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Efficient three-wave mixing in a three-level atomic medium with an assisting microwave driven field*

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The potential for nonlinear conversion between two laser pulses in a three-level V-type medium with assistance of an auxiliary microwave resonant radiation is studied. The results show that microwave driven field can lead to the parametric generation of a new laser pulse with high conversion efficiency when a weak pump laser pulse is applied.

Keywords: three-wave mixing, Rabi frequency, atomic coherence
PACC: 4250, 4265M

1. Introduction

Effects of atomic coherence and interference are currently becoming an important and intensive tool in highly resonant media.[1−21] By use of atomic coherence, the problem of developing resonant nonlinear optics with high efficiency has attracted a lot of interest recently. The conventional system used for this purpose is a four-level double-Λ system, or a four-level cascade-type system based on the effects of atomic coherence and interference. The relevant representative papers are cited in Refs.[1–13]. For example, Zibrov *et al* investigated an efficient nonlinear process in a four-level medium coherently prepared by resonant, counterpropagating fields and showed that such a process could be used for generation of pairs of Stokes and anti-Stokes fields.[1] Very recently, Wu *et al* performed a time-dependent analysis of four-wave mixing (FWM) in a double-Λ system in an ultraslow propagation regime and obtained the analytical expressions of pulsed probe lasers, FWM-generated pulses, phase shifts and absorption coefficients, group velocities, and FWM efficiency.[2] At the same time, they also analysed and discussed a lifetime-broadened cascade-type four-wave-mixing scheme in the ultraslow propagation regime and showed that the generated FWM field could acquire the same group velocity and pulse shape as those obtained by an ultraslow pump field.[3,4] On the other hand, some interesting experiments using these systems have been successfully carried out, which may open up new opportunities for technological applications of these predicted phenomena.[14−18] Kang and co-workers reported an experimental study of resonant six-wave mixing in coherently prepared four-level double-Λ Rb atomic media.[16] Alternatively, they also reported an experimental observation of resonantly enhanced slow-light four-wave mixing in such cold Rb atomic media.[17] For the three-level atomic system, Wang and Yang pointed out that, in the transfer of the quantum state of two signal fields, both the frequency down-conversion and up-conversion were possible.[19]

In this paper, we demonstrate the efficient generation of coherent light in a V-type system by a low-intensity pump pulsed laser radiation assisted by a microwave driven field. We derive the corresponding explicit analytical expressions for the generated three-wave mixing (TWM) field. Dependences of the intensity of the generated TWM field on the microwave driven field, on the propagation distance, and on the photon detuning are investigated. To conclude, we also give a brief discussion on the experimental real-

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ization of the proposed scheme.

This paper is organized as follows. In Section 2, the theoretical model is described and the coupled Schrödinger–Maxwell equations of motion that govern the dynamics of such a three-level system are solved analytically. In Section 3, we analyse and discuss the influence of the microwave driven field, the propagation distance, and the photon detuning on the intensity of the generated TWM field, and present the analytical expression of TWM efficiency as well. Finally, we conclude with a brief summary in Section 4.

2. TWM scheme and solutions of the coupled Schrödinger–Maxwell equations of motion

Let us consider a lifetime broadened three-level V-type atomic system for the TWM generation, as depicted in Fig.1. The atomic levels are denoted by $|1\rangle$, $|2\rangle$, and $|3\rangle$. A pump pulsed laser field $E_{\rm p}$ (angular frequency $\omega_{\rm p}$ and Rabi frequency $2\Omega_{\rm p}$) is applied to the dipole-allowed transition $|1\rangle \leftrightarrow |2\rangle$. In order to generate a TWM field (angular frequency $\omega_{\rm m}$ and Rabi frequency $2\Omega_{\rm m}$), an assisting microwave driven field $E_{\rm d}$ (angular frequency $\omega_{\rm d}$ and Rabi frequency $2\Omega_{\rm d}$) is used to couple Zeeman sublevels $|2\rangle$ and $|3\rangle$ through an allowed magnetic dipole transition and produce dynamically induced maximum coherence between sublevels $|2\rangle$ and $|3\rangle$.

