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Robust fuzzy control for chaotic dynamics in Lorenz systems with uncertainties*

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This paper deals with the robust fuzzy control for chaotic systems in the presence of parametric uncertainties. An uncertain Takagi–Sugeno fuzzy model for a Lorenz chaotic system is first constructed. Then a robust fuzzy state feedback control scheme ensures the control for stable operations under bounded parametric uncertainties. For a chaotic system with known uncertainty bounds, a robust fuzzy regulator is designed by choosing the control parameters satisfying the linear matrix inequality. To verify the validity and effectiveness of the proposed controller design method, an analysis technique is suggested and applied to the control of an uncertain Lorenz chaotic system.

Keywords: chaos, Takagi–Sugeno fuzzy model, robust control, linear matrix inequality
PACC: 0545

1. Introduction

Since the research of OGY method[1] was published, many researchers have attempted to control chaos by means of linear or nonlinear frameworks.[2−8]

In the past decades, fuzzy control has increasingly attracted attention, especially because it can provide an effective solution to the control of plants that are poorly modelled mathematically or its model uncertainty in dynamics is either unknown or impossible.

From a practical point of view, a robust approach is more suitable for the fuzzy feedback control to be free from the uncertainty. In this paper, we study the fuzzy-model-based controller design methodology for the control of uncertain chaotic Lorenz systems. An uncertain Takagi–Sugeno (T–S) fuzzy model for the Lorenz equation is first constituted, then a robust fuzzy feedback regulator is designed. For these structured uncertainty, a sufficient condition for the synthesis of the state feedback fuzzy-model-based controller is derived by applying linear matrix inequalities (LMI) theory which is a new and fast growing field and a valuable alternative to the analytical method.[9]

This paper is organized as follows: In Section 2, the problem under consideration is described and a brief description of T–S fuzzy model is also given, and in Section 3, an alternative representation of the uncertain T–S fuzzy model is presented and the chaotic Lorenz system is modelled. A state feedback fuzzy-model-based regulator is then derived in Section 4. An illustrative example is given in Section 5 and finally some conclusions are drawn in Section 6.

2. Problem description

The T–S fuzzy model describes the local dynamics of a given uncertain system by linear systems, and the overall fuzzy model of the system is obtained by fuzzy blending with all local linear models.[10,11] In view of parametric uncertainties, the ith rule for the T–S fuzzy system is formulated as the following form.[12]

Rule i:

$$\text{IF } z_1(t) \text{ is } N_{i1} \text{ and } \ldots \text{ and } z_n(t) \text{ is } N_{in}$$

$$\text{THEN } \dot{\boldsymbol{x}}(t) = (\boldsymbol{A}_i + \Delta\boldsymbol{A}_i)\boldsymbol{x}(t) + (\boldsymbol{B}_i + \Delta\boldsymbol{B}_i)\boldsymbol{u}(t), \quad i = 1, 2, ..., q, \tag{1}$$

where z_i is the premise variable assumed to be independent of the control input $\boldsymbol{u}$ and N_{ij} is fuzzy sets, $\boldsymbol{x}(t) \in \Re^n$ is the state vector, $\boldsymbol{u}(t) \in \Re^m$, $\boldsymbol{A}_i \in \Re^{n\times n}$

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and $\boldsymbol{B}_i \in \Re^{n\times m}$, and $\Delta\boldsymbol{A}_i$ and $\Delta\boldsymbol{B}_i$ represent the uncertainties in the system. Using the centre-average defuzzification, product inference, and single-tone fuzzifier, the global T–S fuzzy system (1) is obtained as

$$\begin{aligned}\dot{\boldsymbol{x}}(t) =& \sum_{i=1}^{q} \mu_i(z(t))((\boldsymbol{A}_i + \Delta\boldsymbol{A}_i)\boldsymbol{x}(t) \\ &+ (\boldsymbol{B}_i + \Delta\boldsymbol{B}_i)\boldsymbol{u}(t)),\end{aligned} \tag{2}$$

where

$$\mu(z(t)) = \frac{\omega_i(z(t))}{\sum_{i=1}^{q}\omega_i(z(t))},$$

$$\omega_i(z(t)) = \prod_{j=1}^{n} N_{ij}(z_j(t)),$$

and $N_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in N_{ij}.

