_0^2}{\lambda}\mathrm{e}^{2\lambda t} + C_4\right)\right]}{1 + \mathrm{sech}\left[\sqrt{-2b_2}\left(C_1 g_0\mathrm{e}^{\lambda t}x - \frac{2C_1C_2g_0^2}{\lambda}\mathrm{e}^{2\lambda t} + C_4\right)\right]} \cdot \exp\left[\mathrm{i}\left(-\frac{\lambda}{4}x^2 + C_2 g_0\mathrm{e}^{\lambda t}x + \frac{g_0^2(b_2C_1^2 - C_2^2)}{2\lambda}\mathrm{e}^{2\lambda t} + C_3\right)\right]. \tag{33}$$

$$u_4 = C_1\sqrt{\frac{b_2 g_0}{2}}\,\mathrm{e}^{\frac{\lambda t}{2}}\frac{\tanh\left[\sqrt{-2b_2}\left(C_1 g_0\mathrm{e}^{\lambda t}x - \frac{2C_1C_2g_0^2}{\lambda}\mathrm{e}^{2\lambda t} + C_4\right)\right]}{1 - \mathrm{sech}\left[\sqrt{-2b_2}\left(C_1 g_0\mathrm{e}^{\lambda t}x - \frac{2C_1C_2g_0^2}{\lambda}\mathrm{e}^{2\lambda t} + C_4\right)\right]} \cdot \exp\left[\mathrm{i}\left(-\frac{\lambda}{4}x^2 + C_2 g_0\mathrm{e}^{\lambda t}x + \frac{g_0^2(b_2C_1^2 - C_2^2)}{2\lambda}\mathrm{e}^{2\lambda t} + C_3\right)\right]. \tag{34}$$

Family 4 New kink soliton solutions ($b_2 > 0, b_4 > 0, g_0 < 0$)):

$$u_5 = \frac{C_1}{2}\sqrt{-b_2 g_0}\,\mathrm{e}^{\frac{\lambda t}{2}}\left[2 + \tanh\left(\frac{\sqrt{b_2}}{2}(C_1 g_0\mathrm{e}^{\lambda t}x - \frac{2C_1C_2g_0^2}{\lambda}\mathrm{e}^{2\lambda t} + C_4\right)\right] \cdot \exp\left[\mathrm{i}\left(-\frac{\lambda}{4}x^2 + C_2 g_0\mathrm{e}^{\lambda t}x + \frac{g_0^2(b_2C_1^2 - C_2^2)}{2\lambda}\mathrm{e}^{2\lambda t} + C_3\right)\right]. \tag{35}$$

$$u_6 = \frac{C_1}{2}\sqrt{-b_2 g_0}\,\mathrm{e}^{\frac{\lambda t}{2}}\left[5 - \frac{8}{3 - \tanh\left[\frac{\sqrt{b_2}}{2}\left(C_1 g_0\mathrm{e}^{\lambda t}x - \frac{2C_1C_2g_0^2}{\lambda}\mathrm{e}^{2\lambda t} + C_4\right)\right]}\right] \cdot \exp\left[\mathrm{i}\left(-\frac{\lambda}{4}x^2 + C_2 g_0\mathrm{e}^{\lambda t}x + \frac{g_0^2(b_2C_1^2 - C_2^2)}{2\lambda}\mathrm{e}^{2\lambda t} + C_3\right)\right]. \tag{36}$$

In order to understand the main soliton properties of these solutions expressed by Eqs.(31)–(36), we plot them in Figs.1–6. Figure 1 shows the dynamics of the Feshbach-resonance-managed bright soliton in the expulsive parabolic potential. As shown in Fig.1, with increasing absolute value of the scattering length,the bright soliton has an increase in peak value and a decrease in width, which is in agreement with the result

in Ref.[13]. The properties of other solitons expressed by Eqs.(32), (33), (35) and (36) are similar to those of the bright soliton.