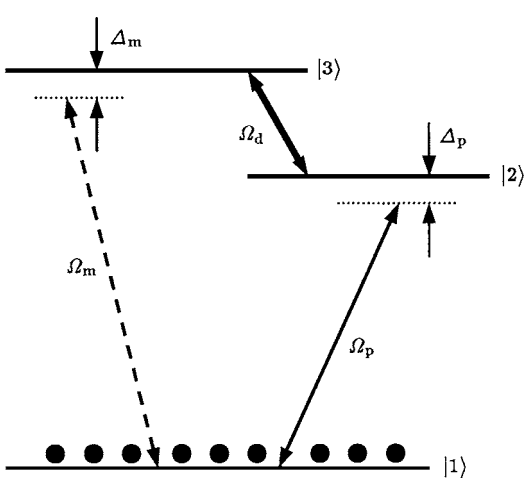


Fig.1. Schematic diagram of lifetime-broadened three-level atoms interacting with a pump pulsed laser field (angular frequency $\omega_{\rm p}$ and Rabi frequency $2\Omega_{\rm p}$) and an assisting microwave driven field (angular frequency $\omega_{\rm d}$ and Rabi frequency $2\Omega_{\rm d}$) to generate a mixing field (angular frequency $\omega_{\rm m}$ and Rabi frequency $2\Omega_{\rm m}$).

In the rotating-wave and electro-dipole approximations, the semiclassical Hamiltonian describing the atom–field interaction for the system under study in the Schrödinger's picture is given by

$$H = \sum_{j=1}^{3} E_j^{(\rm a)} |j\rangle\langle j| - \hbar(\Omega_{\rm p} {\rm e}^{{\rm i}\theta_{\rm p}} |2\rangle\langle 1| + \Omega_{\rm d} {\rm e}^{{\rm i}\theta_{\rm d}} |3\rangle\langle 2| + \Omega_{\rm m} {\rm e}^{{\rm i}\theta_{\rm m}} |3\rangle\langle 1| + {\rm H.c.}), \quad (1)$$

where the symbol H.c. refers to the Hermitian conjugate, $\theta_n = \boldsymbol{k}_n \cdot \boldsymbol{r} - \omega_n t$ corresponds to the positive-frequency part of the respective optical field, Ω_n $(n = p, d, m)$ are one-half Rabi frequencies for the relevant laser-driven transitions, i.e. $\Omega_{\rm p} = \mu_{21} E_{\rm p}/(2\hbar)$, $\Omega_{\rm d} = \mu_{32} E_{\rm d}/(2\hbar)$ and $\Omega_{\rm m} = \mu_{31} E_{\rm m}/(2\hbar)$, with $\mu_{kl} = \boldsymbol{\mu}_{kl} \cdot \boldsymbol{e}_{\rm L}$ ($\boldsymbol{e}_{\rm L}$ is the unit polarization vector of the corresponding laser field) denoting the dipole moment for the transition between levels $|k\rangle$ and $|l\rangle$, and $E_j^{(\rm a)} = \hbar\omega_j (j = 1 - 3)$ is the energy of the atomic state $|j\rangle$. For simplicity, in following analysis we will take $\omega_1 = 0$ for the ground state $|1\rangle$ as the energy origin. Turning to the interaction picture, the resulting Hamiltonian can be rewritten as follows (taking $\hbar = 1$):

$$\begin{aligned} H_0 =& \omega_{\rm p} |2\rangle\langle 2| + \omega_{\rm m} |3\rangle\langle 3|, \\ H_{\rm int} =& \Delta |2\rangle\langle 2| + \Delta |3\rangle\langle 3| \\ & - (\Omega_{\rm p} {\rm exp}\,({\rm i}\boldsymbol{k}_{\rm p} \cdot r) |2\rangle\langle 1| + \Omega_{\rm d} {\rm exp}\,({\rm i}\boldsymbol{k}_{\rm d} \cdot r) |3\rangle\langle 2| \\ & + \Omega_{\rm m} {\rm exp}\,({\rm i}\boldsymbol{k}_{\rm m} \cdot r) |3\rangle\langle 1| + {\rm H.c.}), \end{aligned} \quad (2)$$

where we have assumed that $\Delta_{\rm p} = \Delta_{\rm m} = \Delta$ is satisfied. $\Delta_{\rm p} = \omega_2 - \omega_{\rm p}$ and $\Delta_{\rm m} = \omega_3 - \omega_{\rm m}$ are the corresponding frequency detunings of the pump pulsed and generated TWM fields, as shown in Fig.1.

Let us assume the wavefunction of the form

$$\begin{aligned} |\Psi\rangle =& A_1 |1\rangle + A_2 {\rm exp}\,({\rm i}\boldsymbol{k}_{\rm p} \cdot \boldsymbol{r}) |2\rangle \\ & + A_3 {\rm exp}\,({\rm i}(\boldsymbol{k}_{\rm p} + \boldsymbol{k}_{\rm d}) \cdot \boldsymbol{r}) |3\rangle, \end{aligned} \quad (3)$$

where $A_j (j = 1 - 3)$ stand for the time-dependent probability amplitudes of the atomic states.

