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Two-dimensional relativistic electromagnetic dromion-like soliton in a cold transparent plasma*

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By using a standard multiple scale method, a Davey–Stewartson (DS) equation has been derived and also applied to a multi-dimensional analytical investigation on the interaction of an ultra-intense laser pulse with a cold unmagnetized transparent electron-ion plasma. The regions of instability are found by considering the modulation instability of a plane wave solution of the DS equation. The DS equation is just of the Davey–Stewartson 1 (DS1) type and admits a dromion solution, i.e. a two-dimensional (2D) dromion soliton decaying exponentially in all spatial directions. A 2D relativistic electromagnetic dromion-like soliton (2D REDLS) is derived for a vector potential.

Keywords: relativistic electromagnetic soliton, laser plasma
PACC: 5235S, 5240D

1. Introduction

In the last decade,[1] the rapid development of laser technology, especially CPA (chirped pulse amplification), made available the table top laser sources of femto-second pulses with intensities up to $10^{21}\mathrm{W/cm}^2$, which empowers us to study many nonlinear phenomena, such as high-order harmonic generation,[2,3] plasma channels, vortices, etc. The relativistic electromagnetic solitons (RESes)[4,5] have attracted much attention of all due to their robust characteristics. In the RES, the dispersion effect that is due to the finite electron inertia in response to the electromagnetic field action is balanced by the nonlinearity coming from both the relativistic increase in electron mass and the plasma density redistribution under the action of the ponderomotive forces expelling electrons from the regions of high field intensity.

A relativistic electromagnetic soliton is possessed of a coherent structure that can be generated by the interaction of an ultra-intense laser pulse with an under-dense[6] or ultra-dense[7] plasma, which is found in 1-dimensional(1D),[8] 2-dimensional(2D)[9] and 3-dimensional(3D)[10] Particle-in-cell (PIC) simulations. In an inhomogeneous plasma the RES moves with an acceleration speed that is proportional to the density gradient. At some critical density the soliton emits its energy in the form of a low frequency short electromagnetic burst.[11] However Esirkepov *et al*[12] recently show that the 3D soliton emits high-frequency electromagnetic radiation, whose frequency is much higher than the Langmuir frequency. This radiation emanates from the electron density hump rotating in the wall of the soliton cavity, similar to coherent synchrotron emission.[13]

The analytical investigations dealing with the RES are rapidly conducted by many authors following a pioneer work by Marburger and Tooper,[14] where the exact relativistically intense optical standing waves were considered for the first time and all ingredients describing 1D circularly-polarized RES were given. Electromagnetic solitons with relativistic amplitudes were first investigated by Kozlov *et al*,[15] Their study continued being carried out in Refs.[16–20]. An exact 1D analytical RES solution of the cold fluid equation for the electrons with ions fixed in the

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case of an arbitrary intense laser pulse is given in Ref.[21], which has been extended to the case of a soliton in a uniform magnetized plasma.[22] The 1D solitons moving in a plasma in which electrons and ions are movable have been reconsidered and the new results on the effects of the ion motion on the soliton structure have been found.[23]

But the above analytical investigations are all restricted within 1D geometry, which is not realistic in the sense of laboratory devices. The solitary waves observed in laboratory devices are certainly not restricted within only one dimension. The multi-dimensional analytical analysis of RES, however, is widely open except for a few of articles[24] devoted to the analytical investigations on RES by numerical integration of a generalized nonlinear Schrödinger equation. In this paper, we make a multi-dimensional analytical investigation on the interaction of an intense laser pulse with a cold unmagnetized transparent electron-ion plasma and derive a Davey–Stewartson (DS) equation to describe the system. The DS equation is just the equation of Davey–Stewartson 1 (DS1) type and admits the 2D dromion solution decaying exponentially in all spatial directions. Then we obtain a relativistic electromagnetic dromion soliton (REDS) in the interaction of an intense laser pulse with a plasma. In Section 2, we derive the DS1 equation by a standard multiple scale method. The modulation instability of a plane wave solution of the DS equation is discussed in Section 3. In Section 4 we give a 2D REDS solution. The discussion and conclusion are presented at the end of the article.

