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Mass neutrino oscillations in Robertson–Walker space–time*

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Along the geodesic we calculate the interference phase of the mass neutrinos propagating in the radial direction in Robertson–Walker space–time. Since our universe is expanding, the phase factor Φ is increasing under the condition of the same proper physical distance l. Different values of curvature parameter k in Robertson–Walker metric represent different cosmological models, correspondingly, we obtain the different interference phases.

Keywords: interference phase, mass neutrino, Robertson–Walker space–time
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1. Introduction

Neutrino oscillations have been a hot topic recently. Neutrino oscillations in a flat space–time have been extensively studied in the past by using both plane waves[1] and wave packets.[2] Many authors have calculated gravitational effects on neutrino oscillations in curved space–time. Also, mass neutrino oscillations have aroused great interest in high energy experimental and theoretical physics recently, as a consequence of, in particular, the high confidence atmospheric neutrino experiment of Super-Kamiokande to assure the neutrino mass.[3] In 1996, Fornengo and co-workers[4] studied the interference phase of the high-energy mass neutrinos when propagating in the radial and nonradial directions in Schwarzschild field. In 2001, Zhang and Beesham[5] also studied the propagation of neutrinos in Schwarzschild field with a different method, and they obtained that the phase calculated along the null is equivalent to the half phase along the geodesic in high energy limit, which means that the correct relative phase of the mass neutrinos is either the null phase or the half geodesic phase. However, the factor of 2 will be automatically deleted when considering the two neutrinos arrival time difference.[6] In 2003, they also analysed in detail the mass neutrino phase along the geodesic line and the null line in curved and flat space–time, separately.[7] With the same method, in 2003 and 2004, along the geodesic we calculated the interference phase of mass neutrinos in Kerr space–time[8] and Reissner–Nordstrom space–time,[9] respectively.

In this paper, we will discuss the interference phase of mass neutrinos in the Robertson–Walker (R–W) space–time. From the result, we will study the influences of the cosmological scale factor $R(t)$ and the curvature parameter k on the phase. We set $G = \hbar = c = 1$ throughout this paper.

2. Interference phase in R–W space–time

In a curved space–time, the phase factor is conventionally written as:[10]

$$\Phi = \int m \mathrm{d}s, \tag{1}$$

where the phase factor Φ is also the classical Lagrange function for the particle motion, and the route is along the geodesic determined by the variational principle or Hamilton–Jacob equation,[11] m is the mass of neutrino and $\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu$.

Now we follow the method by Zhang and Beesham[5] to study the phase of the neutrino oscil-

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lating along the radial direction in R–W space–time, which is represented by

$$\mathrm{d}s^2 = \mathrm{d}t^2 + g_{11}\mathrm{d}r^2 + g_{22}\mathrm{d}\theta^2 + g_{33}\mathrm{d}\varphi^2. \tag{2}$$

Here $g_{11} = -\dfrac{R^2(t)}{1-kr^2}$, $g_{22} = -R^2(t)r^2$, $g_{33} = -R^2(t)r^2\sin^2\theta$, the cosmological scale factor $R(t)$ is the arbitrary function of time t, and k =0, ±1 is the curvature parameter.

We consider the phase of the mass neutrinos in R–W space–time by means of the particle geodesic equation

$$\frac{\mathrm{d}^2x^\mu}{\mathrm{d}s^2} + \Gamma^\mu_{\alpha\beta}\frac{\mathrm{d}x^\alpha}{\mathrm{d}s}\frac{\mathrm{d}x^\beta}{\mathrm{d}s} = 0. \tag{3}$$