Assumption 1 The uncertainty matrices $\Delta\boldsymbol{A}_i$ and $\Delta\boldsymbol{B}_i$ are norm-bounded, and in the form

$$[\Delta\boldsymbol{A}_i \quad \Delta\boldsymbol{B}_i\ ] = \boldsymbol{D}_i\boldsymbol{F}_i(t)[\boldsymbol{E}_{1i} \quad \boldsymbol{E}_{2i}\ ],$$

where $\boldsymbol{D}_i$, $\boldsymbol{E}_{1i}$, and $\boldsymbol{E}_{2i}$ are known real constant matrices with appropriate dimensions, presenting the structures of the system uncertainties, and $\boldsymbol{F}_i(t)$ is an unknown matrix-valued function with Lebesgue-measurable elements, satisfying

$$\boldsymbol{F}_i^T(t)\boldsymbol{F}_i(t) \le \boldsymbol{I},$$

where $\boldsymbol{I}$ is the identity matrix with an appropriate dimension.

3. Modelling the Lorenz chaotic systems

In this section, an alternative formulation of the above-described uncertain T–S fuzzy model (2) is proposed, which will be used for the design of state feedback fuzzy-model-based controller below.

3.1. Alternative formulation

It is well known that the uncertain T–S fuzzy model (2) has strong nonlinear interactions among the fuzzy rules, which complicates the analysis and control of the system. In order to overcome these difficulties, the T–S fuzzy model (2) is presented as a set of uncertain linear systems. Define

$$\Phi_i = \{\boldsymbol{x}|\mu_i(\boldsymbol{x}) \ge \mu_l(\boldsymbol{x}), l = 1, 2, ..., q_i\}, \quad i = 1, 2, ..., q, \tag{3}$$

where q_i is the number of the fuzzy rules that are simultaneously fired with the ith fuzzy rule.

The feature of Φ is revealed in the following expression:

$$\lambda_i = \begin{cases} 1 & \boldsymbol{x} \in \Phi_i, \\ 0 & \boldsymbol{x} \notin \Phi_i, \end{cases} \qquad \sum_{i=1}^{q}\lambda_i = 1.$$

In each subspace, the fuzzy model can be presented by an uncertain linear system as follows:

$$\begin{aligned}\dot{\boldsymbol{x}} =& (\boldsymbol{A}_i + \Delta\boldsymbol{A}_i + \Delta\tilde{\boldsymbol{A}}_i + \Delta\bar{\boldsymbol{A}}_i)\boldsymbol{x} \\ &+ (\boldsymbol{B}_i + \Delta\boldsymbol{B}_i + \Delta\tilde{\boldsymbol{B}}_i + \Delta\bar{\boldsymbol{B}}_i)\boldsymbol{u}, \boldsymbol{x} \in \Phi_i,\end{aligned} \tag{4}$$

where

$$\Delta\tilde{\boldsymbol{A}}_i = \sum_{l=1}^{q_i}\mu_l(\boldsymbol{A}_l - \boldsymbol{A}_i),$$

$$\Delta\bar{\boldsymbol{A}}_i = \sum_{l=1}^{q_i}\mu_l(\Delta\boldsymbol{A}_l - \Delta\boldsymbol{A}_i),$$

$$\Delta\tilde{\boldsymbol{B}}_i = \sum_{l=1}^{q_i}\mu_l(\boldsymbol{B}_l - \boldsymbol{B}_i),$$

$$\Delta\bar{\boldsymbol{B}}_i = \sum_{l=1}^{q_i}\mu_l(\Delta\boldsymbol{B}_l - \Delta\boldsymbol{B}_i).$$

It is noted when the LMI-based procedure is utilized, the matrices $\Delta\boldsymbol{A}_i$, $\Delta\tilde{\boldsymbol{A}}_i$, $\Delta\bar{\boldsymbol{A}}_i$, $\Delta\boldsymbol{B}_i$, $\Delta\tilde{\boldsymbol{B}}_i$, $\Delta\bar{\boldsymbol{B}}_i$ must be more carefully considered, for the solvability of the LMIs depends on the matrix structures. That is to say, even though there exists a feasible controller for the given system in theory, the solution may not be found with LMIs constructed by the expression (4) in practice.