2. Basic equation

To start with, we study finite amplitude waves in a cold transparent plasma and assume the ions to be at rest, and consider the model of an unbounded cold collisionless plasma described by a generalized nonlinear Schrödinger equation. Here and in the following collisions and thermal effects are neglected, for the collision time is typically much longer than the laser pulse duration; and the thermal velocity is typically much less than the quiver velocity of an electron in a laser field. The generalized nonlinear Schrödinger equation is normalized by the following conditions, $e\boldsymbol{A}/m_{\mathrm{e}}c \to \boldsymbol{A}$, $\omega_{\mathrm{e}}\boldsymbol{r}/c \to \boldsymbol{r}$, where $\omega_{\mathrm{e}} = (n_0 e^2/\varepsilon_0 m_{\mathrm{e}})^{1/2}$ is the electron frequency, n_0 is the unperturbed number density of electrons.[25]

$$\left(2\mathrm{i}\frac{\partial}{\partial z} + \nabla_\perp^2 + \sigma\right) A = \frac{1 + \nabla_\perp^2 (1 + |A|^2)^{1/2}}{(1 + |A|^2)^{1/2}} A, \quad (1)$$

with $\sigma = (\alpha^2 - 1)/\mu^2\alpha^2$, where the free small parameter α comes from the method of multiple scale. The properties of Eq.(1) are well known in the theory of wave collapse[25] and the self-focusing[26] problem in the cold plasma.

Considering the limit of the weak nonlinearity, we seek for the solution of Eq.(1) that is reduced to

$$\left(2\mathrm{i}\frac{\partial}{\partial z} + \nabla_\perp^2 + \sigma\right) A = f(|A|)\, A, \quad (2)$$

with

$$f(|A|) = \left(1 - \frac{1}{2}|A|^2\right)\left[1 + \left(\frac{\partial |A|}{\partial x}\right)^2 + \left(\frac{\partial |A|}{\partial y}\right)^2 + |A|\left(\frac{\partial^2 |A|}{\partial x^2} + \frac{\partial^2 |A|}{\partial y^2}\right)\right].$$

Noticing that A is complex and taken as $A = P\mathrm{e}^{\mathrm{i}Q}$, where P and Q are real functions, Eq.(2) becomes

$$2\frac{\partial P}{\partial z} + 2\left(\frac{\partial P}{\partial x}\frac{\partial Q}{\partial x} + \frac{\partial P}{\partial y}\frac{\partial Q}{\partial y}\right) + P\left(\frac{\partial^2 Q}{\partial x^2} + \frac{\partial^2 Q}{\partial y^2}\right) = 0, \quad (3)$$

$$2P\frac{\partial Q}{\partial z} - \left(1 - P^2 + \frac{1}{2}P^4\right)\left(\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2}\right) - \sigma P + P - \frac{1}{2}P^3 + P\left[\left(\frac{\partial Q}{\partial x}\right)^2 + \left(\frac{\partial Q}{\partial y}\right)^2\right] + \left(P - \frac{1}{2}P^3\right)\left[\left(\frac{\partial P}{\partial x}\right)^2 + \left(\frac{\partial P}{\partial y}\right)^2\right] = 0. \quad (4)$$