From Eq.(3), we can obtain the time-like nontrivial equation

$$\frac{\mathrm{d}^2t}{\mathrm{d}s^2} - H\left[g_{11}\left(\frac{\mathrm{d}r}{\mathrm{d}s}\right)^2 + g_{22}\left(\frac{\mathrm{d}\theta}{\mathrm{d}s}\right)^2 + g_{33}\left(\frac{\mathrm{d}\varphi}{\mathrm{d}s}\right)^2\right] = 0, \tag{4}$$

where $H = \dfrac{\dot{R}(t)}{R(t)}$ is the Hubble constant. Resorting to the R–W metric, and setting $\mathrm{d}\theta = \mathrm{d}\varphi = 0$, we obtain

$$\frac{\mathrm{d}^2t}{\mathrm{d}s^2} + H\left(\frac{\mathrm{d}t}{\mathrm{d}s}\right)^2 - H = 0, \tag{5}$$

or

$$\frac{\mathrm{d}}{\mathrm{d}s}\left(R\frac{\mathrm{d}t}{\mathrm{d}s}\right) - \frac{\mathrm{d}R}{\mathrm{d}s}\frac{\mathrm{d}s}{\mathrm{d}t} = 0. \tag{6}$$

Letting $\dfrac{\mathrm{d}t}{\mathrm{d}s} = y$, then Eq.(6) becomes

$$\frac{\mathrm{d}(Ry)}{\mathrm{d}s} - \frac{1}{y}\frac{\mathrm{d}R}{\mathrm{d}s} = 0. \tag{7}$$

Solving Eq.(7), we have

$$\frac{\mathrm{d}t}{\mathrm{d}s} = y = \frac{\sqrt{R^2+1}}{R}. \tag{8}$$

On the other hand, from the R–W metric with $\mathrm{d}\theta = \mathrm{d}\varphi = 0$, we can obtain

$$\frac{\mathrm{d}s}{\mathrm{d}r} = \left[\frac{R^2(t)}{1-kr^2}\frac{1}{\left(\dfrac{\mathrm{d}t}{\mathrm{d}s}\right)^2 - 1}\right]^{\frac{1}{2}}. \tag{9}$$

Substituting Eq.(8) into Eq.(9), we have

$$\frac{\mathrm{d}s}{\mathrm{d}r} = \frac{R^2(t)}{\sqrt{1-kr^2}}. \tag{10}$$

Thus the accurate phase factor in R–W space–time along the radial direction will be obtained by the following calculation:

$$\begin{aligned}\Phi = \int m\left(\frac{\mathrm{d}s}{\mathrm{d}r}\right)\mathrm{d}r &= mR^2(t)\int_{r_0}^{r}\frac{\mathrm{d}r}{\sqrt{1-kr^2}} \\ &= mR(t)l,\end{aligned} \tag{11}$$

where $l = R(t)\displaystyle\int_{r_0}^{r}\frac{\mathrm{d}r}{\sqrt{1-kr^2}}$ is the proper physical distance. On the other hand, because the covariant energy P_0 of the neutrino can be defined as $E = P_0 = mg_{00}\dfrac{\mathrm{d}t}{\mathrm{d}s}$,[5] Eq.(11) can be rewritten as

$$\Phi = mR(t)l \approx \frac{m^2}{P_0}l. \tag{12}$$

3. Conclusion

In this paper, we have given the interference phase of mass neutrinos propagating in the radial direction along the geodesic in R–W space–time. For the scale factor $R(t)$ as a function of time t, the phase factor Φ will vary with t. Since our universe is expanding, correspondingly, the phase factor Φ is increasing under the condition of the same proper physical distance l. On the other hand, when the curvature parameter k takes 0, ±1, respectively, the proper physical distance l will have different integral result, which corresponds to the different phase factor Φ. This gravitational effect[12–15] on neutrino oscillations is intimately related to the redshift.

Now, particle physics and cosmology are required to study deeply the properties of neutrino. The establishment of these properties will produce great influence on the research of the origin, the evolvement and the final destiny of the universe. So, as one of the properties, we believe that the study on mass neutrino oscillation is a very important work.

Here we only calculate the interference phase in the radial direction. The calculation in the nonradial direction is also meaningful. So we will keep paying attention to the work in the future.

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