Here, an alternative formulation of the T–S fuzzy model (1) is utilized.[13] In each subspace Φ_i, the T–S fuzzy system is formulated by

$$\begin{aligned}\dot{\boldsymbol{x}} =& (\boldsymbol{A}_i + \Delta\boldsymbol{A}_i)\boldsymbol{x} + \sum_{l=1}^{q_i}\mu_l(\boldsymbol{A}_l - \boldsymbol{A}_i)\boldsymbol{x} \\ &+ \sum_{l=1}^{q_i}\mu_l(\Delta\boldsymbol{A}_l - \Delta\boldsymbol{A}_i)\boldsymbol{x} \\ &+ (\boldsymbol{B}_i + \Delta\boldsymbol{B}_i)\boldsymbol{u} + \sum_{l=1}^{q_i}\mu_l(\boldsymbol{B}_l - \boldsymbol{B}_i)\boldsymbol{u} \\ &+ \sum_{l=1}^{q_i}\mu_l(\Delta\boldsymbol{B}_l - \Delta\boldsymbol{B}_i)\boldsymbol{u} \\ =& \left(\boldsymbol{A}_i + \Delta\boldsymbol{A}_i + \sum_{l=1}^{q_i}\mu_l\tilde{\boldsymbol{D}}_{li}\tilde{\boldsymbol{E}}_{li1} + \sum_{l=1}^{q_i}\mu_l\bar{\boldsymbol{D}}_{li}\bar{\boldsymbol{E}}_{li1}\right)\boldsymbol{x} \\ &+ \left(\boldsymbol{B}_i + \Delta\boldsymbol{B}_i + \sum_{l=1}^{q_i}\mu_l\tilde{\boldsymbol{D}}_{li}\tilde{\boldsymbol{E}}_{li2} + \sum_{l=1}^{q_i}\mu_l\bar{\boldsymbol{D}}_{li}\bar{\boldsymbol{E}}_{li2}\right)\boldsymbol{u} \\ =& (\boldsymbol{A}_i + \boldsymbol{D}_i\boldsymbol{F}_i\boldsymbol{E}_{1i} + \tilde{\boldsymbol{D}}_i\tilde{\boldsymbol{F}}_i\tilde{\boldsymbol{E}}_{i1} + \bar{\boldsymbol{D}}_i\bar{\boldsymbol{F}}_i\bar{\boldsymbol{E}}_{i1})\boldsymbol{x} \\ &+ (\boldsymbol{B}_i + \boldsymbol{D}_i\boldsymbol{F}_i\boldsymbol{E}_{2i} + \tilde{\boldsymbol{D}}_i\tilde{\boldsymbol{F}}_i\tilde{\boldsymbol{E}}_{i2} \\ &+ \bar{\boldsymbol{D}}_i\bar{\boldsymbol{F}}_i\bar{\boldsymbol{E}}_{i2})\boldsymbol{u}, \quad \boldsymbol{x} \in \Phi_i,\end{aligned} \tag{5}$$

where

$$\boldsymbol{A}_l - \boldsymbol{A}_i = \tilde{\boldsymbol{D}}_{li}\tilde{\boldsymbol{E}}_{li1},$$
$$\boldsymbol{B}_l - \boldsymbol{B}_i = \tilde{\boldsymbol{D}}_{li}\tilde{\boldsymbol{E}}_{li2},$$
$$\Delta\boldsymbol{A}_l - \Delta\boldsymbol{A}_i = \bar{\boldsymbol{D}}_{li}\bar{\boldsymbol{F}}_{li}\bar{\boldsymbol{E}}_{li1},$$
$$\Delta\boldsymbol{B}_l - \Delta\boldsymbol{B}_i = \bar{\boldsymbol{D}}_{li}\bar{\boldsymbol{F}}_{li}\bar{\boldsymbol{E}}_{li2},$$
$$\tilde{\boldsymbol{D}}_i = [\tilde{\boldsymbol{D}}_{1i}...\tilde{\boldsymbol{D}}_{q_i i}],$$
$$\tilde{\boldsymbol{F}}_i(t) = \mathrm{diag}[\mu_1(t)\boldsymbol{I}...\mu_{q_i i}(t)\boldsymbol{I}],$$
$$\bar{\boldsymbol{D}}_i = [\bar{\boldsymbol{D}}_{1i}...\bar{\boldsymbol{D}}_{q_i i}],$$
$$\tilde{\boldsymbol{E}}_{i1} = [\tilde{\boldsymbol{E}}_{1i1}^{\mathrm{T}}...\tilde{\boldsymbol{E}}_{q_i i1}^{\mathrm{T}}]^{\mathrm{T}},$$
$$\bar{\boldsymbol{F}}_i(t) = \mathrm{diag}[\mu_1(t)\bar{\boldsymbol{F}}_{1i}...\mu_{q_i}(t)\bar{\boldsymbol{F}}_{q_i i}],$$
$$\tilde{\boldsymbol{E}}_{i2} = [\tilde{\boldsymbol{E}}_{1i2}^{\mathrm{T}}...\tilde{\boldsymbol{E}}_{q_i i2}^{\mathrm{T}}]^{\mathrm{T}},$$
$$\bar{\boldsymbol{E}}_{i1} = [\bar{\boldsymbol{E}}_{1i1}^{\mathrm{T}}...\bar{\boldsymbol{E}}_{q_i i1}^{\mathrm{T}}]^{\mathrm{T}},$$
$$\bar{\boldsymbol{E}}_{i2} = [\bar{\boldsymbol{E}}_{1i2}^{\mathrm{T}}...\bar{\boldsymbol{E}}_{q_i i2}^{\mathrm{T}}]^{\mathrm{T}},$$

