Vol 15 No 1, January 2006
1009-1963/2006/15(01)/0083-06

Chinese Physics

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and IOP Publishing Ltd

A new output feedback synchronization theorem for a class of chaotic systems with a scalar transmitted signal*

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(Received 22 October 2004)

This paper proposes a new, simple and yet applicable output feedback synchronization theorem for a large class of chaotic systems. We take a linear combination of drive system state variables as a scale-driving signal. It is proved that synchronization between the drive and the response systems can be obtained via a simple linear output error feedback control. The linear feedback gain is a function of a free parameter. The approach is illustrated using the Rössler hyperchaotic systems and Chua's chaotic oscillators.

Keywords: chaos, hyperchaos, synchronization
PACC: 0545

1. Introduction

Starting with Pecora and Carroll's observation about the possibility of synchronizing two chaotic systems in a drive-response configuration,[1] synchronization in chaotic systems has become an area of active research.[1−10]

Due to the simplicity in configuration and implementation, the linear output feedback scheme in which the output (or driving signal) is a scalar signal and the linear combination of the drive system state variables is especially attractive and has been commonly adopted for practical implementation.[4−7] For example, Peng *et al*[4] shows that synchronization of hyperchaotic systems can be attained with a suitably scalar driving signal and uses the negativity of the conditional Lyapunov exponents as a criterion for synchronization. However, Synchronization of two chaotic systems is not guaranteed by having only negative conditional or transverse Lyapunov exponents. Intervals of desynchronized bursting behaviour, called attractor bubbling, can appear even when the largest transverse Lyapunov exponent is negative.[11] It is also difficult how to find K–B parameters such that the conditional Lyapunov exponents are negative. Later, Refs.[5–7] proposed global synchronization theorems for a subclass of chaotic Lur'e systems. But, the proposed approaches require that both the linear part and nonlinear part of the chaotic systems to have a special structure.

In this paper, a new, simple and yet applicable synchronization theorem is presented for a large class of chaotic systems which is more general than that considered in Refs.[5, 6]. Taking the linear combination of the drive system state variables as a scale-driving signal, we prove that asymptotic synchronization can be obtained through a simple linear output error feedback control. The linear feedback gain is a function of a free parameter. The proposed approach is simple and easy to realize. We use the three-dimensional Chua's chaotic oscillator[12] and the four-dimensional Rössler hyperchaotic system[13] to illustrate the applicability of our synchronization scheme.

2. A new output feedback synchronization theorem

We consider a drive system described by the state equation

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + Bf(\boldsymbol{x}) + C, \tag{1}$$

where $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)^{\mathrm{T}} \in R^{n\times 1}$ is the state vector, $f : R^n \to R$ is the nonlinear function whereas

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$A \in R^{n\times n}$, $B \in R^{n\times 1}$ and $C \in R^{n\times 1}$ are the constant matrices. We assume (A, B) to be a controllable pair. $(\cdot)^{\mathrm{T}}$ denotes transpose of a matrix.

Remark 1 This class of chaotic systems in the form of Eq.(1) is more general than that considered in Refs [5, 6]. Several well-known (hyper) chaotic systems belong to the class individualized by Eq.(1), for example, Chua's circuit,[12] the Rössler hyperchaotic system,[13] the Saito hyperchaotic circuit,[14] the TNC hyperchaotic circuit,[15] the n-dimensional Chua circuit.[16]

It is worth noting that the synchronizing signal is an artificial output of the system (1) that can be properly designed to drive a response system. Since the adoption of a scalar signal is a suitable feature and preferable to secure communication applications, the transmitted signal is chosen as a scalar variable in the form

$$y(t) = \boldsymbol{H}^{\mathrm{T}}\boldsymbol{x}(t) = h_1x_1 + h_2x_2 + \cdots + h_nx_n, \quad (2)$$