Making use of the Schrödinger equation in the interaction picture ${\rm i}\dfrac{\partial |\Psi\rangle}{\partial t} = H_{\rm int} |\Psi\rangle$ and the Maxwell's equation that governs the propagation of the optical pulse, the equations of motion for the probability amplitudes of the atomic wavefunctions and wave equations for electromagnetic fields $\Omega_{\rm p,m}$ can be readily

obtained as follows:

$$\frac{\partial A_1}{\partial t} = \mathrm{i}\Omega_{\mathrm{p}}^* A_2 + \mathrm{i}\Omega_{\mathrm{m}}^* A_3, \tag{4a}$$

$$\frac{\partial A_2}{\partial t} = -(\gamma_2 + \mathrm{i}\Delta)A_2 + \mathrm{i}\Omega_{\mathrm{p}} A_1 + \mathrm{i}\Omega_{\mathrm{d}}^* A_3, \tag{4b}$$

$$\frac{\partial A_3}{\partial t} = -(\gamma_3 + \mathrm{i}\Delta)A_3 + \mathrm{i}\Omega_{\mathrm{d}} A_2 + \mathrm{i}\Omega_{\mathrm{m}} A_1, \tag{4c}$$

$$\frac{\partial \Omega_{\mathrm{p}}}{\partial z} + \frac{1}{c}\frac{\partial \Omega_{\mathrm{p}}}{\partial t} = \mathrm{i}\kappa_{12} A_2 A_1^*, \tag{4d}$$

$$\frac{\partial \Omega_{\mathrm{m}}}{\partial z} + \frac{1}{c}\frac{\partial \Omega_{\mathrm{m}}}{\partial t} = \mathrm{i}\kappa_{13} A_3 A_1^*, \tag{4e}$$

where γ_2 and γ_3 denote the decay rates of states $|2\rangle$ and $|3\rangle$, respectively. The quantity $\kappa_{12(13)} = 2\pi N\omega_{\mathrm{p(m)}}|\mu_{12(13)}|^2/(\hbar c)$ is related to the frequently used oscillator strength of the transition $|1\rangle \leftrightarrow |2\rangle(|1\rangle \leftrightarrow |3\rangle)$, with N being the atomic number density in the medium. Above we have utilized the $\boldsymbol{k}$-wave-vector matching condition $\boldsymbol{k}_{\mathrm{m}} = \boldsymbol{k}_{\mathrm{p}} + \boldsymbol{k}_{\mathrm{d}}$. In the following discussion, retaining the first order of the optical fields $\Omega_{\mathrm{p,m}}$ and assuming that the atomic system is initially in the ground state $|1\rangle$ ($A_1 \simeq 1$) under the condition that the laser–atom interaction is weak, then carrying out Fourier transformations $\alpha_j(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} A_j(t)\mathrm{e}^{\mathrm{i}\omega t}\mathrm{d}t$ and $\Lambda_{\mathrm{p,m}}(z,\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} \Omega_{\mathrm{p,m}}(z,t)\mathrm{e}^{\mathrm{i}\omega t}\mathrm{d}t$ for Eqs.(4b)–(4e), we have the results

$$(\omega + \mathrm{i}\tilde{\gamma}_2)\alpha_2 + \Omega_{\mathrm{d}}^*\alpha_3 = -\Lambda_{\mathrm{p}}, \tag{5a}$$

$$(\omega + \mathrm{i}\tilde{\gamma}_3)\alpha_3 + \Omega_{\mathrm{d}}\alpha_2 = -\Lambda_{\mathrm{m}}, \tag{5b}$$

$$\frac{\partial \Lambda_{\mathrm{p}}}{\partial z} - \mathrm{i}\frac{\omega}{c}\Lambda_{\mathrm{p}} = \mathrm{i}\kappa_{12}\alpha_2, \tag{5c}$$

$$\frac{\partial \Lambda_{\mathrm{m}}}{\partial z} - \mathrm{i}\frac{\omega}{c}\Lambda_{\mathrm{m}} = \mathrm{i}\kappa_{13}\alpha_3, \tag{5d}$$

where we have introduced the definitions $\tilde{\gamma}_2 = \gamma_2 + \mathrm{i}\Delta$ and $\tilde{\gamma}_3 = \gamma_3 + \mathrm{i}\Delta$, respectively.

