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Rapid Communication

Single-mode quadrature-amplitude measurement for generalized two-mode squeezed states studied by virtue of the entangled state representation

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The result of one-mode quadrature-amplitude measurement for some generalized two-mode squeezed states has been studied by virtue of the entangled state representation of the corresponding two-mode squeezing operators. We find that the remaining field-mode simultaneously collapses to the single-mode squeezed state with more stronger squeezing. The measurement result caused by a single-mode squeezed state projector is also calculated, which indicates quantum entanglement in squeezing.

Keywords: entangled state representation, quadrature-amplitude measurement, two-mode squeezed states

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1. Introduction

Since the publication of the paper of Einstein, Podolsky and Rosen (EPR) in 1935,[1] arguing the incompleteness of quantum mechanics, the conception of entanglement has become more and more fascinating and important as it plays a central role in quantum communication and quantum computation.[2−5] As is known, by entanglement one means that two-particle state does not factor into a product of single particle states, but is a sum of at least two terms, each of which is a product. One may recapitulate the entanglement in more mathematical language: A bipartite pure state is said to be entangled if its Schmidt number is greater than one. When two particles are so entangled, neither particle separately has a state, because the particles in an entanglement do not have states or even some properties, independent of each other. One thus must consider a two-particle system as a simple entity. The two-mode squeezed state itself is an entangled state which entangles the idle mode and signal mode in the frequency domain as an outcome of a parametric-down conversion process (in a non-degenerate parametric amplifier).[6] To put it in another way, the correlations between the idler mode and signal mode give rise to two-mode squeezing. This is an evidence that in two-mode systems there are a richer variety of quantum phenomena since there exists the possibility of quantum correlations between the modes. Theoretically, squeezed states are generated by operating the two-mode squeezing operator on the vacuum state.[7] In Ref.[8] a one-sided two-mode squeezed state is constructed and in Ref.[9] the one- and two-mode combinatorial squeezed state is proposed which may exhibit more stronger squeezing. A question thus naturally arises: If one makes a single-mode quadrature-amplitude measurement, say $|x\rangle_{11}\langle x|$, $|x\rangle_1$ is the eigenvector of the quadrature operator $\frac{a_1+a_1^\dagger}{\sqrt{2}}=X_1$, on the two-mode squeezed state or on a generalized two-mode squeezed state, then what does the another field-mode become after the collapse? This problem has physical meaning, since the two field modes in a squeezed state are originally

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entangled. To our knowledge, this problem has not been fully solved and the exact result has not been reported in the literature before, because some complicated forms of generalized squeezing operators hinder the processing. In this paper we shall show that the calculation can be greatly simplified by virtue of the entangled state representation[10] (denoted by $|\eta\rangle$) or the coordinate representation of the various generalized squeezing operators, or the coherent state representation [11,12] accompanied with a mathematical integration formula, and the exact solutions can be derived exactly and analysed physically. This paper is arranged as follows. In Section 2, after explaining that the two-mode squeezing operator has a natural representation in $|\eta\rangle \equiv |\eta_1, \eta_2\rangle$ representation[13]

$$\begin{aligned} S =& \exp[\lambda(a_1 a_2 - a_1^\dagger a_2^\dagger)] \\ =& \mu \int \frac{\mathrm{d}^2\eta}{\pi} |\mu\eta\rangle \langle\eta| , \\ \mu =& \mathrm{e}^\lambda, \eta = \eta_1 + \mathrm{i}\eta_2, \end{aligned} \tag{1}$$

where μ is the squeezing parameter, we introduce a squeezed state $U(\mu,\nu)|00\rangle$ generated by the generalized squeezing operator

$$U(\mu,\nu) = \sqrt{\mu\nu} \int \frac{\mathrm{d}^2\eta}{\pi} |\mu\eta_1, \nu\eta_2\rangle \langle\eta| , \tag{2}$$

