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Full-vectorial analysis of optical waveguides by the finite difference method based on polynomial interpolation

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Based on the polynomial interpolation, a new finite difference (FD) method in solving the full-vectorial guided-modes for step-index optical waveguides is proposed. The discontinuities of the normal components of the electric field across abrupt dielectric interfaces are considered in the absence of the limitations of scalar and semivectorial approximation, and the present FD scheme can be applied to both uniform and non-uniform mesh grids. The modal propagation constants and field distributions for buried rectangular waveguides and optical rib waveguides are presented. The hybrid nature of the vectorial modes is demonstrated and the singular behaviours of the minor field components in the corners are observed. Moreover, solutions are in good agreement with those published early, which tests the validity of the present approach.

Keywords: polynomial interpolation, finite difference, full vectorial mode solver, optical waveguides, photonic integrated circuits

PACC: 4280L, 4110H, 4282

1. Introduction

With the rapid development of photonic integrated circuits/planar lightwave circuits (PICs/PLCs), the importance of optical waveguide, such as buried rectangular waveguides and rib waveguides, has been widely recognized since either PICs or PLCs are designed around these waveguides. Accurate calculation of modal fields and propagation constants for optical waveguides is perhaps the most basic and indispensable for optimizing the PICs/PLCs. It is almost impossible, however, to obtain their analytical solutions. Therefore, the use of numerical (or approximate) analysis becomes necessary.[1−3] To date, various numerical techniques, e.g., finite difference (FD) method,[4] finite element (FE) method,[5] imagery distance beam propagation method (ID-BPM),[6] and mapped Galerkin method (MGM),[7] have been proposed to perform this task. Among them, the FD is an attractive candidate because of the simplicity of its implementation and the sparsity of its resulting matrix.

The early application of the FD was based on the scalar wave equation[8] which is valid only for the optical waveguides with very small refractive index difference (say, weakly guiding approximation). In recent years, the FD has been employed to determine semivectorial guided modes where the polarization dependence is taken into account.[9] Yet the polarization coupling is ignored. Therefore the full vectorial characteristics of the guided-modes are not clearly demonstrated. More recently, the FD has been applied to analysing the full vectorial modes for the optical waveguides with large refractive index differences in regions of high field intensity.[10] Taylor series expansion (TSE) is often used to approximate the resulting vectorial wave equation,[4,10,11] However, the TSE is not easily adaptable to non-uniform grid sizes and extended to account for the discontinuities of the normal components of the electric field across abrupt dielectric interfaces.

In this paper, a new finite difference scheme based on the polynomial interpolation is proposed for full vectorial analysis of optical waveguides. The afore-

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mentioned discontinuities are considered in our FD scheme, and full vectorial distributions of the modal fields can be generated. Moreover, the present FD scheme can be applied to both uniform and non-uniform grid sizes. In addition, three adjacent grid points are used to approximate each differential operator, so the solution is more accurate than that obtained by using two adjacent grid points. The modal propagation constants and field distributions for both buried waveguides and optical rib waveguides are presented in this paper.

2. Description of the method

2.1. Vectorial wave equation

The vectorial wave equation based on electric fields can be derived from the Maxwell's equations

$$\nabla \times \boldsymbol{E} = -j\omega\mu_0 \boldsymbol{H}, \tag{1}$$

$$\nabla \times \boldsymbol{H} = j\omega n^2 \varepsilon_0 \boldsymbol{E}, \tag{2}$$

where $\boldsymbol{E}$ and $\boldsymbol{H}$ are the electric and magnetic field vectors respectively, ε_0 and μ_0 are the electric permittivity and the magnetic permeability in vacuum, respectively, and $n = n(x, y)$ is the refractive index of the guiding medium. Through a simple derivation,[1] we have the following vectorial wave equation:

$$\nabla^2 \boldsymbol{E} + \nabla\left(\boldsymbol{E} \cdot \frac{\nabla n^2}{n^2}\right) + n^2 k_0^2 \boldsymbol{E} = 0, \tag{3}$$