where $\boldsymbol{H} = (h_1, \ldots, h_n)^{\mathrm{T}} \in R^{n\times 1}$ is a constant vector. In our synchronization scheme, we use a linear output error feedback approach to synchronize the drive system with the response system. We take $y(t)$ in the expression (2) as a scalar-driving signal. The state equation of the response system is

$$\dot{\hat{\boldsymbol{x}}} = A\hat{\boldsymbol{x}} + Bf(\hat{\boldsymbol{x}}) + C + \boldsymbol{K}(y - \hat{y}), \quad (3)$$

where $\hat{\boldsymbol{x}} = (\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n)^{\mathrm{T}} \in R^{n\times 1}$ is the state vector of the response system, $\hat{y}(t) = \boldsymbol{H}^{\mathrm{T}}\hat{x}$, and $\boldsymbol{K} = (k_1, \ldots, k_n)^{\mathrm{T}} \in R^{n\times 1}$ is the linear output error feedback gain.

The design procedure of the output error feedback gain vector $\boldsymbol{K}$ and output gain vector $\boldsymbol{H}$ is given as follows:

1) Construct a constant matrix $T \in R^{n\times n}$ as follows:

$$T = QW, \quad (4)$$

where $Q = [\ A^{n-1}B \ \cdots \ AB \ \ B\ ]$ and

$$W = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ a_1 & 1 & \ddots & & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ a_{n-2} & a_{n-3} & \cdots & 1 & 0 \\ a_{n-1} & a_{n-2} & \cdots & a_1 & 1 \end{bmatrix}. \quad (5)$$

Here $\{a_1, a_2, \ldots, a_n\}$ are the coefficients of the characteristic polynomial of A, i.e.

$$|sI - A| = s^n + a_1s^{n-1} + \cdots + a_n.$$

Moreover, we denote $S = [-a_n, -a_{n-1}, \ldots, -a_1]^{\mathrm{T}}$.

2) Calculate the output gain vector $\boldsymbol{H}$ in the expression (2)

$$\boldsymbol{H}^{\mathrm{T}} = (1,\ 0,\ \ldots,\ 0)T^{-1}. \quad (6)$$

3) For a given positive constant θ, calculate the feedback gain vector $\boldsymbol{K}$ in Eq.(3)

$$\boldsymbol{K} = TL(\theta), \quad (7)$$

where

$$L(\theta) = [\ \Delta_n^1\theta \ \ \cdots \ \ \Delta_n^{n-1}\theta^{n-1} \ \ \theta^n\ ]^{\mathrm{T}}, \quad (8)$$

with the combination $\Delta_n^i = \dfrac{n!}{(n-i)!\ i!}$.

Remark 2 Since (A, B) is a controllable pair, Q is a nonsingular matrix. Moreover, T is also a nonsingular matrix.

Before giving our main theorem, we first introduce two lemmas.

Lemma 1[5] Denote

$$D = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \vdots & \vdots & & \ddots & 1 \\ 0 & 0 & \cdots & \cdots & 0 \end{bmatrix} \in R^{n\times n},$$

and

$$d = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \in R^{n\times 1}. \quad (9)$$

There exists a unique positive definite matrix $P(\theta) = (p_{ij}(\theta))$ that satisfies the matrix equation

$$\theta P + D^{\mathrm{T}}P + PD = dd^{\mathrm{T}}, \quad \theta > 0. \quad (10)$$

Moreover, we have

$$1)\quad p_{ij}(\theta) = \frac{1}{\theta^{i+j-1}}p_{ij}(1), \quad i, j = 1,\ 2,\ \ldots,\ n. \quad (11)$$

$$2)\quad p_{1j}(1) = (-1)^{j+1}, \quad j \geq 1;$$
$$p_{ij}(1) = -[p_{i,j-1}(1) + p_{i-1,j}(1)], \quad i > 1, \quad j \geq i. \quad (12)$$