and then discuss how this squeezed state collapses when a measurement $|x\rangle_{11}\langle x|$ is operating on it. We see that when $\mu = \nu$ the collapse results in enhancement of squeezing of the remaining field mode. In Section 3 we carry out calculations for the case of the so-called one- and two-combinatorial squeezed state generated by the squeezing operator

$$V(\lambda_1, \lambda_2) = \exp[-\mathrm{i}(\lambda_1 X_1 P_2 + \lambda_2 X_2 P_1)], \tag{3}$$

and discuss the collapse of $V(\lambda_1,\lambda_2)|00\rangle$, and

$$X_i \equiv \frac{a_i + a_i^\dagger}{2^{1/2}}, \quad P_i \equiv \frac{a_i - a_i^\dagger}{2^{1/2} i}. \tag{4}$$

In Section 4 we examine the collapse of the general 3-parameter SU(1,1) squeezed state which undergoes a measurement $|x\rangle_{11}\langle x|$. The measurement result caused by a single-mode squeezed state projector is also calculated, which indicates that quantum entanglement in squeezing, i.e. the second-mode field will be controlled by the first mode's squeezing parameter. For details of squeezed state see Refs.[14–16].

2. Brief review of the relation between $|\eta\rangle$ and the squeezing operators

Based on EPR's idea that two particles' relative coordinate $X_1 - X_2$ and the total momentum $P_1 + P_2$ are commutative, so they possess the common eigenvector

$$|\eta\rangle = \exp[-\frac{1}{2}|\eta|^2 + \eta a_1^\dagger - \eta^* a_2^\dagger + a_1^\dagger a_2^\dagger]|00\rangle \equiv |\eta_1, \eta_2\rangle , \tag{5}$$

where $\eta = \eta_1 + \mathrm{i}\eta_2$ is a complex number, $|00\rangle$ is the two-mode vacuum state, $|\eta\rangle$ obeys

$$(X_1 - X_2)|\eta\rangle = \sqrt{2}\eta_1 |\eta\rangle , \quad (P_1 + P_2)|\eta\rangle = \sqrt{2}\eta_2 |\eta\rangle . \tag{6}$$

Note that for two-mode optical fields, the quadrature operators characteristic of quantum fluctuations of various states are

$$\begin{aligned} \frac{a_1 + a_2 + a_1^\dagger + a_2^\dagger}{2^{3/2}} &\equiv \frac{1}{2}(X_1 + X_2), \\ \frac{a_1 + a_2 - a_1^\dagger - a_2^\dagger}{2^{3/2} i} &\equiv \frac{1}{2}(P_1 + P_2). \end{aligned} \tag{7}$$

It just happens that the two-mode quadrature operator $(P_1 + P_2)/2$ is one of the object operators discussed by EPR, and the two-mode squeezing operator squeezes in a standard way, i.e.

$$\begin{aligned} S\frac{1}{2}(X_1 + X_2)S^{-1} =& \frac{1}{2}(X_1 + X_2)\mathrm{e}^\lambda, \\ S\frac{1}{2}(P_1 + P_2)S^{-1} =& \frac{1}{2}(P_1 + P_2)\mathrm{e}^{-\lambda}. \end{aligned} \tag{8}$$

Thus there must exist an intrinsic relation between S and the common eigenvector $|\eta\rangle$ of $X_1 - X_2$ and $P_1 + P_2$. In Ref.[13] such a remarkable relation was found as shown in Eq.(1), which indicates that S actually squeezes the EPR entangled state $|\eta\rangle$ to $\mu|\mu\eta\rangle$ in a natural way, i.e.