To derive the DS equation, we employ a standard multiple scale method. We introduce the following multiple scale variables $\xi = \varepsilon(x - v_{\mathrm{g}}z)$, $\eta = \varepsilon y$ and $T = \varepsilon^2 z$, where $\theta = kx - \omega z$, ε is a small parameter, and v_{g} is a constant that can be determined by a solvability condition. We then expand the variables P and Q in terms of the expansion parameter ε as

$$P = P^{(0)} + \varepsilon P^{(1)} + \varepsilon^2 P^{(2)} + \cdots, \quad (5)$$

$$Q = \varepsilon Q^{(1)} + \varepsilon^2 Q^{(2)} + \cdots. \tag{6}$$

The constant term $P^{(0)}$ in the expansion of P denotes the continuous wave (cw) background, which is much smaller than 1 because of the limit of weak nonlinearity just under consideration here. It can be seen that the presence of the cw background is critical to the formation of the stable REDS. The variables are the real functions of the multiple scale variables $x, \xi; y, T$; and z, T, where x, y, z are called fast variables and ξ, η, T are called slow variables. Substituting the expansions (5) and (6) into Eqs.(3) and (4), we obtain

$$2\frac{\partial P^{(n)}}{\partial z} + P^{(0)}\left(\frac{\partial^2 Q^{(n)}}{\partial x^2} + \frac{\partial^2 Q^{(n)}}{\partial y^2}\right) = \chi_1^{(n)}, \tag{7}$$

$$\begin{aligned} &2P^{(0)}\frac{\partial Q^{(n)}}{\partial z} - \left(1 - P^{(0)2} + \frac{1}{2}P^{(0)4}\right) \\ &\times \left(\frac{\partial^2 P^{(n)}}{\partial x^2} + \frac{\partial^2 P^{(n)}}{\partial y^2}\right) \\ &+ \left(1 - \sigma - \frac{3}{2}P^{(0)2}\right)P^{(n)} = \chi_2^{(n)}, \end{aligned} \tag{8}$$

where $n = 1, 2, 3\ldots$ and the explicit expressions of $\chi_1^{(n)}$ and $\chi_2^{(n)}$ are

$$\chi_1^{(1)} = 0, \tag{9}$$

$$\begin{aligned} \chi_1^{(2)} =& 2v_g\frac{\partial P^{(1)}}{\partial \xi} - 2\left(\frac{\partial P^{(1)}}{\partial x}\frac{\partial Q^{(1)}}{\partial x} + \frac{\partial P^{(1)}}{\partial y}\frac{\partial Q^{(1)}}{\partial y}\right) \\ &- P^{(1)}\left(\frac{\partial^2 Q^{(1)}}{\partial x^2} + \frac{\partial^2 Q^{(2)}}{\partial y^2}\right) \\ &- P^{(0)}\left(2\frac{\partial^2 Q^{(1)}}{\partial x \partial \xi} + 2\frac{\partial^2 Q^{(1)}}{\partial y \partial \eta}\right), \end{aligned} \tag{10}$$

$$\chi_2^{(1)} = 0, \tag{11}$$

$$\begin{aligned} \chi_2^{(2)} =& 2P^{(0)}v_g\frac{\partial Q^{(1)}}{\partial \xi} - 2P^{(1)}\frac{\partial Q^{(1)}}{\partial z} \\ &+ 2\left(1 - P^{(0)2} + P^{(0)4}/2\right) \\ &\times \left(\frac{\partial^2 P^{(1)}}{\partial x \partial \xi} + \frac{\partial^2 P^{(1)}}{\partial y \partial \eta}\right) \\ &+ \left(P^{(0)3}/2 - 2P^{(0)}\right) \\ &\times \left[\left(\frac{\partial P^{(1)}}{\partial x}\right)^2 + \left(\frac{\partial P^{(1)}}{\partial y}\right)^2\right] \\ &+ \left(2P^{(0)3}P^{(1)} - 2P^{(0)}P^{(1)}\right) \\ &\times \left(\frac{\partial^2 P^{(1)}}{\partial x^2} + \frac{\partial^2 P^{(1)}}{\partial y^2}\right) + \frac{3}{2}P^{(0)}P^{(1)2}. \end{aligned} \tag{12}$$

The other explicit expressions of $\chi_1^{(n)}$ and $\chi_2^{(n)}$ are very tedious and are omitted here.