in which k_0 is the wave number in free space. In the case where the refractive index profile of an optical waveguide does not vary along the propagation direction (z direction), and the electric field can be written in a separable form as

$$\boldsymbol{E}(x, y, z) = \boldsymbol{u}(x, y)\exp(-j\beta z), \tag{4}$$

where $\beta = k_0 n_{\text{eff}}$ is the propagation constant and n_{eff} is the effective index. By setting

$$\boldsymbol{u}(x, y) = u_x(x, y)\hat{x} + u_y(x, y)\hat{y} + u_z(x, y)\hat{z} \tag{5}$$

in which $\hat{x}$, $\hat{y}$ and $\hat{z}$ are unit vectors in x, y and z directions, respectively, we have

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \beta^2. \tag{6}$$

Thus Eq.(3) is reduced to two coupled equations for u_x and u_y as follows:

$$\begin{bmatrix} A_{xx} & A_{xy} \\ A_{yx} & A_{yy} \end{bmatrix} \begin{bmatrix} u_x \\ u_y \end{bmatrix} = \beta^2 \begin{bmatrix} u_x \\ u_y \end{bmatrix}, \tag{7}$$

where the differential operators are defined as follows:

$$A_{xx}u_x = \frac{\partial}{\partial x}\left[\frac{1}{n^2}\frac{\partial(n^2 u_x)}{\partial x}\right] + \frac{\partial^2 u_x}{\partial y^2} + k_0^2 n^2 u_x, \tag{8a}$$

$$A_{yy}u_y = \frac{\partial^2 u_y}{\partial x^2} + \frac{\partial}{\partial y}\left[\frac{1}{n^2}\frac{\partial(n^2 u_y)}{\partial y}\right] + k_0^2 n^2 u_y, \tag{8b}$$

$$A_{xy}u_y = \frac{\partial}{\partial x}\left[\frac{1}{n^2}\frac{\partial(n^2 u_y)}{\partial y}\right] - \frac{\partial^2 u_y}{\partial x \partial y}, \tag{8c}$$

$$A_{yx}u_x = \frac{\partial}{\partial y}\left[\frac{1}{n^2}\frac{\partial(n^2 u_x)}{\partial x}\right] - \frac{\partial^2 u_x}{\partial y \partial x}. \tag{8d}$$

2.2. Finite difference scheme

Figure 1 shows the sketch of the finite difference mesh for a rib waveguide. The structure is scanned with the small rectangular sub-regions with a size of $\Delta x \times \Delta y$. Within each sub-region the refractive index is constant, so the discontinuities of the refractive index profile occur only at the boundaries between adjacent sub-regions. In our finite difference approach, each of discrete points where fields are sampled is chosen to be located at the centre of its corresponding sub-region, while some finite difference scheme selected the grid points located at the vertices of each sub-region.[11] This is valid for the magnetic field which is continuous across all dielectric interfaces. For the normal component of the electric fields which is discontinuous across an abrupt dielectric interface, it will lead to vagueness. The general situation for an arbitrary grid point P with its neighbouring points, N, S,W, E, NW, NE, SW, and SE is illustrated in Fig.2. The refractive indices for these points are labelled here as n_P, n_N, n_S, n_W, n_E, n_{NW}, n_{NE}, n_{SW}, and n_{SE}, respectively.

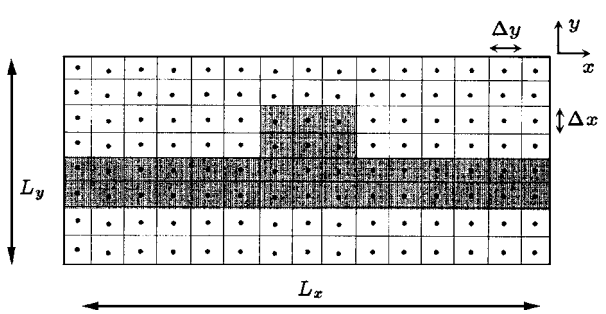


Fig.1. Sketch of the finite difference mesh for a rib waveguide.