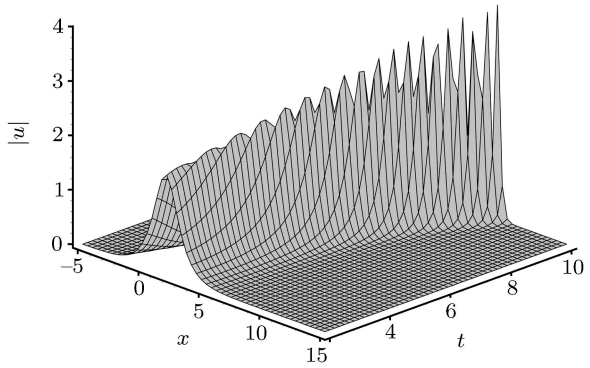


Fig.1. The dynamics of the Feshbach-resonance-managed soliton in the expulsive potential given by $|u_1|$. The parameters are given as follows: $C_1 = 3$, $C_2 = 1$, $C_4 = 0$, $b_2 = 1$, $g_0 = \frac{1}{4}, \lambda = \frac{1}{5}$.

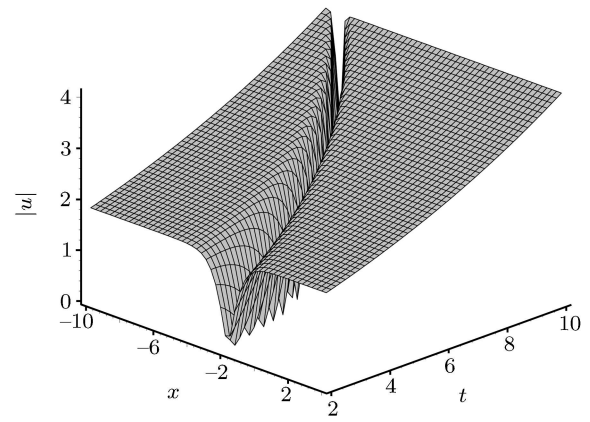


Fig.2. The dynamics of the Feshbach-resonance-managed soliton in the expulsive potential given by $|u_2|$. The parameters are given as follows: $C_1 = 3$, $C_2 = 1$, $C_4 = 0$, $b_2 = -2$, $g_0 = -\frac{1}{4}, \lambda = \frac{1}{5}$.

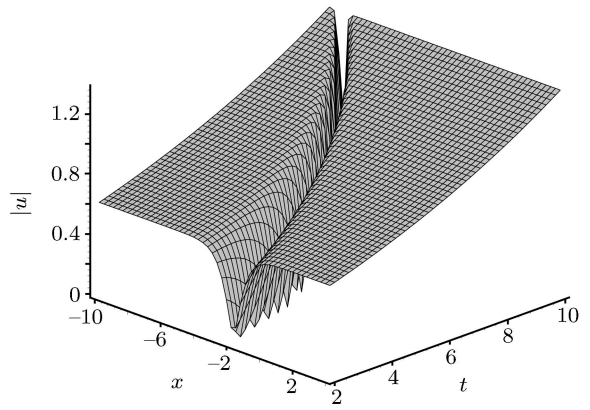


Fig.3. The dynamics of the Feshbach-resonance-managed soliton in the expulsive potential given by $|u_3|$. The parameters are given as follows: $C_1 = -3$, $C_2 = 1$, $C_4 = 0$, $b_2 = -2$, $g_0 = -\frac{1}{4}, \lambda = \frac{1}{5}$.

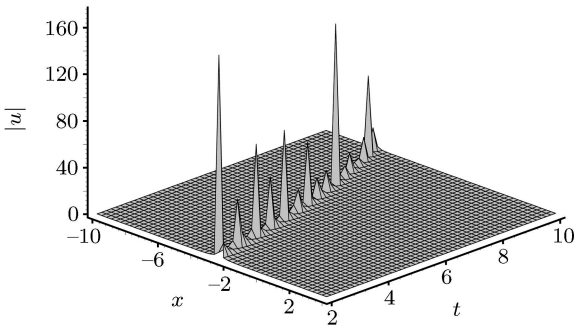


Fig.4. The dynamics of the Feshbach-resonance-managed soliton in the expulsive potential given by $|u_4|$. The parameters are given as follows: $C_1 = -3$, $C_2 = 1$, $C_4 = 0$, $b_2 = -2$, $g_0 = -\frac{1}{4}, \lambda = \frac{1}{8}$.