3) $L(\theta)$ defined in (8) is just the first column of $P(\theta)^{-1}$, i.e.,

$$L(\theta) = P(\theta)^{-1}d. \tag{13}$$

Lemma 2[17] Denote $B_c = [\ 0 \quad \cdots \quad 0 \quad 1\ ]^T$, then

$$T^{-1}AT = D + B_c S^T, \tag{14}$$

$$T^{-1}B = B_c. \tag{15}$$

Theorem 1 Consider the drive system (1) and response system (3) where $\boldsymbol{H}$ and $\boldsymbol{K}$ are described by the expressions (6) and (7), respectively. Suppose that $f(\cdot)$ is a Lipschitz nonlinear function with the Lipschitz constant $\gamma > 0$ (since the state of a chaotic system varies in a bounded region and $f(\cdot)$ is a smooth function, this assumption holds true at least in this region). If

$$\theta > \mu \equiv 2\beta\sqrt{\frac{p_{nn}(1)}{\lambda_{\min}(P(1))}},$$

where $\beta = \left\|S^T\right\| + \gamma\|T\|$ and $\lambda_{\min}(P(1))$ denote the smallest eigenvalue of $P(1)$, then the drive system (1) and the response system (3) are asymptotically synchronized with each other.

Proof Define the coordinate transformation $z = T^{-1}x$, we transform the drive system (1) into the following equation:

$$\dot{z} = T^{-1}ATz + T^{-1}Bf(Tz) + T^{-1}C. \tag{16}$$

From Lemma 2, we can obtain

$$\begin{aligned}\dot{z} =& Dz + B_c S^T z + B_c f(Tz) + T^{-1}C\\ =& Dz + B_c(S^T z + f(Tz)) + T^{-1}C.\end{aligned} \tag{17}$$

Define $\hat{z} = T^{-1}\hat{x}$. Noting that

$$y - \hat{y} = (1, 0, \ldots, 0)T^{-1}(x - \hat{x}) = d^T(z - \hat{z}), \tag{18}$$

we have

$$\begin{aligned}\dot{\hat{z}} =& D\hat{z} + B_c(S^T\hat{z} + f(T\hat{z}))\\ &+ L(\theta)d^T(z - \hat{z}) + T^{-1}C\\ =& D\hat{z} + B_c(S^T\hat{z} + f(T\hat{z}))\\ &+ P(\theta)^{-1}dd^T(z - \hat{z}) + T^{-1}C.\end{aligned} \tag{19}$$

Denoting $e = \hat{z} - z$ and substituting for the expression (19) from the expression (17), we have the equation

$$\dot{e} = (D - P(\theta)^{-1}dd^T)e + B_c(S^T e + f(T\hat{z}) - f(Tz)). \tag{20}$$

We introduce the Lyapunov function

$$V = e^T P(\theta)e = (\hat{x} - x)^T(T^{-1})^T P(\theta)T(\hat{x} - x). \tag{21}$$

$\dot{V}$ will then be

$$\begin{aligned}\dot{V} =& e^T(D^T P(\theta) + P(\theta)D - 2dd^T)e\\ &+ 2e^T P(\theta)(B_c(S^T e + f(T\hat{z}) - f(Tz)))\\ =& -\theta\, e^T P(\theta)e - (e^T d)^2 + 2e^T P(\theta)(B_c(S^T e\\ &+ f(T\hat{z}) - f(Tz)))\\ \le& -\theta\, e^T P(\theta)e + 2e^T P(\theta)(B_c(S^T e\\ &+ f(T\hat{z}) - f(Tz))).\end{aligned} \tag{22}$$

Noting that

$$\begin{aligned}&\left\|S^T e + f(T\hat{z}) - f(Tz)\right\|\\ \le& \left\|S^T e\right\| + \|f(T\hat{z}) - f(Tz)\|\\ \le& \left\|S^T\right\|\|e\| + \gamma\|T\hat{z} - Tz\|\\ \le& (\left\|S^T\right\| + \gamma\|T\|)\ \|e\| = \beta\|e\|,\end{aligned} \tag{23}$$