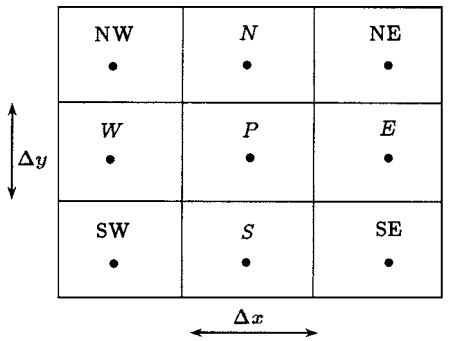


Fig.2. Point P and its neighbouring points.

Now we should convert the partial differential equation (7) into a corresponding finite difference equation. We first deal with the differential operator A_{xx} which operates on u_x. Because u_x is continuous across a horizontal interface, the term $\dfrac{\partial^2 u_x}{\partial y^2}$ can be approximated with a conventional three-point difference, that is,

$$\left.\frac{\partial^2 u_x}{\partial y^2}\right|^P \approx \frac{1}{(\Delta y)^2}\left(u_x^N - 2u_x^P + u_x^S\right), \tag{9}$$

where u_x^P stands for the field u_x at a point P. As we know, when there is an index discontinuity between W and P or between E and P, u_x will be discontinuous, so the finite difference equations must be modified in order to consider this discontinuity. We introduce a polynomial interpolation procedure to do this. A piecewise quadratic polynomial, defined as follows, is used to fit u_x^W,u_x^P, and u_x^E,

$$f(x) = \begin{cases} a^W + bx + cx^2, \text{in sub} - \text{region } W, \\ a^P + bx + cx^2, \text{in sub} - \text{region } P, \\ a^E + bx + cx^2, \text{in sub} - \text{region } E. \end{cases} \tag{10}$$

The function $f(x)$ is satisfied with the following conditions:

$$\begin{aligned} u_x^W &= a^W - b\Delta x + c(\Delta x)^2, \\ u_x^P &= a^P, \\ u_x^E &= a^E + b\Delta x + c(\Delta x)^2. \end{aligned} \tag{11}$$

Considering that the $n^2 u_x$ should be continuous at $x = \pm\dfrac{\Delta x}{2}$ (local coordinate system), that is,

$$\begin{aligned} n_W^2\left[a^W - \frac{1}{2}b\Delta x + \frac{1}{4}c(\Delta x)^2\right] &= n_P^2\left[a^P - \frac{1}{2}b\Delta x + \frac{1}{4}c(\Delta x)^2\right], \\ n_P^2\left[a^W + \frac{1}{2}b\Delta x + \frac{1}{4}c(\Delta x)^2\right] &= n_E^2\left[a^E + \frac{1}{2}b\Delta x + \frac{1}{4}c(\Delta x)^2\right]. \end{aligned} \tag{12}$$

We have

$$\frac{\partial^2 f(x)}{\partial x^2} = \frac{1}{(\Delta x)^2}\left(\delta_W u_x^W - \delta_P u_x^P + \delta_E u_x^E\right) \tag{13}$$

with

$$\begin{aligned} \delta_W &= \frac{4\left(n_W^2 n_P^2 + n_W^2 n_E^2\right)}{n_P^4 + 2n_E^2 n_P^2 + 2n_W^2 n_P^2 + 3n_E^2 n_W^2}, \\ \delta_P &= \frac{4\left(2n_P^4 + n_E^2 n_P^2 + n_W^2 n_P^2\right)}{n_P^4 + 2n_E^2 n_P^2 + 2n_W^2 n_P^2 + 3n_E^2 n_W^2}, \\ \delta_E &= \frac{4\left(n_E^2 n_P^2 + n_W^2 n_E^2\right)}{n_P^4 + 2n_E^2 n_P^2 + 2n_W^2 n_P^2 + 3n_E^2 n_W^2}. \end{aligned} \tag{14}$$