For the lowest-order solution, we obtain

$$P^{(1)} = P_{11}e^{i\theta} + \text{c.c.}, \tag{13}$$

$$Q^{(1)} = Q_{10} + (Q_{11}e^{i\theta} + \text{c.c.}), \tag{14}$$

with the linear dispersion relation

$$\omega = (k/2)\left[(1 - P^{(0)2} + P^{(0)4}/2)k^2 + 1 - \sigma + 3/2\right]^{1/2}$$

and the relationship

$$P_{11} = iP^{(0)}k^2Q_{11}/2\omega.$$

From the second-order of Eqs.(7) and (8), a solvability condition gives the constant $v_g = k(1 - P^{(0)2} + P^{(0)4}/2)^{1/2}$. We obtain the solution

$$P^{(2)} = P_{20} + (P_{21}e^{i\theta} + P_{22}e^{2i\theta} + \text{c.c.}), \tag{15}$$

$$Q^{(2)} = Q_{22}e^{2i\theta} + \text{c.c.}. \tag{16}$$

The absence of the secular terms in Eqs.(7) and (8) for n=2 requires that

$$P_{20} = \frac{2P^{(0)}v_g}{1 - \sigma - 3P^{(0)2}/2}\frac{\partial Q_{10}}{\partial \xi} + \beta_1\left|Q_{11}\right|^2, \tag{17}$$

and

$$P_{21} = -\frac{P^{(0)}}{2\omega}\left[\frac{v_g k^2}{\omega} - 2k\right]\frac{\partial Q_{11}}{\partial \xi}. \tag{18}$$

Then we obtain

$$Q_{22} = i\gamma_1 Q_{11}^2, \tag{19}$$

$$P_{22} = \gamma_2 Q_{11}^2, \tag{20}$$

with

$$\beta_1 = P^{(0)}k^2\frac{8\omega^2 - 3P^{(0)4}k^4 + 3P^{(0)2}k^2}{4\omega^2\left(1 - \sigma - 3P^{(0)2}/2\right)}, \tag{21}$$

$$\beta_2 = -P^{(0)}k^2\left[1 - \frac{P^{(0)2}k^2}{8\omega^2}\left(5P^{(0)}k^2 - 8k^2 - 3\right)\right], \tag{22}$$

$$\gamma_1 = \frac{4k^2P^{(0)}[-4k^2(1 - P^{(0)2} + P^{(0)4}/2) + 1 - \sigma - 3P^{(0)2}/2] - 16\omega^2P^{(0)}}{3P^{(0)}k^4[-4k^2(1 - P^{(0)2} + P^{(0)4}/2) + 1 - \sigma - 3P^{(0)2}/2]/2\omega + 4\omega\beta_2}, \tag{23}$$

$$\gamma_2 = \frac{\beta_2 k^2 + 3P^{(0)}k^4}{k^2[-4k^2(1 - P^{(0)2} + P^{(0)4}/2) + 1 - \sigma - 3P^{(0)2}/2] - 4\omega^2}. \tag{24}$$

In the third-order, we obtain the closed equations for Q_{10} and Q_{11} from the solvability condition of Eqs.(7) and (8)

$$\mathrm{i}\frac{\partial\phi}{\partial T}+\sigma_1\frac{\partial^2\phi}{\partial\xi^2}+\sigma_2\frac{\partial^2\phi}{\partial\eta^2}+\sigma_3\left|\phi\right|^2\phi+\sigma_4\psi\phi=0, \quad (25)$$

$$\sigma_5\frac{\partial^2\psi}{\partial\xi^2}+\sigma_6\frac{\partial^2\psi}{\partial\eta^2}+\sigma_7\frac{\partial^2}{\partial\xi^2}\left|\phi\right|^2=0, \quad (26)$$