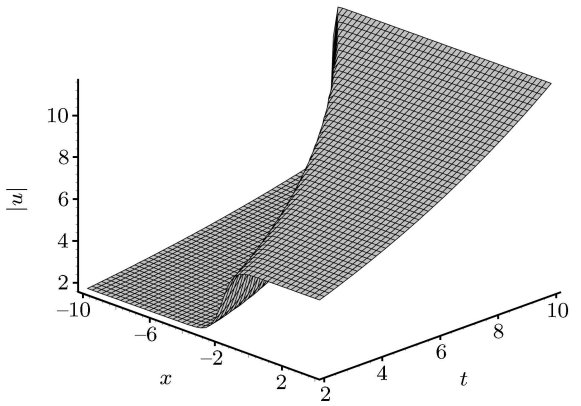


Fig.5. The dynamics of the Feshbach-resonance-managed soliton in the expulsive potential given by $|u_5|$. The parameters are given as follows: $C_1 = -4$, $C_2 = 1$, $C_4 = 0$, $b_2 = 2$, $g_0 = -\frac{1}{4}, \lambda = \frac{1}{5}$.

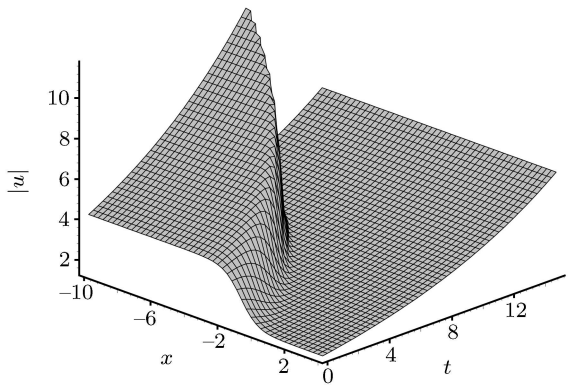


Fig.6. The dynamics of the Feshbach-resonance-managed soliton in the expulsive potential given by $|u_6|$. The parameters are given as follows: $C_1 = -4$, $C_2 = 1$, $C_4 = 0$, $b_2 = 2$, $g_0 = -\frac{1}{4}, \lambda = \frac{1}{5}$.

As scattering length $a(t)$ contains two arbitrary constants, we are inclined to consider that besides selecting the scattering length $a(t) = g_0 \exp(\lambda t)$ in

the experiment, we may also select other scattering length $a(t)$ expressed in Eq.(21). For example, when we select $C_5 = -C_6 = 1$, the scattering length is $a(t) = \lambda\mathrm{sech}(\lambda t)$. Meanwhile, the bright soliton solution of Eq.(1) reads

$$\begin{aligned} u_7 =& C_1\sqrt{b_2\lambda\mathrm{sech}(\lambda t)}\mathrm{sech}[\sqrt{b_2}(C_1\lambda\mathrm{sech}(\lambda t)x \\ & - 2C_1C_2\lambda\tanh(\lambda t) + C_4)] \\ & \cdot \exp\left[\mathrm{i}\left(\frac{\lambda}{4}\tanh(\lambda t)x^2 + C_2\lambda\mathrm{sech}(\lambda t)x \right.\right. \\ & \left.\left. + (b_2C_1^2 - C_2^2)\lambda\tanh(\lambda t) + C_3\right)\right]. \end{aligned} \tag{37}$$

Figure 7 shows the dynamics of bright soliton with the scattering length $a(t) = \lambda\mathrm{sech}(\lambda t)$ in the expulsive parabolic potential. As shown in Fig.7, with increasing absolute value of the scattering length, the bright soliton has a decrease in peak value and an expansion in width. This result is contrary to that in Ref.[13].

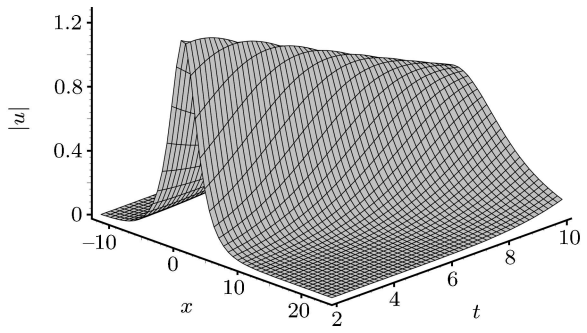


Fig.7. The dynamics of the Feshbach-resonance-managed soliton in the expulsive potential given by $|u_7|$. The parameters are given as follows: $C_1 = 3$, $C_2 = 1$, $C_4 = 0$, $b_2 = 1$, $g_0 = \frac{1}{4}, \lambda = \frac{1}{5}$.