and

$$\tilde{\boldsymbol{F}}_i(t)^{\mathrm{T}}\tilde{\boldsymbol{F}}_i(t) \le \tilde{\boldsymbol{R}}_i, \quad \bar{\boldsymbol{F}}_i(t)^{\mathrm{T}}\bar{\boldsymbol{F}}_i(t) \le \bar{\boldsymbol{R}}_i.$$

Therefore, the global dynamics is constructed by

$$\dot{\boldsymbol{x}} = \sum_{i=1}^{q}\{\lambda_i(\boldsymbol{A}_i + \boldsymbol{D}_i\boldsymbol{F}_i\boldsymbol{E}_{1i} + \tilde{\boldsymbol{D}}_i\tilde{\boldsymbol{F}}_i\tilde{\boldsymbol{E}}_{i1} + \bar{\boldsymbol{D}}_i\bar{\boldsymbol{F}}_i\bar{\boldsymbol{E}}_{i1})\boldsymbol{x} + (\boldsymbol{B}_i + \boldsymbol{D}_i\boldsymbol{F}_i\boldsymbol{E}_{2i} + \tilde{\boldsymbol{D}}_i\tilde{\boldsymbol{F}}_i\tilde{\boldsymbol{E}}_{i2} + \bar{\boldsymbol{D}}_i\bar{\boldsymbol{F}}_i\bar{\boldsymbol{E}}_{i2})\boldsymbol{u}\}. \quad (6)$$

The system (6) is mathematically identical to the T–S fuzzy system (2), but structurally more convenient for the design purpose.

It may be noticed that the proposed representation is different from that in Ref.[14], in that the resulting representation is much more suitable for solving the relevant problems with LMIs, as will be further demonstrated by the simulation example studied below.

3.2. Modelling the Lorenz chaotic system

The Lorenz equations are as follows:

$$\begin{cases} \dot{x}_1(t) = -a(x_2(t) - x_1(t)), \\ \dot{x}_2(t) = bx_1(t) - x_2(t) - x_1(t)x_3(t), \\ \dot{x}_3(t) = x_1(t)x_2(t) - cx_3(t). \end{cases} \quad (7)$$

Figure 1 shows the phase portrait of its attractor with system parameters (a, b, c)=(10, 28, 8/3) from time step $t_0 = 0$ to $t_{\mathrm{f}} = 10$, where initial condition is $(x_1, x_2, x_3) = (10, -10, -10)$.

To construct a T–S fuzzy model for the Lorenz system, the quadratic nonlinear terms x_1x_2 and $-x_1x_3$ in Eq.(7) must be expressed as linear weighted sums of some linear functions.