with $\sigma_1=\omega(v_{\mathrm{g}}k/\omega-1)^2/k^2$, $\sigma_2=\omega/k^2$, $\sigma_3=\omega(3k^2P^{(0)}\gamma_1/2\omega+3\gamma_2-\beta)/P^{(0)}$, $\sigma_4=-\omega(k^3/\omega+2k^2v_{\mathrm{g}}/(1-\sigma-3P^{(0)2}/2))/k^2$, $\sigma_5=4P^{(0)}v_{\mathrm{g}}^2/(1-\sigma-3P^{(0)2}/2)-P^{(0)}$, $\sigma_6=-P^{(0)}$, $\sigma_7=2v_{\mathrm{g}}\beta-2P^{(0)}k^3/\omega+P^{(0)}v_{\mathrm{g}}k^4/2\omega^2$, $\phi=Q_{11}$ and $\psi=\partial Q_{10}/\partial\xi$. The coupled Eqs.(25) and (26) describing the nonlinear evolution of the intense laser pulse propagating in the cold unmagnetized plasma are in a general form of the DS equation, and thus describe this physical situation when the resonant coupling occurs between high- and low-frequency waves. From Eqs.(25) and (26) we can see that the long wave component ψ is nonlinearly coupled with the short wave component ϕ. Generally in a 1D system, this effect is described by the so-called Zakharov model that reduces to an effective nonlinear Schrödinger (NLS) equation if the nonlinear wave is independent of the variable η in our situation. The property of the DS equation depends strongly on the values of their coefficients σ_i $(i=1,2,\ldots 7)$ that depend on the free parameters $P^{(0)}$, σ and wave number k. It can be shown that the nonvanishing cw background is unstable in this model, therefore, dark solitons do not survive on the wave background due to the background instability.[27] Here we give the exact solution of bright soliton, which is the dromion solution, i.e. a 2D RES decaying exponentially in all spatial directions. In the following section, we will consider the modulation instability of a plane wave solution of Eqs.(25) and (26), which can give the instability regions of parameters k and σ. Then we can obtain the condition of the REDS of the system.

3. Instability of a plane wave solution

We here consider a stability problem of the plane wave solution of Eqs.(25) and (26). Accordingly the amplitudes of ϕ and ψ are separated into two parts each as follows:

$$\phi=(\phi_0+\delta\phi)\mathrm{e}^{-\mathrm{i}\Delta T}, \quad (27)$$

$$\psi=\psi_0+\delta\psi. \quad (28)$$

Substituting Eqs.(27) and (28) into Eqs.(25) and (26), we obtain

$$\mathrm{i}\frac{\partial\delta\phi}{\partial T}+\sigma_1\frac{\partial^2\delta\phi}{\partial\xi^2}+\sigma_2\frac{\partial^2\delta\phi}{\partial\eta^2}+\sigma_3\left|\phi_0\right|^2(\delta\phi^*+\delta\phi)+\sigma_4\phi_0\delta\psi=0, \quad (29)$$

$$\sigma_5\frac{\partial^2\delta\psi}{\partial\xi^2}+\sigma_6\frac{\partial^2\delta\psi}{\partial\eta^2}+\sigma_7\phi_0\frac{\partial^2}{\partial\xi^2}(\delta\phi^*+\delta\phi)=0. \quad (30)$$

Introducing $\delta\phi=U+\mathrm{i}V$ and assuming $\delta\phi$ and $\delta\psi$ to vary as $\exp[\mathrm{i}(K_1\xi+K_2\eta-\Omega T)]$ with $K_1\ll k$, $K_2\ll k$, and $\Omega\ll\omega$, we obtain from Eqs.(29) and (30) the following relationships:

$$\frac{\partial U}{\partial T}-\left(\sigma_1K_1^2+\sigma_2K_2^2\right)V=0, \quad (31)$$

$$-\frac{\partial V}{\partial T}-\left(\sigma_1K_1^2+\sigma_2K_2^2-2\sigma_3\phi_0^2\right)U+\sigma_4\phi_0\delta\psi=0, \quad (32)$$

$$\sigma_5\frac{\partial^2\delta\psi}{\partial\xi^2}+\sigma_6\frac{\partial^2\delta\psi}{\partial\eta^2}+2\sigma_7\phi_0\frac{\partial^2U}{\partial\xi^2}=0. \quad (33)$$