$$S|\eta\rangle = \mu|\mu\eta\rangle , \quad \mu = \mathrm{e}^\lambda. \tag{9}$$

Using the normal product form of the vacuum projector $|00\rangle\langle 00| =: \exp[-a_1^\dagger a_1 - a_2^\dagger a_2] :$ and the technique of integration within an ordered product (IWOP) of operators[17,18] to the integration in Eq.(1) we obtain the normally ordered expansion of S,

$$
\begin{aligned}
&\mu\int\frac{\mathrm{d}^2\eta}{\pi}\,|\mu\eta\rangle\langle\eta| \\
=&\mu\int\frac{\mathrm{d}^2\eta}{\pi}:\exp\Big\{-\frac{|\eta|^2}{2}(1+\mu^2) \\
&+\eta(\mu a_1^\dagger-a_2)+\eta^*(a_1-\mu a_2^\dagger) \\
&+a_1^\dagger a_2^\dagger+a_1a_2-a_1^\dagger a_1-a_2^\dagger a_2\Big\}: \\
=&\frac{2\mu}{1+\mu^2}:\exp\Big\{\frac{\mu^2}{1+\mu^2}(a_1^\dagger\mu-a_2)(a_1-\mu a_2^\dagger) \\
&-(a_1-a_2^\dagger)(a_1^\dagger-a_2)\Big\}: \\
=&\mathrm{e}^{-a_1^\dagger a_2^\dagger\tanh\lambda}\mathrm{e}^{(a_1^\dagger a_1+a_2^\dagger a_2+1)\ln\mathrm{sech}\lambda}\mathrm{e}^{a_1a_2\tanh\lambda}, \qquad (10)
\end{aligned}
$$

where :: denotes normal ordering. When $\mu=1$,

$$\int\frac{\mathrm{d}^2\eta}{\pi}\,|\eta\rangle\langle\eta|=1, \qquad (11)$$

so the set of $|\eta\rangle$ is complete. The two-mode squeezed vacuum state is

$$S\,|00\rangle=\mathrm{sech}\lambda\exp[a_1^\dagger a_2^\dagger\tanh\lambda]\,|00\rangle\,. \qquad (12)$$

The more generalized two-mode squeezed state is in the form of

$$C\exp[A(a_1^{\dagger2}+a_2^{\dagger2})+Ba_1^\dagger a_2^\dagger]\,|00\rangle\,, \qquad (13)$$

where C is the normalization constant determined by A and B. Then an interesting question naturally arises: what is the second-mode field after the measurement of this state by $|x\rangle_{11}\langle x|$?

To solve this problem, we replace A and B by another two independent parameters μ and ν in the way of

$$
\begin{aligned}
&A=\frac{\mu^2-\nu^2}{L},\ B=\frac{2(1-\mu^2\nu^2)}{L}, \\
&L=2(1+\mu^2)(1+\nu^2),\ \ C=2\sqrt{\frac{\mu\nu}{L}}, \qquad (14)
\end{aligned}
$$

since from Eq.(14) one can solve μ and ν from the known A and B. Then Eq.(13) can be recast into the form

$$
\begin{aligned}
&2\sqrt{\frac{\mu\nu}{L}}\exp\Big[\frac{\mu^2-\nu^2}{L}(a_1^{\dagger2}+a_2^{\dagger2}) \\
&+\frac{2(1-\mu^2\nu^2)}{L}a_1^\dagger a_2^\dagger\Big]\,|00\rangle\equiv|\,\rangle_{\mu,\nu}\,. \qquad (15)
\end{aligned}
$$

It seems hard to calculate the result of measurement by $|x\rangle_{11}\langle x|$, since ${}_1\langle x|\,|\,\rangle_{\mu,\nu}$ is difficult to evaluate.

3. Measurement performed on $U(\mu,\nu)|00\rangle$ by $|x\rangle_{11}\langle x|$

The difficulty can be solved when we notice that $|\,\rangle_{\mu,\nu}$ can be expressed as[14]

$$
\begin{aligned}
|\,\rangle_{\mu,\nu}&=U(\mu,\nu)\,|00\rangle \\
&=\sqrt{\mu\nu}\int\frac{\mathrm{d}^2\eta}{\pi}\,|\mu\eta_1,\nu\eta_2\rangle\langle\eta|\,00\rangle\,. \qquad (16)
\end{aligned}
$$