Case 1 The nonlinear term $f(\boldsymbol{x}(t)) = x_1(t)x_2(t)$ can be represented as

$$f(\boldsymbol{x}(t)) = \sum_{i=1}^{2}\mu_i g_i(\boldsymbol{x}(t))x_2(t),$$

where

$$g_1(\boldsymbol{x}(t)) = M_1, \quad g_2(\boldsymbol{x}(t)) = M_2$$

and

$$\mu_1(\boldsymbol{x}(t)) = \frac{-x_1 + M_2}{M_2 - M_1},$$
$$\mu_2(\boldsymbol{x}(t)) = \frac{x_1 - M_1}{M_2 - M_1}. \quad (8)$$

Therefore, we can construct an exact T–S fuzzy model of the system (7) as follows:

Plant Rules:

$$\text{Rule 1 : IF } x_1(t) \text{ is } N_{\min}, \text{ THEN } \dot{\boldsymbol{x}} = \boldsymbol{A}_1\boldsymbol{x};$$
$$\text{Rule 2 : IF } x_1(t) \text{ is } N_{\max}, \text{ THEN } \dot{\boldsymbol{x}} = \boldsymbol{A}_2\boldsymbol{x}; \quad (9)$$

where

$$\boldsymbol{A}_1 = \begin{bmatrix} -a & a & 0 \\ b & -1 & -x_{\min} \\ 0 & x_{\min} & -c \end{bmatrix},$$

$$\boldsymbol{A}_2 = \begin{bmatrix} -a & a & 0 \\ b & -1 & -x_{\max} \\ 0 & x_{\max} & -c \end{bmatrix}.$$

Setting $[x_{\min}, x_{\max}]$ as [–20, 30], because $x_1(t)$ is bounded within [–20, 30] in Fig.1, the membership functions are

$$N_{x_{\min}} = \frac{-x_1 + x_{\max}}{x_{\max} - x_{\min}},$$
$$N_{x_{\max}} = \frac{x_1 - x_{\min}}{x_{\max} - x_{\min}}. \quad (10)$$

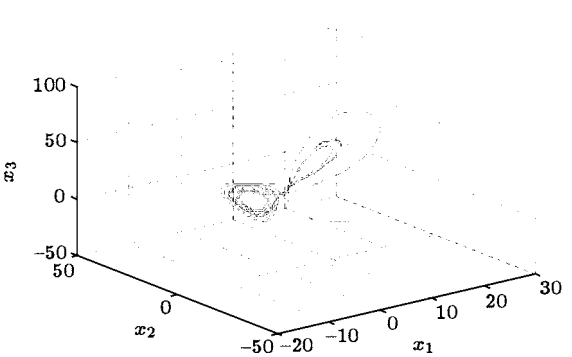


Fig.1. Phase portrait of Lorenz chaotic attractor under initial condition $x(0) = (10, -10, -10)$.

Figure 2 shows the membership functions for the plant rules.

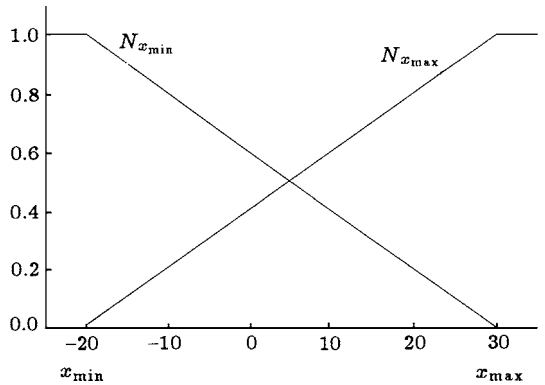


Fig.2. The membership functions for the T–S fuzzy model of Lorenz system.

The defuzzified output of the T–S fuzzy model (9) with membership functions (10) can be calculated. Thus, the T–S fuzzy model of the controlled system with uncertain parameter values is:

Rule 1: IF $x_1(t)$ is $N_{x_{\min}}$,
THEN $\dot{\boldsymbol{x}} = (\boldsymbol{A}_1 + \Delta\boldsymbol{A}_1)\boldsymbol{x} + (\boldsymbol{B}_1 + \Delta\boldsymbol{B}_1)\boldsymbol{u}$;
Rule 2: IF $x_1(t)$ is $N_{x_{\max}}$,
THEN $\dot{\boldsymbol{x}} = (\boldsymbol{A}_2 + \Delta\boldsymbol{A}_2)\boldsymbol{x} + (\boldsymbol{B}_2 + \Delta\boldsymbol{B}_2)\boldsymbol{u}$.