Considering that U and V vary as $\exp[\mathrm{i}(K_1\xi+K_2\eta-\Omega T)]$ we can obtain a nonlinear disperse relation for the amplitude modulation of the system from the above equation

$$\Omega^2=\left(\sigma_1K_1^2+\sigma_2K_2^2\right)\times\left[\sigma_1K_1^2+\sigma_2K_2^2-2\sigma_3\phi_0^2+2\sigma_4\sigma_7\phi_0^2K_1^2/\left(\sigma_5K_1^2+\sigma_6K_2^2\right)\right]. \quad (34)$$

If the right-hand side of the equation (RHS) (34) is negative, Ω should have an imaginary part and then the plane wave solution becomes unstable under the modulations (27) and (28). Then the stability condition is RHS >0. Considering the relationship $K_1\ll k$, $K_2\ll k$, $K_1>0$, $K_2>0$ and $\phi_0>0$ the stability condition can be approximately determined by

$$f(\sigma,k)=(\sigma_1+\sigma_2)\left[-\sigma_3+\sigma_4\sigma_7/(\sigma_5+\sigma_6)\right]. \quad (35)$$

The instability regions of parameters σ and k determined by a function $f(\sigma,k)$ are Regions 2 and 3 as illustrated in Fig.1. If a nonlinear saturation occurs, the formation of a localized structure may be expected. Thus the REDS will appear.

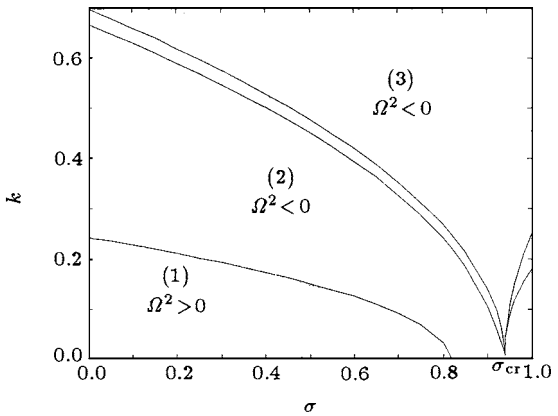


Fig.1. The space of parameters σ and k illustrates regions of the modulation stability and instability at $P^{(0)}$=0.203.

4. Analytical dromion-like solution of the system

To derive an explicit expression of REDS we should give a standard form of DS1 equation by introducing new coordinates and variables. Because the coefficients σ_i $(i = 1, 2, \ldots, 7)$ are rather complicated in general, to give an explicit condition for DS1 is difficult. Therefore we present only two particular cases of parameters for which the conditions are satisfied: one is $\sigma = 0.2228$, $k = 0.4$, and $P^{(0)} = 0.3960$, the other is $\sigma = 0.7876$, $k = 0.199$, and $P^{(0)} = 0.2031$. With these parameter values, the DS1 equations (25) and (26) have a standard form

$$\mathrm{i}\frac{\partial\phi}{\partial z'} + \frac{\partial^2\phi}{\partial x'^2} + \frac{\partial^2\phi}{\partial y'^2} + 2\left|\phi\right|^2\phi + \psi\phi = 0, \quad (36)$$

$$\frac{\partial^2\psi}{\partial x'^2} - \frac{\partial^2\psi}{\partial y'^2} + 4\frac{\partial^2}{\partial x'^2}\left|\phi\right|^2 = 0, \quad (37)$$

where we have introduced new coordinates $x' = (\sigma_3/2\sigma_1)^{1/2}(x - v_g z)$, $y' = (k^2\sigma_3/2\omega)^{1/2}y$ and $z' = (\sigma_3/2)z$, and new variables $Q_{11} = \varepsilon^{-1}\phi$, $\frac{2\sigma_4}{\sigma_3}\frac{\partial Q_{10}}{\partial\xi} = \varepsilon^{-2}\psi$ as well. Then the single dromion solution of DS1 equations (36) and (37) reads[28]