Note that $U(\mu,\nu)$ is unitary, since

$$
\begin{aligned}
&U^\dagger(\mu,\nu)U(\mu,\nu)=1, \\
&U^\dagger(\mu,\nu)=U(\frac{1}{\mu},\frac{1}{\nu})=U^{-1}(\mu,\nu). \qquad (17)
\end{aligned}
$$

U squeezes the two-mode quadratures as

$$
\begin{aligned}
U(X_1+X_2)U^{-1}&=\nu(X_1+X_2), \\
U(P_1+P_2)U^{-1}&=\frac{P_1+P_2}{\nu}, \qquad (18)
\end{aligned}
$$

and

$$
\begin{aligned}
&U(P_1-P_2)U^{-1}=\mu(P_1-P_2), \\
&U(X_1-X_2)U^{-1}=\frac{X_1-X_2}{\mu}. \qquad (19)
\end{aligned}
$$

Now we can directly calculate ${}_1\langle x|\,U|00\rangle$ (i.e. its collapse result after the measurement). Using the Schmidt decomposition[19]

$$
\begin{aligned}
|\eta&=\eta_1+\mathrm{i}\eta_2\rangle \\
&=\mathrm{e}^{-\mathrm{i}\eta_2\eta_1}\int_{-\infty}^{\infty}\mathrm{d}x\,|x\rangle_1\otimes\Big|x-\sqrt{2}\eta_1\Big\rangle_2\mathrm{e}^{\mathrm{i}\sqrt{2}\eta_2 x}, \qquad (20)
\end{aligned}
$$

where

$$|x\rangle_i=\Big(\frac{1}{\pi}\Big)^{1/4}\exp\Big\{-\frac{1}{2}x^2+\sqrt{2}xa_i^\dagger-\frac{a_i^{\dagger2}}{2}\Big\}|0\rangle_i \qquad (21)$$

is the coordinate eigenvector in Fock space, we derive

$$
\begin{aligned}
&{}_1\langle x|\,U|00\rangle=\sqrt{\mu\nu}\,{}_1\langle x|\int\frac{\mathrm{d}^2\eta}{\pi}\,|\mu\eta_1,\nu\eta_2\rangle\langle\eta|\,|00\rangle \\
=&\sqrt{\mu\nu}\,{}_1\langle x|\int\frac{\mathrm{d}^2\eta}{\pi}\,|\mu\eta_1,\nu\eta_2\rangle\,\mathrm{e}^{-(\eta_1^2+\eta_2^2)/2} \\
=&\sqrt{\mu\nu}\int\frac{\mathrm{d}^2\eta}{\pi}\Big|x-\sqrt{2}\mu\eta_1\Big\rangle_2 \\
&\times\mathrm{e}^{\mathrm{i}\sqrt{2}\nu\eta_2 x}\mathrm{e}^{-(\eta_1^2+\eta_2^2)/2}\mathrm{e}^{-\mathrm{i}\eta_2\eta_1\mu\nu}. \qquad (22)
\end{aligned}
$$

Performing the integration over $\mathrm{d}\eta_2$,

$$
\begin{aligned}
&\int_{-\infty}^{\infty}\mathrm{d}\eta_2\exp[-\eta_2^2/2-\mathrm{i}\eta_2\eta_1\mu v+\mathrm{i}\sqrt{2}\eta_2 x\nu] \\
=&\sqrt{2\pi}\exp[-\frac{1}{2}(\sqrt{2}x\nu-\eta_1\mu\nu)^2] \qquad (23)
\end{aligned}
$$