So the term $\dfrac{\partial}{\partial x}\left[\dfrac{1}{n^2}\dfrac{\partial(n^2 u_x)}{\partial x}\right]$ is approximated as

$$\begin{aligned} &\left.\frac{\partial}{\partial x}\left[\frac{1}{n^2}\frac{\partial(n^2 u_x)}{\partial x}\right]\right|^P \\ &\approx \frac{1}{(\Delta x)^2}\left(\delta_W u_x^W - \delta_P u_x^P + \delta_E u_x^E\right). \end{aligned} \tag{15}$$

Now we summarize the finite difference equation for the $A_{xx}u_x$ as follows:

$$\begin{aligned} A_{xx}u_x|^P =& \frac{1}{(\Delta x)^2}\left(\delta_W u_x^W - \delta_P u_x^P + \delta_E u_x^E\right) \\ &+ \frac{1}{(\Delta y)^2}\left(u_x^N - 2u_x^P + u_x^S\right) + k_0^2 n_P^2. \end{aligned} \tag{16}$$

A similar procedure can also be used to derive the finite difference equations for $A_{xy}u_y$, $A_{yy}u_y$, and $A_{yx}u_x$, and then Eq.(7) is converted into a standard matrix eigenvalue equation, which can be solved by using LAPACK subroutines.[12] It is noticed that the grid points on the edge of the computational window $L_x \times L_y$ should be dealt with specially. In order to avoid the nonphysical reflection, the absorbing boundary conditions[13] are introduced to approximate these edge points. Moreover, Eq.(7) can be easily modified to fit with the non-uniform mesh grids. In addition, for the rib waveguide whose refractive profile is symmetric about the y axis, only half of this structure needs to be included. While for the buried rectangu-

lar waveguides whose refractive profile is symmetric about the x and y axes, only one quarter of this structure needs to be considered.

3. Results and discussions

The first structure studied is of the buried rectangular waveguide whose cross section is shown in Fig.3, where w and h are the width and height of the core, respectively. n_1 is the index of the core and n_2 is the index of the cladding. The normalized propagation constant B is defined as

$$B = \frac{n_{\text{eff}}^2 - n_2^2}{n_1^2 - n_2^2}. \tag{17}$$

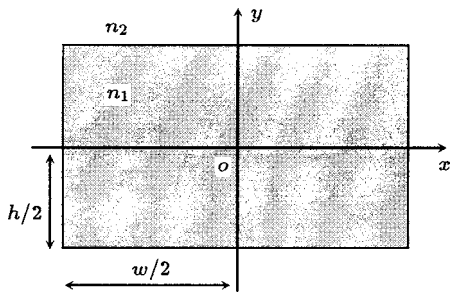


Fig.3. Cross section of a buried rectangular waveguide.