The solutions to Eqs.(5a) and (5b) can be found to be

$$\alpha_2 = -\frac{\Omega_{\mathrm{d}}^*}{D_1}\Lambda_{\mathrm{m}} + \frac{D_3}{D_1}\Lambda_{\mathrm{p}}, \tag{6a}$$

$$\alpha_3 = -\frac{\Omega_{\mathrm{d}}}{D_1}\Lambda_{\mathrm{p}} + \frac{D_2}{D_1}\Lambda_{\mathrm{m}}, \tag{6b}$$

where we have introduced the notations $D_1 = |\Omega_{\mathrm{d}}|^2 - D_2 D_3$, $D_2 = \omega + \mathrm{i}\tilde{\gamma}_2$ and $D_3 = \omega + \mathrm{i}\tilde{\gamma}_3$.

Substituting Eqs.(6a) and (6b) into Eqs.(5c) and (5d), we then obtain directly the coupled wave equations for the pump and TWM fields as follows:

$$\frac{\partial \Lambda_{\mathrm{p}}}{\partial z} - \mathrm{i}\left(\kappa_{12}\frac{D_3}{D_1} + \frac{\omega}{c}\right)\Lambda_{\mathrm{p}} = -\mathrm{i}\kappa_{12}\frac{\Omega_{\mathrm{d}}^*}{D_1}\Lambda_{\mathrm{m}}, \tag{7a}$$

$$\frac{\partial \Lambda_{\mathrm{m}}}{\partial z} - \mathrm{i}\left(\kappa_{13}\frac{D_2}{D_1} + \frac{\omega}{c}\right)\Lambda_{\mathrm{m}} = -\mathrm{i}\kappa_{13}\frac{\Omega_{\mathrm{d}}}{D_1}\Lambda_{\mathrm{p}}. \tag{7b}$$

After some algebra but with no further assumptions, for a given $\Lambda_{\mathrm{p}}(\omega,0)$ and TMW emission process $\Lambda_{\mathrm{m}}(\omega,0) = 0$, the analytical solutions to Eqs.(7a) and (7b) are

$$\Lambda_{\mathrm{p}}(\omega,z) = \Lambda_{\mathrm{p}}(\omega,0)(S_+\mathrm{e}^{\mathrm{i}\lambda_- z} - S_-\mathrm{e}^{\mathrm{i}\lambda_+ z}), \tag{8a}$$

$$\Lambda_{\mathrm{m}}(\omega,z) = \Lambda_{\mathrm{p}}(\omega,0)S(\mathrm{e}^{\mathrm{i}\lambda_- z} - \mathrm{e}^{\mathrm{i}\lambda_+ z}), \tag{8b}$$

with

$$G \equiv G(\omega) = \sqrt{(\kappa_{12}D_3 - \kappa_{13}D_2)^2 + 4\kappa_{12}\kappa_{13}|\Omega_{\mathrm{d}}|^2},$$

$$\lambda_+ \equiv \lambda_+(\omega) = \frac{\omega}{c} + \frac{\kappa_{12}D_3 + \kappa_{13}D_2 + G}{2D_1},$$

$$\lambda_- \equiv \lambda_-(\omega) = \frac{\omega}{c} + \frac{\kappa_{12}D_3 + \kappa_{13}D_2 - G}{2D_1},$$

$$S_+ \equiv S_+(\omega) = \frac{\kappa_{13}D_2 - \kappa_{12}D_3 + G}{2G},$$

$$S_- \equiv S_-(\omega) = \frac{\kappa_{13}D_2 - \kappa_{12}D_3 - G}{2G},$$

$$S \equiv S(\omega) = \frac{\kappa_{13}\Omega_{\mathrm{d}}}{G}.$$

where Eqs.(8a) and (8b) are the main results of the present study. If we use a Gaussian pulse shape for the pump pulsed laser at the entrance to the medium, i.e. $\Omega_{\mathrm{p}}(0,t) = \Omega_{\mathrm{p}}(0,0)\mathrm{e}^{-(t/\tau)^2}$ (τ is the pulse width), then by carrying out Fourier transformation we can find $\Lambda_{\mathrm{p}}(0,\omega) = \Omega_{\mathrm{p}}(0,0)\frac{\tau}{\sqrt{2}}\mathrm{e}^{-(\omega\tau)^2/4}$.