and then substituting Eq.(23) into Eq.(22), we have

$$
\begin{aligned}
{}_1\langle x|\,U|00\rangle =&\sqrt{2\mu\nu}\int\frac{\mathrm{d}\eta_1}{\pi^{1/4}\sqrt{\pi}}\exp\Big\{-\frac{\eta_1^2}{2}(1+\mu^2\nu^2+2\mu^2)+\sqrt{2}\eta_1\mu(\nu^2x+x-\sqrt{2}a_2^\dagger)\\
&+\sqrt{2}xa_2^\dagger-x^2\nu^2-\frac{1}{2}x^2-\frac{a_2^{\dagger2}}{2}\Big\}\,|0\rangle_2\\
=&\left(\frac{\mu\nu}{\mu^2\nu^2+2\mu^2+1}\right)^{\frac{1}{2}}\frac{2}{\pi^{1/4}}\exp\Big\{\frac{\mu^2}{1+\mu^2\nu^2+2\mu^2}(\nu^2x+x-\sqrt{2}a_2^\dagger)^2\\
&+\sqrt{2}xa_2^\dagger-x^2\nu^2-\frac{1}{2}x^2-\frac{a_2^{\dagger2}}{2}\Big\}\,|0\rangle_2\\
=&\frac{2}{\pi^{1/4}}\sqrt{\frac{\mu\nu}{\mu^2\nu^2+2\mu^2+1}}\exp\Big\{-\frac{1}{2(\mu^2\nu^2+2\mu^2+1)}[(2\nu^2+\mu^2\nu^2+1)x^2\\
&-2(1-\mu^2\nu^2)\sqrt{2}xa_2^\dagger-(2\mu^2-1-\mu^2\nu^2)a_2^{\dagger2}\Big\}\,|0\rangle_2\,,
\end{aligned}
\tag{24}
$$

which shows that after the collapse the second-mode field becomes a single-mode squeezed state which is related with both μ and ν. In particular, when $\mu=\nu$, $U(\mu,\nu)\to S$, Eq.(24) becomes

$$
\begin{aligned}
{}_1\langle x|\,S|00\rangle =&\frac{1}{\pi^{1/4}}\frac{2\mu}{1+\mu^2}\exp\Big\{-\frac{x^2}{2}+\frac{\sqrt{2}xa_2^\dagger(1-\mu^2)}{\mu^2+1}-\frac{(1-\mu^2)^2a_2^{\dagger2}}{2(\mu^2+1)^2}\Big\}\,|0\rangle_2\\
=&\pi^{-1/4}\mathrm{sech}\lambda\exp\Big\{-\frac{x^2}{2}-\sqrt{2}xa_2^\dagger\tanh\lambda-\frac{a_2^{\dagger2}}{2}\tanh^2\lambda\Big\}\,|0\rangle_2\,,
\end{aligned}
\tag{25}
$$

where in the last step we have set

$$
\mu=\mathrm{e}^\lambda,\ \tanh\lambda=\frac{\mu^2-1}{\mu^2+1},\ \mathrm{sech}\lambda=\frac{2\mu}{\mu^2+1}. \tag{26}
$$

Equation (25) indicates that the second-mode field is a single-mode squeezed state, and the squeezing is enhanced from $\tanh\lambda\ \to\tanh^2\lambda$, referring to the term $\frac{a_2^{\dagger2}}{2}\tanh^2\lambda$ in Eq.(25) (note that the original two-mode squeezed state has a term $a_1^\dagger a_2^\dagger\tanh\lambda$, see Eq.(12)). The result of Eq.(25) can be further checked by recalling the coordinate representation of the two-mode squeezing operator[20]

$$
S^{-1}=\int\int_{-\infty}^{\infty}\mathrm{d}x_1\mathrm{d}x_2\,|x_1\cosh\lambda+x_2\sinh\lambda,x_2\cosh\lambda+x_1\sinh\lambda\rangle\,\langle x_1,x_2|\,, \tag{27}
$$

so

$$
\begin{aligned}
{}_1\langle x|\,S^{-1}|00\rangle =&\int\int_{-\infty}^{\infty}\mathrm{d}x_1'\mathrm{d}x_2'\delta(x-x_1'\cosh\lambda-x_2'\sinh\lambda)\,|x_2'\cosh\lambda+x_1'\sinh\lambda\rangle_2\ \ \langle x_1',x_2'|\,00\rangle\\
=&\mathrm{sech}\lambda\int\mathrm{d}x_2'\,|x_2'\mathrm{sech}\lambda+x\tanh\lambda\rangle_2\ \ \langle(x\mathrm{sech}\lambda-x_2'\tanh\lambda),x_2'|\,00\rangle\,.
\end{aligned}
\tag{28}
$$