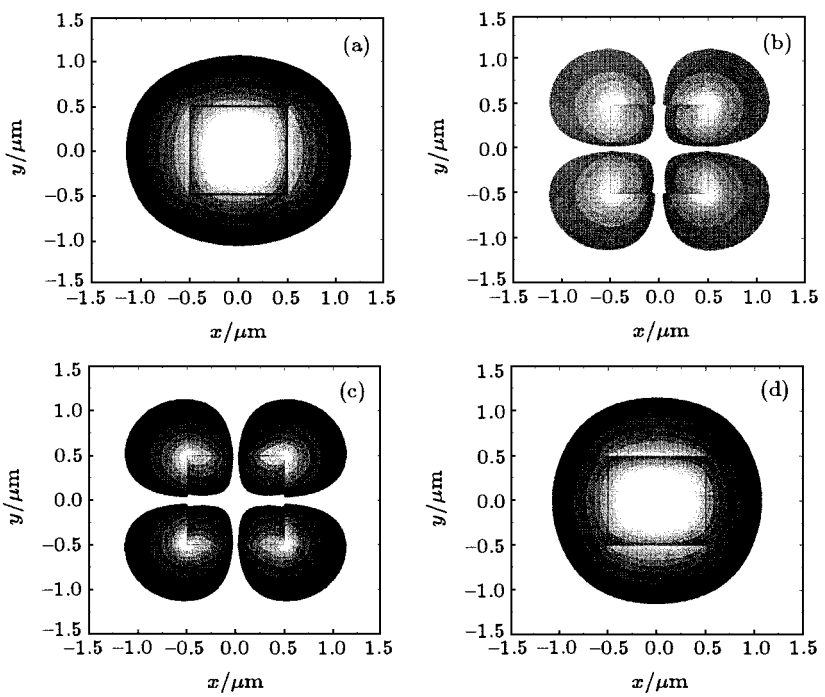


Fig.4. Field patterns of the transverse E-fields for the fundamental modes of a buried rectangular waveguide with $w = h$=1.0μm, n_1=1.5, and n_2=1.0 at λ=1.5μm: (a) u_x and (b) u_y for x polarized mode; (c) u_x and (d) u_y for y polarized mode.

The calculated parameters are the same as those considered in Ref.[14], where $w = h$=1.0μm, n_1=1.5, n_2=1.0, and wavelength λ=1.5μm. The computational window is 3μm×3μm in size, the grid size in 2μm×2μm (covering the core and its neighbouring cladding) is $\Delta x = \Delta y$=0.05μm, and the grid size in other claddings is $\Delta x = \Delta y$=0.1μm. Figure 4 gives the field patterns of the transverse E-fields of the fundamental modes (x and y polarization modes, also known as quasi-TE and quasi-TM modes). It is noted that the major component has a single peak around the centre of the core, while the minor component has four peaks located very close to the corners. This accords with the fact that both derivatives, with respect

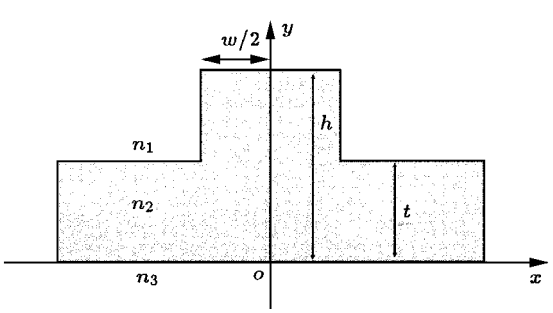


Fig.5. Cross section of an optical rib waveguide.

to x and y, of the refractive index distribution are non-zero close to the four corners of the waveguide. Notice that the discontinuity of u_x across the dielec-

tric/air along the x direction is clearly observed, while the discontinuity of u_y across the dielectric/air along the y direction is also observed. So, the hybrid nature of the vectorial mode is clearly evident. The normalized propagation constants for x and y polarization modes are identical, that is, B=0.4903. The result in Ref.[10] for the same structure, obtained by nonlinear iterative method (NLIM), is 0.4909. Our result is in good agreement with that obtained by this NLIM.

Table 1. Effective indices n_{eff} and normalized propagation constants B of a typical optical rib waveguide computed by different methods.

Method	x polarization		y polarization	
	n_{eff}	B	n_{eff}	B
SV-FDM[15]	3.3882623	0.4789	3.387543	0.4718
SIM[16]	3.3887400	0.4837	3.38788	0.4751
V-BPM[17]	3.3882530	0.4788	3.387512	0.4714
V-LM[18]	3.3896940	0.4932	3.388183	0.4871
MGM[7]	3.3882640	0.4790	3.387391	0.4702
Present Method	3.3887000	0.4833	3.387840	0.4747