3. Analysis and discussion for the generated TWM field

In Fig.2, we plot the relative intensity of the generated TWM field $\left|\frac{\Lambda_{\mathrm{m}}(\omega,L)}{\tau\Omega_{\mathrm{p}}(0,0)/\sqrt{2}}\right|^2$ as a function of ω/γ_2 for different values of the microwave driven field Ω_{d}. We find from Fig. 2 that the TWM signal intensity is quite sensitive to the slight change of the microwave driven field. As a matter of fact, it can be easily seen from Eq.(7b) that the generated TWM field is closely associated with the microwave driven field because the driving source of the generated TWM field is "$-\mathrm{i}\kappa_{13}\frac{\Omega_{\mathrm{d}}}{D_1}\Lambda_{\mathrm{p}}$" in the wave-mixing process. When no microwave driven field occurs and only a pump pulsed laser field is applied, no TWM field is generated in such a system. Consequently, the coherence between Zeeman sublevels $|2\rangle$ and $|3\rangle$ produced by the microwave driven field plays an important role in generating a coherent TWM field. The relative intensity of the generated TWM field $\left|\frac{\Lambda_{\mathrm{m}}(\omega,L)}{\tau\Omega_{\mathrm{p}}(0,0)/\sqrt{2}}\right|^2$ versus

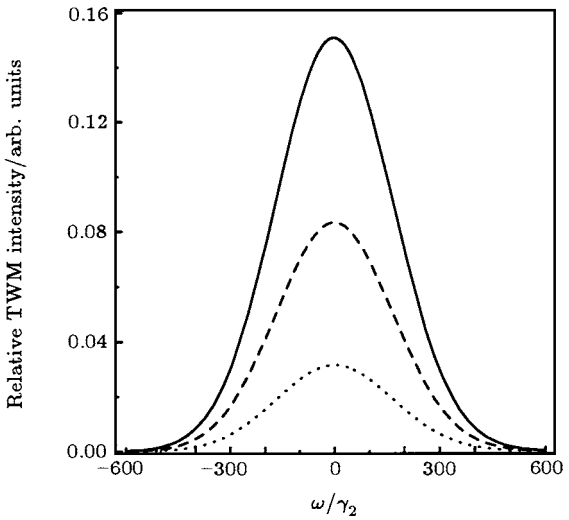


Fig.2. The relative intensity of the generated TWM field $\left|\frac{\Lambda_{\rm m}(\omega,L)}{\tau\Omega_{\rm p}(0,0)/\sqrt{2}}\right|^2$ as a function of ω/γ_2 for different values of $\Omega_{\rm d}$, with $\Omega_{\rm d}=2\gamma_2$ (solid curve), $\Omega_{\rm d}=1.5\gamma_2$ (dashed curve), and $\Omega_{\rm d}=\gamma_2$ (dotted curve). The initial incident pulse is $\Omega_{\rm p}(z=0,t)=\Omega_{\rm p}(0,0)\exp(-t^2/\tau^2)$. The fitting parameters are: $\kappa_{12}=\kappa_{13}=2\times10^9{\rm cm}^{-1}\cdot{\rm s}^{-1}$, $\gamma_2=0.6\times10^8{\rm s}^{-1}$, $\gamma_3=1.25\times10^8{\rm s}^{-1}$, $\Delta=0$, $\tau=10^{-6}$s, and $L=1$mm.

the amplitude of the microwave driven field $\Omega_{\rm d}$ is shown in Fig.3, in which we find that the intensity of the generated TWM field first increases to a maximum point, then decreases with the increase of the amplitude of the microwave driven field, finally down to a negligible steady value. In Fig.4, we plot the relative intensity of the generated TWM field $\left|\frac{\Lambda_{\rm m}(\omega,z)}{\tau\Omega_{\rm p}(0,0)/\sqrt{2}}\right|^2$ as a function of propagation distance z. We observe that the intensity of the generated TWM field firstly increases dramatically to a maximum value, then decreases gradually and approaches rapidly to a zero value with propagation distance z increasing. It is easy to see from the expression (8b)(also see the expression (11b) below) that this decrease with propagation distance z in Fig. 4 results from the atomic decay of two upper levels $|2\rangle$ and $|3\rangle$. This means that the generated TWM field has a better conversion efficiency in an optically thin medium (which can always take place at a sufficiently low density) and for a longer interaction length the TWM field $\Lambda_{\rm m}(\omega,z)$ is completely absorbed. In order to analyse the influence of the photon detuning on the generated TWM field, the corresponding curve in Fig.5 is clearly presented. We find that the relative intensity of the generated TWM field $\left|\frac{\Lambda_{\rm m}(\omega,L)}{\tau\Omega_{\rm p}(0,0)/\sqrt{2}}\right|^2$ versus the detuning Δ exhibits a double-wing profile. This result can be understood by employing dressed atomic states. The coupling of the atom to the microwave driven field $\Omega_{\rm d}$ gives rise to the splitting of the upper level $|3\rangle$ into two sublevels $|3\pm\rangle$ ($|3+\rangle=\frac{1}{\sqrt{2}}(|3\rangle+|2\rangle)$ and $|3-\rangle=\frac{1}{\sqrt{2}}(|3\rangle-|2\rangle)$). The two TWM peaks correspond to the dressed-state transitions $|3+\rangle\leftrightarrow|1\rangle$ and $|3-\rangle\leftrightarrow|1\rangle$. The hollow