Using Eq.(21) we have

$$
\begin{aligned}
{}_1\langle x|\,S^{-1}|00\rangle =&\mathrm{sech}\lambda(\frac{1}{\pi})^{1/4}\int_{-\infty}^{\infty}\frac{\mathrm{d}x_2'}{\sqrt{\pi}}\exp\Big\{-x_2'^2+\sqrt{2}a_2^\dagger\mathrm{sech}\lambda x_2'-\frac{1}{2}x^2+\sqrt{2}xa_2^\dagger\tanh\lambda-\frac{a_2^{\dagger2}}{2}\Big\}\,|0\rangle_2\\
=&\pi^{-1/4}\mathrm{sech}\lambda\exp\Big\{-\frac{x^2}{2}+\sqrt{2}xa_2^\dagger\tanh\lambda-\frac{a_2^{\dagger2}}{2}\tanh^2\lambda\Big\}\,|0\rangle_2\,.
\end{aligned}
\tag{29}
$$

Let $S^{-1} \to S$, $\tanh\lambda \to -\tanh\lambda$, Eq.(29) turns to be the same as Eq.(25).

4. Measurement performed on $V|00\rangle$ by $|x\rangle_{11}\langle x|$

Now we consider the one- and two-mode combinatorial state in Eq.(3). Using $\lambda_1 = \lambda e^{\gamma}$, $\lambda_2 = \lambda e^{-\gamma}$, Eq.(3) becomes

$$V|00\rangle = \exp[-i\lambda(e^{\gamma}X_1P_2 + e^{-\gamma}X_2P_1)]|00\rangle . \quad (30)$$

It seems difficult to calculate ${}_1\langle x|V|00\rangle$. However, as shown in Ref.[9], V has its representation in the two-mode coordinate eigenstate space,

$$\begin{aligned} V =& \exp[-i\lambda(e^{\gamma}X_1P_2 + e^{-\gamma}X_2P_1)] \\ =& \int\int_{-\infty}^{\infty} dx_1 dx_2 \, |x_1\cosh\lambda + x_2 e^{-\gamma}\sinh\lambda, x_2\cosh\lambda + x_1 e^{\gamma}\sinh\lambda\rangle\langle x_1, x_2| . \end{aligned} \quad (31)$$