The present FDM is also applied to analyse the typical deep-etched optical rib waveguide whose geometry is illustrated in Fig.5, in which the rib width w=2.0μm, the rib height h=1.3μm, the slab waveguide height t=0.2μm, the superstratum index n_1=1.0, the core index n_2=3.44 and the substratum index n_3=3.34 with the wavelength λ=1.55μm. In the superstratum (air), coarse grid size, $\Delta x = \Delta y$=0.1μm, is used, while in the core and the substratum, fine grid size, $\Delta x = \Delta y$=0.05μm, is used. Table 1 shows the effective indices n_{eff} and normalized propagation constants B of the fundamental modes for x and y polarization modes. In order to demonstrate the accuracy of our approach, the results of the present analysis and those of semivectorial FDM (SV-FDM),[15] spectral index method (SIM),[16] vectorial beam propagation method (V-BPM),[17] vectorial Lanczos method (V-LM),[18] and MGM[7] are all listed in this table. Clearly, the results obtained by the present method from the polarized solution in each case are good agreement with those obtained by the methods abovementioned. Figure 6 plots the distributions of the transverse E-fields of the fundamental modes (x and y polarization modes). Here again the major component

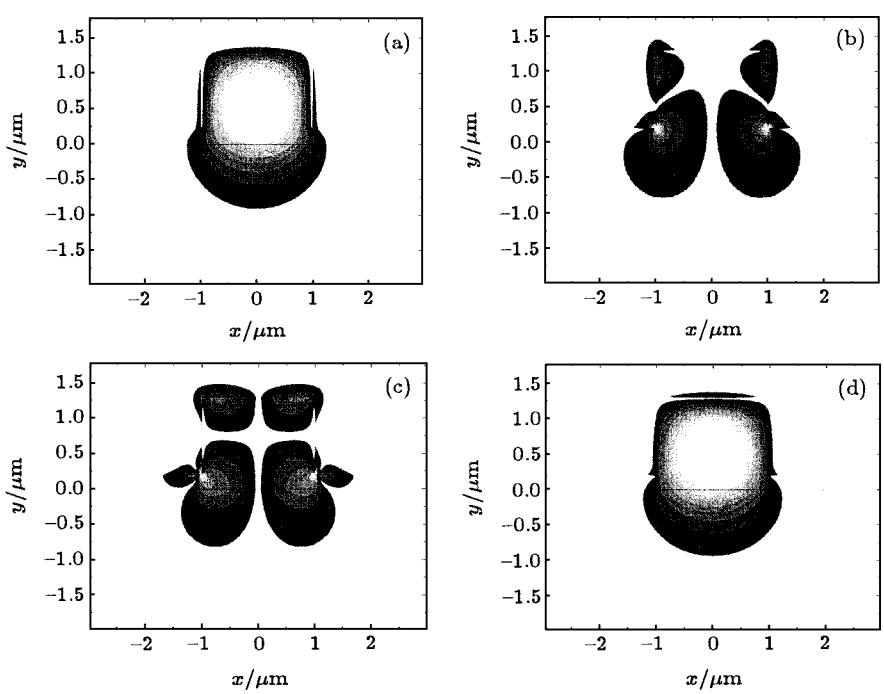


Fig.6. Field patterns of the transverse E-fields for the fundamental modes of an optical rib waveguide with w=2.0μm, h=1.3μm, t=0.2 μm, n_1=1.0, n_2=3.44, n_3=3.34 at λ=1.55μm: (a) u_x and (b) u_y for x polarized mode; (c) u_x and (d) u_y for y polarized mode.

has a single peak around the centre while the minor component has four peaks located very close to the corners. Moreover, the expected discontinuities, u_x across core/air along the x direction, and u_y across core/air and core/substrate along the y direction, are also clearly observed.

4. Conclusions

Based on the polynomial interpolations, a new FD method in solving full-vectorial modes for optical waveguides with step-index profiles is derived. The discontinuities of the normal components of the electric field across abrupt dielectric interfaces are taken into account, and this FD scheme can be applied to both uniform and non-uniform mesh grids. Results for the buried rectangular waveguide and optical rib waveguide are presented, and the hybrid nature of the vectorial guided-modes is demonstrated. Solutions are in good agreement with the results obtained by other efficient methods. We conclude that the present method is powerful for the analysing, modelling and designing of optical waveguides used for PICs/PLCs.

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