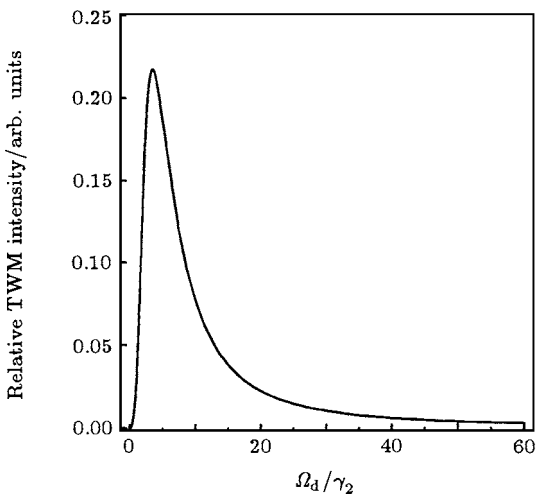


Fig.3. The relative intensity of the generated TWM field $\left|\frac{\Lambda_{\rm m}(\omega,L)}{\tau\Omega_{\rm p}(0,0)/\sqrt{2}}\right|^2$ as a function of the microwave driven field $\Omega_{\rm d}/\gamma_2$. The initial incident pulse is $\Omega_{\rm p}(z=0,t)=\Omega_{\rm p}(0,0)\exp(-t^2/\tau^2)$. The fitting parameters are: $\kappa_{12}=\kappa_{13}=2\times10^9{\rm cm}^{-1}\cdot{\rm s}^{-1}$, $\gamma_2=0.6\times10^8{\rm s}^{-1}$, $\gamma_3=1.25\times10^8{\rm s}^{-1}$, $\Delta=0$, $\tau=10^{-6}$s, $\omega=0.01\gamma_2$, and $L=1$mm.

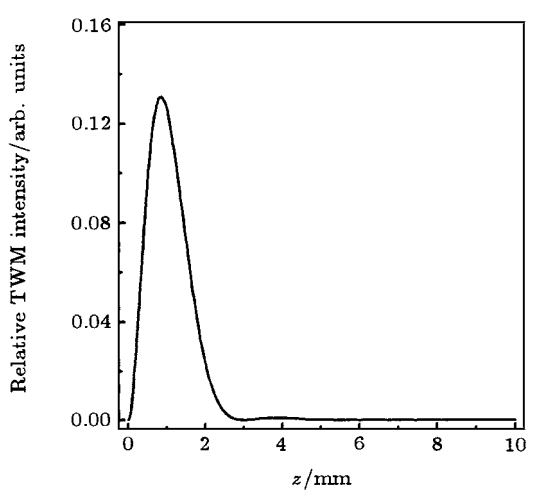


Fig.4. The relative intensity of the generated TWM field $\left|\frac{\Lambda_{\rm m}(\omega,z)}{\tau\Omega_{\rm p}(0,0)/\sqrt{2}}\right|^2$ as a function of propagation distance z. The initial incident pulse is $\Omega_{\rm p}(z=0,t)=\Omega_{\rm p}(0,0)\exp(-t^2/\tau^2)$. The fitting parameters: $\Omega_{\rm d}=2\gamma_2$, $\kappa_{12}=\kappa_{13}=2\times10^9{\rm cm}^{-1}\cdot{\rm s}^{-1}$, $\gamma_2=0.6\times10^8{\rm s}^{-1}$, $\gamma_3=1.25\times10^8{\rm s}^{-1}$, $\Delta=0$, $\tau=10^{-6}$s, and $\omega=0.01\gamma_2$.

valley near the resonant point $\Delta = 0$ (see Fig.5) appears due to quantum interference between the two different quantum pathways: $|3+\rangle \leftrightarrow |1\rangle$ and $|3-\rangle \leftrightarrow |1\rangle$, yet this interference makes a negative contribution to the generated TWM field.