The uncertainty matrices $\Delta\boldsymbol{A}_i$ and $\Delta\boldsymbol{B}_i$ are represented as

$$\Delta\boldsymbol{A}_i = \boldsymbol{D}_i\boldsymbol{F}_i(t)\boldsymbol{E}_{1i}$$

$$\text{and} \quad \Delta\boldsymbol{B}_i = \boldsymbol{D}_i\boldsymbol{F}_i(t)\boldsymbol{E}_{2i}, \quad i = 1, 2,$$

where $\boldsymbol{D}_1 = \boldsymbol{D}_2 = \text{diag}[\ -0.3 \quad 0.3 \quad 0.3\ ]$,

$$\boldsymbol{E}_{11} = \boldsymbol{E}_{12} = \begin{bmatrix} a & -a & 0 \\ b & 0 & 0 \\ 0 & 0 & c \end{bmatrix} = \begin{bmatrix} 10 & -10 & 0 \\ 28 & 0 & 0 \\ 0 & 0 & 8/3 \end{bmatrix},$$

$$\boldsymbol{E}_{12} = \boldsymbol{E}_{22} = \boldsymbol{0}$$

In the ith subspace Φ_i, the T–S fuzzy model of Lorenz system is formulated by

$$\begin{aligned}\dot{\boldsymbol{x}} =& (\boldsymbol{A}_i + \boldsymbol{D}_i\boldsymbol{F}_i\boldsymbol{E}_{1i} + \tilde{\boldsymbol{D}}_i\tilde{\boldsymbol{F}}_i\tilde{\boldsymbol{E}}_{i1} + \bar{\boldsymbol{D}}_i\bar{\boldsymbol{F}}_i\bar{\boldsymbol{E}}_{i1})\boldsymbol{x} \\ &+ (\boldsymbol{B}_i + \boldsymbol{D}_i\boldsymbol{F}_i\boldsymbol{E}_{2i} + \tilde{\boldsymbol{D}}_i\tilde{\boldsymbol{F}}_i\tilde{\boldsymbol{E}}_{i2} \\ &+ \bar{\boldsymbol{D}}_i\bar{\boldsymbol{F}}_i\bar{\boldsymbol{E}}_{i2})\boldsymbol{u}, \quad \boldsymbol{x} \in \Phi_i,\end{aligned}$$

where

$$\tilde{\boldsymbol{D}}_1 = \tilde{\boldsymbol{D}}_2 = [\ 1 \quad 1 \quad 1\ ]^{\mathrm{T}},$$

$$\tilde{\boldsymbol{E}}_{11} = \tilde{\boldsymbol{E}}_{12} = [\ 0 \quad 10 \quad -10\ ],$$

$$\tilde{\boldsymbol{E}}_{12} = \tilde{\boldsymbol{E}}_{22} = \boldsymbol{0}, \quad \bar{\boldsymbol{D}}_1 = \bar{\boldsymbol{D}}_2 = \boldsymbol{0},$$

$$\bar{\boldsymbol{E}}_{11} = \bar{\boldsymbol{E}}_{21} = \boldsymbol{0}, \quad \bar{\boldsymbol{E}}_{12} = \bar{\boldsymbol{E}}_{22} = \boldsymbol{0}.$$

4. Design of robust fuzzy-model-based controller

In this section, a controller design method is developed for the fuzzy system (1). The basic idea involves dividing the overall T–S fuzzy system into uncertain linear subsystems and designing a local state feedback controller for each subsystem. The overall fuzzy-model-based controller is then constructed by combining all the local controllers with a switching rule. For the synthesis of the controller, the design problem is transformed into the LMI format, then the fuzzy-model-based controller is obtained by solving the resulting LMIs.

Here, a state feedback fuzzy-model-based controller is derived for the uncertain T–S fuzzy model with the proposed representation method and LMIs involved. The controller is of the form

$$\boldsymbol{u} = -\sum_{i=1}^{q} \lambda_i \boldsymbol{k}_i \boldsymbol{x}, \tag{11}$$

where $\boldsymbol{k}_i$ is the local state feedback gain in subspace Φ_i.