Using Eqs.(31) and (21), we have

$$\begin{aligned} & {}_1\langle x|\exp[-i\lambda(e^{\gamma}X_1P_2 + e^{-\gamma}X_2P_1)]|00\rangle \\ =& \int\int_{-\infty}^{\infty} dx_1' dx_2' \delta(x - x_1'\cosh\lambda - x_2' e^{-\gamma}\sinh\lambda)\,|x_2'\cosh\lambda + x_1' e^{\gamma}\sinh\lambda\rangle_2\langle x_1', x_2'|\,00\rangle \\ =& \mathrm{sech}\lambda \int_{-\infty}^{\infty} dx_2' \, |x_2'\cosh\lambda + (x e^{\gamma} - x_2'\sinh\lambda)\tanh\lambda\rangle_2 \;\; \langle (x\,\mathrm{sech}\lambda - x_2' e^{-\gamma}\tanh\lambda), x_2'|\,00\rangle \\ =& \mathrm{sech}\lambda \int_{-\infty}^{\infty} dx_2' \, |x_2'\mathrm{sech}\lambda + x e^{\gamma}\tanh\lambda\rangle_2 \;\; \langle (x\,\mathrm{sech}\lambda - x_2' e^{-\gamma}\tanh\lambda), x_2'|\,00\rangle \\ =& \mathrm{sech}\lambda \frac{1}{\pi^{1/4}} \int_{-\infty}^{\infty} \frac{dx_2'}{\sqrt{\pi}} \exp\Big\{ -\frac{1}{2}x_2'^2(1 + \mathrm{sech}^2\lambda + e^{-2\gamma}\tanh^2\lambda) + \sqrt{2}x_2' a_2^{\dagger}\mathrm{sech}\lambda \\ & + x x_2'\mathrm{sech}\lambda\tanh\lambda(e^{-\gamma} - e^{\gamma}) + \sqrt{2}x a_2^{\dagger} e^{\gamma}\tanh\lambda - \frac{a_2^{\dagger 2}}{2} - \frac{1}{2}x^2(\mathrm{sech}^2\lambda + e^{2\gamma}\tanh^2\lambda)\Big\}|0\rangle_2 \\ =& \frac{1}{\pi^{1/4}}\sqrt{\frac{2}{1 + \cosh^2\lambda + e^{-2\gamma}\sinh^2\lambda}} \exp\Big\{ \frac{(a_2^{\dagger} - \sqrt{2}x\tanh\lambda\sinh\gamma)^2}{1 + \cosh^2\lambda + e^{-2\gamma}\sinh^2\lambda} - \frac{a_2^{\dagger 2}}{2} \\ & + \sqrt{2}x a_2^{\dagger} e^{\gamma}\tanh\lambda - \frac{1}{2}x^2(\mathrm{sech}^2\lambda + e^{2\gamma}\tanh^2\lambda)\Big\}|0\rangle_2 . \end{aligned} \quad (32)$$

This is also a single-mode squeezed state, related to both λ and γ; in particular, when $\gamma = 0$, $\lambda_1 = \lambda_2 = \lambda$, Eq.(32) reduces to Eq.(29).

5. Measurement performed on 3-parameter SU(1,1) squeezed state by $|x\rangle_{11}\langle x|$ and the single-mode squeezed state projector

In some cases we can also employ the coherent state representation. For example, the 3-parameter SU(1,1) squeezing operator is

$$\begin{aligned} F^{(r,s)} =& \exp\left(\frac{r}{s^*}a_1^{\dagger}a_2^{\dagger}\right)\exp[(a_1^{\dagger}a_1 + a_2^{\dagger}a_2 + 1) \\ & \times \ln(s^*)^{-1}]\exp\left(-\frac{r^*}{s^*}a_1 a_2\right), \end{aligned} \quad (33)$$

where $|s|^2 - |r|^2 = 1$ guarantees the unitarity of $F^{(r,s)}$. The corresponding state is

$$\frac{1}{s^*}\exp\left(\frac{r}{s^*}a_1^{\dagger}a_2^{\dagger}\right)|00\rangle . \quad (34)$$

Using the completeness relation of the coherent state [11,12]

$$\int \frac{d^2z_1 d^2z_2}{\pi^2}\,|z_1, z_2\rangle\langle z_1, z_2| = 1 \quad (35)$$

and the mathematical formula [20]

$$\begin{aligned} & \int \frac{d^2z}{\pi}\exp(\zeta|z|^2 + \xi z + \eta z^* + f z^2 + g z^{*2}) \\ =& \frac{1}{\sqrt{\zeta^2 - 4fg}}\exp\left[\frac{-\zeta\xi\eta + \xi^2 g + \eta^2 f}{\zeta^2 - 4fg}\right], \end{aligned} \quad (36)$$

whose convergent condition is either

$$\mathrm{Re}(\zeta + f + g) < 0, \;\; \mathrm{Re}\left(\frac{\zeta^2 - 4fg}{\zeta + f + g}\right) < 0, \quad (37)$$

or

$$\mathrm{Re}(\zeta - f - g) < 0, \quad \mathrm{Re}\left(\frac{\zeta^2 - 4fg}{\zeta - f - g}\right) < 0, \tag{38}$$