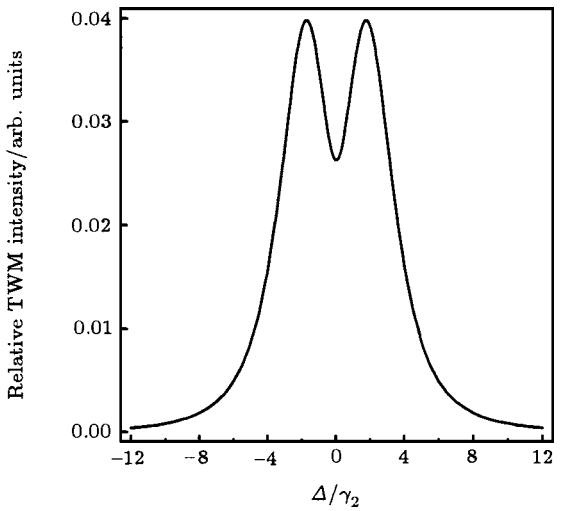


Fig.5. The relative intensity of the generated TWM field $\left|\dfrac{\Lambda_{\rm m}(\omega,L)}{\tau\Omega_{\rm p}(0,0)/\sqrt{2}}\right|^2$ as a function of Δ/γ_2. The initial incident pulse is $\Omega_{\rm p}(z=0,t) = \Omega_{\rm p}(0,0)\exp(-t^2/\tau^2)$. The fitting parameters are: $\Omega_{\rm d} = \gamma_2$, $\kappa_{12} = \kappa_{13} = 2\times10^9{\rm cm}^{-1}\cdot{\rm s}^{-1}$, $\gamma_2 = 0.6\times10^8{\rm s}^{-1}$, $\gamma_3 = 1.25\times10^8{\rm s}^{-1}$, $\tau = 10^{-6}$s, $\omega = 0.01\gamma_2$, and $L = 1$mm.

In what follows, we will turn our attention to an adiabatic regime, where $\lambda_\pm$, $S_\pm$ and S can be expressed by a rapidly converging power series of ω. This permits analytical evaluation of Eqs.(8a) and (8b) so that a clear physical picture of the process can be gained. Specifically, we take that $\lambda_\pm(\omega) = \beta_\pm(0) + \beta'_\pm(0)\omega + O(\omega)$, $S_\pm(\omega) = W_\pm(0) + O(\omega)$ and $S(\omega) = W(0) + O(\omega)$. That is to say, we can make the approximation of neglecting both $O(\omega)$ in $S_\pm(\omega)$ and $S(\omega)$ and $O(\omega^2)$ in $\lambda_\pm(\omega)$. It is shown that under these conditions, the linearized treatment introduced above is well justified. This treatment can accurately describe the process at hand. Applying the inverse Fourier transformation to Eqs.(8a) and (8b), i.e.

$$\Omega_{\rm p}(z,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} \Lambda_{\rm p}(\omega,0) \times (S_+ {\rm e}^{{\rm i}\lambda_- z} - S_- {\rm e}^{{\rm i}\lambda_+ z}){\rm e}^{-{\rm i}\omega t}{\rm d}\omega, \quad (9a)$$

$$\Omega_{\rm m}(z,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} \Lambda_{\rm p}(\omega,0) \times S({\rm e}^{{\rm i}\lambda_- z} - {\rm e}^{{\rm i}\lambda_+ z}){\rm e}^{-{\rm i}\omega t}{\rm d}\omega, \quad (9b)$$

under the above-described conditions, we can obtain

$$\Omega_{\rm p}(z,t) = W_+\,\Omega_{\rm p}(\eta_-)\,{\rm e}^{{\rm i}\beta_-(0)z} - W_-\,\Omega_{\rm p}(\eta_+)\,{\rm e}^{{\rm i}\beta_+(0)z}, \quad (10a)$$

$$\Omega_{\rm m}(z,t) = W\,\Omega_{\rm p}(\eta_-){\rm e}^{{\rm i}\beta_-(0)z} - W\,\Omega_{\rm p}(\eta_+){\rm e}^{{\rm i}\beta_+(0)z}, \quad (10b)$$

where $\eta_\pm = t - z/V_{\rm g\pm}$. The physical interpretation of the expansion expression $\lambda_\pm(\omega)$ is quite clear. The group velocity $V_{\rm g\pm}$ is determined by $V_{\rm g\pm} = {\rm Re}[1/\beta'_\pm(0)]$. $\phi_\pm = {\rm Re}[\beta_\pm(0)]$ denote the phase shifts per unit length, and $\alpha_\pm = {\rm Im}[\beta_\pm(0)]$ are absorption coefficients. For a detailed discussion on the meaning of these parameters on the right-hand side of $\lambda_\pm(\omega)$, it can be found in Ref.[21]. For the typical parameters and conditions for hyperfine-split alkaline-earth-metal atom D lines (e.g. the parameters given in Fig.2,), it is found that the absorption coefficient α_+ is usually much larger than another coefficient α_-. This suggests that the component η_+ in Eqs.(10a) and (10b) decays much faster than another coefficient η_-. Consequently, after a characteristic propagation distance z we have