In each subspace, the state feedback fuzzy-model-based controller is denoted by

$$\boldsymbol{u} = -\boldsymbol{k}_i\boldsymbol{x}, \quad \boldsymbol{x} \in \Phi_i, \tag{12}$$

then the closed-loop system of the expressions (6) and (11) is

$$\begin{aligned}\dot{\boldsymbol{x}} =& \sum_{i=1}^{q} \lambda_i (\boldsymbol{A}_i - \boldsymbol{B}_i\boldsymbol{k}_i + \boldsymbol{D}_i\boldsymbol{F}_i(\boldsymbol{E}_{1i} - \boldsymbol{E}_{2i}\boldsymbol{k}_i) \\ &+ \tilde{\boldsymbol{D}}_i\tilde{\boldsymbol{F}}_i(\tilde{\boldsymbol{E}}_{i1} - \tilde{\boldsymbol{E}}_{i2}\boldsymbol{k}_i) \\ &+ \bar{\boldsymbol{D}}_i\bar{\boldsymbol{F}}_i(\bar{\boldsymbol{E}}_{i1} - \bar{\boldsymbol{E}}_{i2}\boldsymbol{k}_i))\boldsymbol{x}.\end{aligned} \tag{13}$$

In the following, Theorem 1 provides a design algorithm for the state feedback fuzzy-model-based controller (11).

Theorem 1 The uncertain T–S fuzzy model (1) can be stabilized with the state feedback fuzzy-model-based controller (11), if there exist a symmetric and positive definite matrix $\boldsymbol{P}_i$, and matrix $\boldsymbol{N}_i$, such that

$$\begin{bmatrix} \Psi_i & * & * & * \\ \boldsymbol{E}_{1i}\boldsymbol{P}_i - \boldsymbol{E}_{2i}\boldsymbol{N}_i & -\varepsilon_i\boldsymbol{I} & * & * \\ \tilde{\boldsymbol{E}}_{i1}\boldsymbol{P}_i - \tilde{\boldsymbol{E}}_{i2}\boldsymbol{N}_i & \boldsymbol{0} & -\tilde{\varepsilon}_i\tilde{\boldsymbol{R}}_i & * \\ \bar{\boldsymbol{E}}_{i1}\boldsymbol{P}_i - \bar{\boldsymbol{E}}_{i2}\boldsymbol{N}_i & \boldsymbol{0} & \boldsymbol{0} & -\bar{\varepsilon}_i\bar{\boldsymbol{R}}_i \end{bmatrix} < 0, \tag{14}$$

where

$$\begin{aligned}\Psi_i =&(\boldsymbol{A}_i\boldsymbol{P}_i - \boldsymbol{B}_i\boldsymbol{N}_i)^{\mathrm{T}} \\ &+ (\boldsymbol{A}_i\boldsymbol{P}_i - \boldsymbol{B}_i\boldsymbol{N}_i) + \varepsilon_i\boldsymbol{D}_i\boldsymbol{D}_i^{\mathrm{T}} \\ &+ \tilde{\varepsilon}_i\tilde{\boldsymbol{D}}_i\tilde{\boldsymbol{D}}_i^{\mathrm{T}} + \bar{\varepsilon}_i\bar{\boldsymbol{D}}_i\bar{\boldsymbol{D}}_i^{\mathrm{T}}\end{aligned}$$

and $\boldsymbol{k}_i = \boldsymbol{N}_i\boldsymbol{P}_i^{-1}$, and the symbol "*" denotes the transposed elements in the symmetric positions. The proof is omitted here.

5. Computer simulations

In this section, simulation results of the controlled Lorenz chaotic attractor are given. The local state feedback gains $\boldsymbol{k}_i$ are obtained by applying Theorem 1. And by solving the LMIs with $\varepsilon_i = \tilde{\varepsilon}_i = \bar{\varepsilon}_i = 1$ and $\boldsymbol{R}_i = \tilde{\boldsymbol{R}}_i = \bar{\boldsymbol{R}}_i = 0.5\boldsymbol{I}$, $i = 1, 2$, we obtain

$$\boldsymbol{N}_1 = \begin{bmatrix} -7.9505 & -10.3433 & 0.5063 \\ -2.8276 & -9.4183 & 9.5871 \\ -10.9292 & -9.8381 & -9.7044 \end{bmatrix},$$

$$\boldsymbol{N}_2 = \begin{bmatrix} -13.9354 & -29.7576 & 5.0325 \\ 14.6514 & -20.5906 & 55.3673 \\ -40.0608 & -43.1635 & -37.5934 \end{bmatrix},$$

$$\boldsymbol{P}_1 = \begin{bmatrix} 0.0986 & 0.0732 & 0.0716 \\ 0.0732 & 0.0904 & 0.0486 \\ 0.0716 & 0.0486 & 0.1228 \end{bmatrix},$$

$$\boldsymbol{P}_2 = \begin{bmatrix} 0.1606 & 0.1371 & 0.1597 \\ 0.1371 & 0.1726 & 0.1142 \\ 0.1597 & 0.1142 & 0.2253 \end{bmatrix}.$$