we have

$$\begin{aligned}
{}_1\langle x|\, F^{(r,s)}\, |00\rangle =& \frac{1}{s^*}\, {}_1\langle x| \int \frac{\mathrm{d}^2 z_1 \mathrm{d}^2 z_2}{\pi^2} |z_1, z_2\rangle \langle z_1, z_2| \exp\left(\frac{r}{s^*} a_1^\dagger a_2^\dagger\right) |00\rangle \\
=& \frac{\pi^{-1/4}}{s^*} \int \frac{\mathrm{d}^2 z_1 \mathrm{d}^2 z_2}{\pi^2} |z_2\rangle \exp\left\{ -\frac{x^2}{2} + \sqrt{2} x z_1 - \frac{z_1^2}{2} - |z_1|^2 + \frac{r}{s^*} z_1^* z_2^* - \frac{1}{2}|z_2|^2 \right\} \\
=& \frac{\pi^{-1/4}}{s^*} \exp\left\{ -\frac{x^2}{2} + \sqrt{2} x \frac{r}{s^*} a_2^\dagger - \frac{1}{2}\left(\frac{r}{s^*}\right)^2 a_2^{\dagger 2} \right\} |0\rangle ,
\end{aligned} \tag{39}$$

which is the generalization of Eq.(29).

Finally, we consider the case when a measurement for the state (34) is made with the result of first-mode field being in the single-mode squeezed state $\mathrm{sech}^{1/2}\tau \exp\left(\frac{a_1^{\dagger 2}}{2} \tanh\tau\right) |0\rangle_1$, then the second-mode field must be in the state

$$\begin{aligned}
& \frac{1}{s^*} \mathrm{sech}^{1/2}\tau\, {}_1\langle 0| \exp\left(\frac{a_1^2}{2}\tanh\tau\right) \exp\left(\frac{r}{s^*} a_1^\dagger a_2^\dagger\right) |00\rangle \\
=& \frac{1}{s^*} \mathrm{sech}^{1/2}\tau\, {}_1\langle 0| \exp\left(\frac{a_1^2}{2}\tanh\tau\right) \int \frac{\mathrm{d}^2 z_1 \mathrm{d}^2 z_2}{\pi^2} |z_1, z_2\rangle \langle z_1, z_2| \exp\left(\frac{r}{s^*} a_1^\dagger a_2^\dagger\right) |00\rangle \\
=& \frac{1}{s^*} \mathrm{sech}^{1/2}\tau \int \frac{\mathrm{d}^2 z_1 \mathrm{d}^2 z_2}{\pi^2} |z_2\rangle \exp\left\{ -|z_1|^2 + z_1^2 \frac{\tanh\tau}{2} + \frac{r}{s^*} z_1^* z_2^* - \frac{1}{2}|z_2|^2 \right\} \\
=& \frac{1}{s^*} \mathrm{sech}^{1/2}\tau \int \frac{\mathrm{d}^2 z_2}{\pi} \exp\left\{ -|z_2|^2 + z_2 a_2^\dagger + \frac{\tanh\tau}{2}\left(\frac{r}{s^*}\right)^2 (z_2^*)^2 \right\} |0\rangle_2 \\
=& \frac{1}{s^*} \mathrm{sech}^{1/2}\tau \exp\left\{ \frac{\tanh\tau}{2}\left(\frac{r}{s^*}\right)^2 a_2^{\dagger 2} \right\} |0\rangle_2 ,
\end{aligned} \tag{40}$$

which tells us that the second-mode field becomes a single-mode squeezed state too whose squeezing not only depends on the original squeezing parameter $\frac{r}{s^*}$ (the square of $\frac{r}{s^*}$), but also the squeezing parameter of the first-mode field. This means that the quantum entanglement is in squeezing. In this sense, the second mode can be controlled by the first mode.

In summary, we have calculated the result of one-mode quadrature-amplitude measurement for various generalized two-mode squeezed states by virtue of the entangled state representation of the corresponding two-mode squeezing operators. We find that the remaining field-mode simultaneously collapses to the single-mode squeezed state whose squeezing is enhanced, and may be controlled by the measurement itself.

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