$$\phi = G/F,$$
$$\psi = 4\frac{\partial^2}{\partial x'^2}\ln F,$$

with

$$\begin{aligned} F =& 1 + \gamma_1\exp(\eta_1 + \eta_1^*) + \gamma_2\exp(\eta_2 + \eta_2^*) \\ &+ \gamma_3\exp(\eta_1 + \eta_1^* + \eta_2 + \eta_2^*), \\ G =& \rho\exp(\eta_1 + \eta_2), \end{aligned} \quad (38)$$

where $\eta_1 = (k_{1r} + \mathrm{i}k_{1i})x'' + (\omega_{1r} + \mathrm{i}\omega_{1i})z'$ and $\eta_2 = (k_{2r} + \mathrm{i}k_{2i})y'' + (\omega_{2r} + \mathrm{i}\omega_{2i})z'$ are free complex parameters.

According to the equation $Q_{11} = \varepsilon^{-1}\phi$, we obtain the amplitude of the vector potential $|A|$ as follows:

$$\begin{aligned} |A| =& P^{(0)} - \frac{\sqrt{2k_{1r}k_{2r}(\gamma_3 - 1)}P^{(0)}k^2}{\omega} \\ &\times \frac{\sin(\delta_0 + \theta)}{\sqrt{\gamma}\cosh(\delta_1) + \cosh(\delta_2)}, \end{aligned} \quad (39)$$

with

$$\begin{aligned} \delta_0 &= k_{1i}x'' + k_{2i}y'' + \omega_{1i}z' + \omega_{2i}z' + \varphi_\rho, \\ \delta_1 &= k_{1r}x'' + k_{2r}y'' + \omega_{1r}z' + \omega_{2r}z' + \varphi_\gamma, \\ \delta_2 &= k_{1r}x'' - k_{2r}y'' + \omega_{1r}z' - \omega_{2r}z', \end{aligned} \quad (40)$$

and $\gamma_1 = 1$, $\gamma_2 = 1$,$\gamma_3 = \exp(2\varphi_\gamma)$, $\rho = |\rho|\exp(\mathrm{i}\varphi_\rho)$, $x'' = (x' + y')/2$, $y'' = (y' - x')/2$. The explicit expression of $|A|$ shows a dromion-like soliton due to the appearance of the function of $\sin(\delta + \theta)$ in it. The profile of vector potential $|A|$ illustrated in Fig.2 shows that a dromion-like soliton is excited from the cw background with parameters being $k_{1r} = 1$, $k_{1i} = 0.6$, $k_{2r} = 1$, $k_{2i} = -0.6$, $\varphi_\rho = 0$, $\varphi_\gamma = 1$, $P^{(0)} = 0.203$, $k = 0.199$ and $\sigma = 0.7876$ at $z = 0$. In Fig.3 a local electron density depression appears and the electrons accumulate on the edges, creating the so called electron "wall". The nonlinear ponderomotive force creates the depression of the electron density by pushing the electrons, and then the electromagnetic wave becomes trapped inside this depression. The physical mechanism of producing this soliton is different from the standard process where the nonlinear steepening of the wave is counterbalanced by the effect of dispersion. In the RES, the dispersion effects come into play because of the frequency downshift of the laser pulse, which is due to the following two effects. One is the laser pulse losing its energy. This is due to the fact that the laser pulse generates electromagnetic wake fields behind it and these fields interact with the underdense plasma, causing the laser pulse to lose its energy. The frequency of these fields is much lower than the carrier frequency of the laser pulse. On the other hand, the interaction of laser with plasma is adiabatic and then the number of quanta of the electromagnetic field is adiabatically conserved, which leads to the downshift of the carrier frequency because of the decrease in the radiation energy. The other is the dispersion effect that is due to the finite electron inertia in response to the electromagnetic field action. The dispersion is balanced by the nonlinearity coming from both the relativistic increase in electron mass and

the plasma density redistribution under the action of the ponderomotive forces expelling electrons from the regions of high field intensity.