Thus, the controller gain matrices are:

$$\boldsymbol{k}_1 = \begin{bmatrix} -56.0003 & -112.8532 & 81.4376 \\ -3.1593 & -183.6663 & 152.6015 \\ -52.1426 & -51.4032 & -28.2802 \end{bmatrix},$$

$$\boldsymbol{k}_2 = \begin{bmatrix} -80.1213 & -241.5372 & 200.6689 \\ -28.2139 & -408.2525 & 470.5945 \\ -81.5049 & -170.3629 & -22.63198 \end{bmatrix}.$$

The system parameters a, b, c are randomly varied within 30% of their nominal values. The initial values are $x(0) = (10, -10, -10)$. The simulation time is 20s.

The control result is shown in Fig.3. The control input is activated at t=10s for comparison. Before the control input is added, the phase trajectory of the Lorenz system is chaotic. However, after the control input is activated, the chaotic state is quickly directed to the origin. Figure 4 shows the simulation result for the Lorenz system without parametric uncertainties but with the same feedback controller.

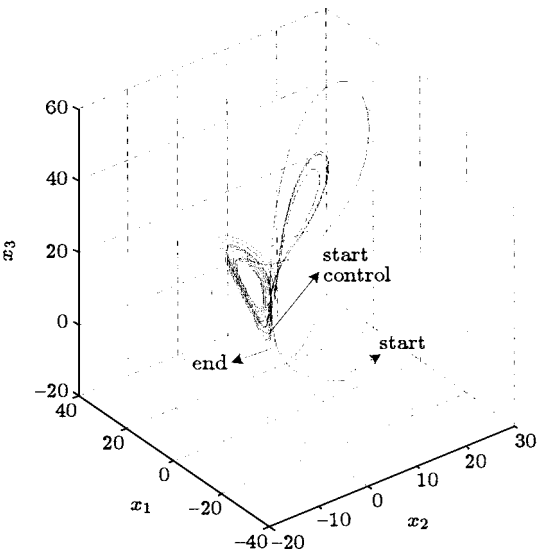


Fig.3. The controlled phase trajectory of the Lorenz system with parametric uncertainties.

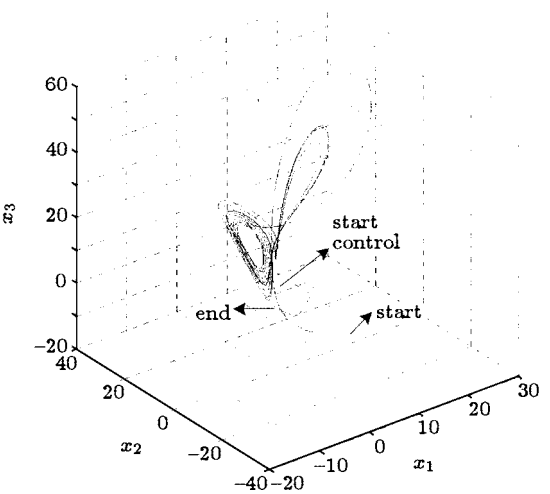


Fig.4. The phase trajectory of the controlled Lorenz system without parametric uncertainties.

The control input is also activated at t=10s. Similar to the case of the parametric uncertain system, the system state is rapidly guided to the origin after control input is activated. Two simulation results show that the T–S fuzzy model-based controller, designed by using Theorem 1, is robust against norm-bounded parametric uncertainties.

6. Conclusions

In this paper, a novel robust fuzzy-model-based controller design methodology for the control of

chaotic systems subject to parametric uncertainties is presented using LMIs. A well-known uncertain T–S fuzzy model is utilized to construct a nonlinear Lorenz chaotic model. Also, an alternative representation is developed and used to cast the uncertain T–S fuzzy model into a set of uncertain linear systems in several subspaces. The state feedback controller is then derived by transforming the design problem into an LMI framework. Finally, the effectiveness of the proposed control technique is demonstrated by computer simulations.

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