4. Summary

In this paper, making use of the extended hyperbolic function method, we have successfully obtained four families of exact solutions of the BECs with time-dependent nonlinear coefficient in an expulsive parabolic potential, including the results in Ref.[16]. As an example for discussion, we select the Feshbach-managed nonlinear coefficient as $a(t) = g_0\exp(\lambda t)$, and obtain the results which can include those in Ref.[13]. We also select other scattering length $a(t) = \lambda\mathrm{sech}(\lambda t)$ and obtain some new soliton solutions. To our knowledge,the new soliton solutions have not been found before. The properties of the soliton solutions for Eq.(1) are shown by Figs.1–7.

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References

[1] Dalfovo F, Giorgini S, Pitaevskii L P and Stringari S 1999 *Rev. Mod. Phys.* **71** 463
[2] Khaykovich L, Schreck F and Ferrari G 2002 *Science* **296** 1290
[3] Strecker K E, Partridge G B and Truscott A G 2002 *Nature* **417** 150
[4] Konotop V V and Salerno M 2002 *Phys. Rev. A* **65** 021602
[5] Roberts J L, Claussen N R, Burke James P, Jr. and Greene Chris H 1998 *Phys. Rev. Lett.* **81** 5109
[6] Cornish S L, Claussen N R, Roberts J L, Cornell E A, and Wieman C E 2000 *Phys. Rev. Lett.* **85** 1795
[7] Donley E A, Claussen N R, and Cornish S L 2001 *Nature* **412** 295
[8] Abdullaev F Kh, Kamchatnov A M, Konotop V V and Brazhnyi V A 2003 *Phys. Rev. Lett.* **90** 230402
[9] Perez-Garcia V M, Michinel H and Herrer H 1998 *Phys. Rev.* A **57** 3837
[10] Moll K D, Gaeta A L and Fibich G 2003 *Phys. Rev. Lett.* **90** 203902
[11] Kevrekidis P G and Frantzeskakis D J 2004 *Mod. Phys. Lett.* B **18** 173
[12] Brazhnyi V A and Konotop V V 2004 *Mod. Phys. Lett.* B **18** 627
[13] Liang Z X, Zhong Z D and Liu W M 2005 *Phys. Rev. Lett.* **94** 050402
[14] Matveev V B and Salli M A 1991 *Darboux Transformations and Solitons, Springer Series in Nonliear Dynamics* (Berlin: Springer)
[15] Hu H C and Lou S Y 2004 *Chin. Phys. Lett.* **21** 2073
[16] Zhang J F and Yang Q 2005 *Chin. Phys. Lett.* **22** 1855
[17] Yang X and Wu Y 2002 *Commun. Theor. Phys.* **38** 615
[18] Chong G S, Hai W H and Xie Q T 2003 *Chin. Phys. Lett.* **20** 2098
[19] Hai W H, Lee C and Chong G S 2004 *Phys. Rev.* A **70** 053621
[20] He H S, Chen J and Yang K Q 2005 *Chin. Phys.* **14** 1710
[21] Zheng Q, Yue P and Gong L X 2005 *Chin. Phys.* **15** 35
[22] Li H M and Wu F M 2004 *Chin. Phys. Lett.* **21** 1425
[23] Li H M, Lin J and Xu Y S 2004 *Acta Phys. Sin.* **53** 349 (in Chinese)
[24] Li H M 2005 *Chin. Phys.* **14** 0251
[25] Lian Z J and Lou S Y 2005 *Chin. Phys.* **14** 1486
[26] Ma H C and Lou S Y 2005 *Chin. Phys.* **14** 1495
[27] Liu S C 2005 *Acta Phys. Sin.* **54** 4506 (in Chinese)
[28] Xia T C, Zhang Y F and Chen D Y 2005 *Chin. Phys.* **14** 247