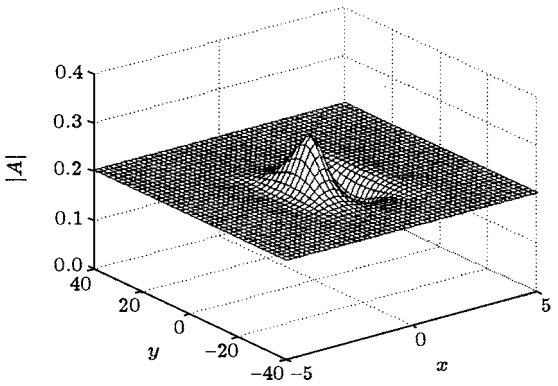


Fig.2. The vector potential $|A|$ in the case of REDS, created from the interaction between an intense laser pulse and a plasma with parameters being $k_{1r} = 1$, $k_{1i} = 0.6$, $k_{2r} = 1$, $k_{2i} = -0.6$, $\varphi_\rho = 0$, $\varphi_\gamma = 1$, and $P^{(0)} = 0.203$.

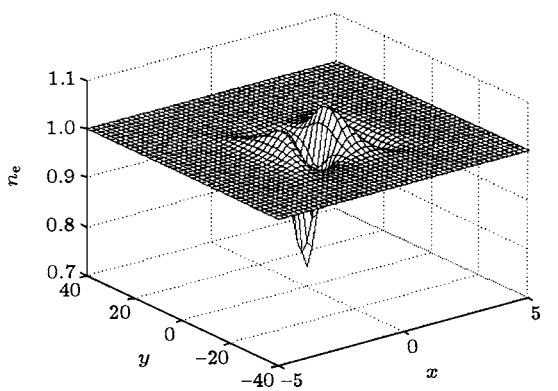


Fig.3. The profile of electron density with the same parameters as those in Fig.2.

Obviously, we have analytically obtained a stable relativistic electromagnetic dromion-like soliton (REDLS) in the presence of cw background, which gives a possible method to obtain the stable RES to control such extremely energetic objects and to use them for certain purposes. Recent 2D[7] and 3D[8] PIC simulations and a new powerful diagnostic tool-proton imaging[3] experiment on RES show that the asymptotic soliton dynamics is abundant in new physical significance and no longer a stable one. Here the cw background $P^{(0)}$ is critical to the formation of the stable REDLS in our system.

It should be mentioned that because of the coefficients σ_i $(i = 1, 2, \ldots 7)$ being very complicated in general, we present only two particular cases of parameters so as to conveniently give the explicit expressions of the solution of the system. Consider directly a general form of the DS equation, then obtaining the explicit analytical solution will become a formidable task. The problem should be solved by numerical method to obtain abundant time evolution behaviours, which is our future work. On the other hand, the explicit solution is stable, which is very important and can give us a new mechanism for the formation of the stable REDLS. Then it is possible to provide us a way to control such extremely energetic objects and to use them for practical purposes.

5. Discussion and conclusion

We have analytically presented a multi-dimensional investigation of the interaction of an ultra-intense laser pulse with a cold unmagnetized transparent electron-ion plasma. By using a standard multiple scale method, we have derived a DS equation for the system, which is just the equation of the DS1 type because of $\sigma_1\sigma_2 > 0$ and admits a dromion solution, i.e. 2D soliton decaying exponentially in all spatial directions. We have discussed the modulation instability of the plane wave solution of the DS equation. The results illustrated in Fig.1 show that the modulation instability occurs in Regions 2 and 3 of parameters σ and k. If the instability grows simultaneously, then nonlinear saturation occurs and the formation of a localized structure may be expected. Then we have obtained a 2D REDLS solution of the amplitude of the vector potential $|A|$. It should be mentioned that the 2D REDLS is generated on a cw background. Because of the stability of the 2D REDLS, it provides us a possible way to control such extremely energetic objects and to use them for certain purposes.

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