$$\Omega_{\rm p}(z,t) = W_+\,\Omega_{\rm p}\left(t - \frac{z}{V_{\rm g-}}\right){\rm e}^{{\rm i}\phi_- z - \alpha_- z}, \quad (11a)$$

$$\Omega_{\rm m}(z,t) = W\,\Omega_{\rm p}\left(t - \frac{z}{V_{\rm g-}}\right){\rm e}^{{\rm i}\phi_- z - \alpha_- z}. \quad (11b)$$

Equations (11) demonstrate that the generated TWM field and the pump pulsed field can, after a characteristic propagation distance, travel through the cold atom medium with the same temporal profile and group velocity. We define the TWM efficiency ϵ as the intensity ratio between the output generated-TWM field and the input pump pulsed field, i.e. $\epsilon = \dfrac{\int_{-\infty}^{+\infty}|E_{\rm m}^{(\rm out)}(L,t)|^2{\rm d}t}{\int_{-\infty}^{+\infty}|E_{\rm p}^{(\rm in)}(0,t)|^2{\rm d}t}$. Here $E_{\rm m}^{(\rm out)}$ is the electric field $E_{\rm m}$ ($|E_{\rm m}|^2 = 4\hbar^2\,|\Omega_{\rm m}|^2\,/\,|\mu_{31}|^2$) of the TWM-generated field at the exit of the medium $z = L$ and $E_{\rm p}^{(\rm in)}$ is the electric field $E_{\rm p}$ ($|E_{\rm p}|^2 = 4\hbar^2\,|\Omega_{\rm p}|^2\,/\,|\mu_{21}|^2$) of the pump pulsed field at the entrance to the medium $z = 0$. According to Eqs.(11), the efficiency ϵ takes the form

$$\epsilon = \frac{\kappa_{12}\omega_{\rm m}\int_{-\infty}^{+\infty}|\Omega_{\rm m}^{(\rm out)}(L,t)|^2{\rm d}t}{\kappa_{13}\omega_{\rm p}\int_{-\infty}^{+\infty}|\Omega_{\rm p}^{(\rm in)}(0,t)|^2{\rm d}t} = \frac{\kappa_{12}\omega_{\rm m}}{\kappa_{13}\omega_{\rm p}}\frac{|W|^2}{|W_+|^2}{\rm e}^{-\Gamma L}, \quad (12)$$

where $\Gamma = 2\alpha_-$. In the derivation of the above expression ϵ, we have used the relation $\frac{\kappa_{12}}{\kappa_{13}} = \frac{\omega_{\rm p}|\mu_{12}|^2}{\omega_{\rm m}|\mu_{13}|^2}$.

Now we give a brief discussion on the experimental realization of our scheme. We consider a system in which the decay rates are $\Gamma_2 = 2\gamma_2 \simeq 1.2 \times 10^8 {\rm s}^{-1}$ and $\Gamma_3 = 2\gamma_3 \simeq 2.5 \times 10^8 {\rm s}^{-1}$, typical for transitions in hyperfine-split ^{87}Rb$--D_1$ line (nuclear spin $I = 3/2$) of cold atoms with the designated states chosen as follows: $|1\rangle = |5{\rm S}_{1/2}, F = 1, m = 1\rangle$, $|2\rangle = |5{\rm P}_{1/2}, F = 1, m = 0\rangle$, and $|3\rangle = |5{\rm P}_{1/2}, F = 2, m = 0\rangle$, all accessible with diode lasers. Firstly, an assisting microwave driven field $\Omega_{\rm d}$ is applied to coupled Zeeman sublevels $|3\rangle$ and $|2\rangle$, between which there will appear a quantum coherence. Subsequently, the incident pump pulse is applied to the $|1\rangle \to |2\rangle$ transition and propagates in a relatively thin Rb atomic medium. Then the TWM field between levels $|3\rangle \leftrightarrow |1\rangle$ is generated spontaneously in such a three-level medium.

4. Conclusions

In conclusion, using the coupled Schrödinger–Maxwell equations of a three-level atomic system and a pulsed laser field with assistance of a stationary microwave driven field, we have proposed and analysed a new scheme to achieve the parametric generation of a new laser pulse with high conversion efficiency in highly resonant media. Our scheme may provide new possibilities for technological applications.

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