where $\beta = \left\|S^T\right\| + \gamma\|T\|$, we have

$$\dot{V} \le -\theta\,\|e\|^2_{P(\theta)} + 2\beta\|e\|_{P(\theta)}\sqrt{p_{nn}(\theta)}\,\|e\|, \tag{24}$$

where $\|e\|_{P(\theta)} \equiv \sqrt{e^T P(\theta)e}$. Supposing that $\theta > 1$, and denoting the smallest eigenvalue of $P(1)$, with $\lambda_{\min}(P(1))$ from Ref.[5] we have

$$\begin{aligned}\|e\|^2_{P(1)} =& \sum_{i,j=1}^{n} e_i p_{ij}(1)e_j \le \theta^{2n-1}\\ &\times \sum_{i,j=1}^{n} e_i p_{ij}(1)e_j \frac{1}{\theta^{i+j-1}} = \theta^{2n-1}\|e\|^2_{P(\theta)},\end{aligned} \tag{25}$$

$$\|e\| \le \sqrt{\frac{1}{\lambda_{\min}(P(1))}}\,\|e\|_{P(1)}, \tag{26}$$

and then

$$\|e\| \le \sqrt{\frac{\theta^{2n-1}}{\lambda_{\min}(P(1))}}\,\|e\|_{P(\theta)}. \tag{27}$$

Since

$$p_{nn}(\theta) = \frac{p_{nn}(1)}{\theta^{2n-1}}, \tag{28}$$

we obtain

$$\begin{aligned}&2\beta\|e\|_{P(\theta)}\sqrt{p_{nn}(\theta)}\,\|e\|\\ =& 2\beta\|e\|_{P(\theta)}\sqrt{\frac{p_{nn}(1)}{\theta^{2n-1}}}\,\|e\|\\ \le& 2\beta\|e\|_{P(\theta)}\sqrt{\frac{p_{nn}(1)}{\theta^{2n-1}}}\sqrt{\frac{\theta^{2n-1}}{\lambda_{\min}(P(1))}}\,\|e\|_{P(\theta)}\\ =& 2\beta\sqrt{\frac{p_{nn}(1)}{\lambda_{\min}(P(1))}}\,\|e\|^2_{P(\theta)}.\end{aligned} \tag{29}$$

Therefore

$$\frac{\mathrm{d}}{\mathrm{d}t}(\|e\|^2_{P(\theta)}) \leq -\left(\theta - 2\beta\sqrt{\frac{p_{nn}(1)}{\lambda_{\min}(P(1))}}\right)\|e\|^2_{P(\theta)}. \tag{30}$$

We define

$$\mu \equiv 2\beta\sqrt{\frac{p_{nn}(1)}{\lambda_{\min}(P(1))}}. \tag{31}$$

Note that μ is a constant that is unrelated with θ. If $\theta > \mu$, then the inequality (30) implies that

$$\|e\|^2_{P(\theta)} \leq \|e(0)\|^2_{P(\vartheta)} \exp\{-(\theta-\mu)t\} \to 0, \tag{32}$$

that is

$$\|e(t)\|_{P(\theta)} \leq \|e(0)\|_{P(\theta)} \exp\left\{-\frac{1}{2}(\theta-\mu)t\right\} \to 0. \tag{33}$$

Let $P(\theta)^{1/2}$ be the square root of $P(\theta)$, the expression (33) implies that

$$\|\hat{x}(t) - x(t)\| \leq \|T\| \cdot \|e(t)\| \leq \|T\| \cdot \left\|P(\theta)^{1/2}\right\|^{-1} \cdot \|e(0)\|_{P(\theta)} \exp\left\{-\frac{1}{2}(\theta-\mu)t\right\} \to 0.$$

This ends the proof of Theorem 1.

3. Simulation examples

In this section, numerical simulations are used to illustrate the theoretical results presented in the previous sections.

Example 1 Synchronization of the Rössler hyperchaotic systems

The Rössler hyperchaotic system[13] is described by

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{pmatrix} = \begin{pmatrix} -x_2 - x_3 \\ x_1 + 0.25x_2 + x_4 \\ 3.0 + x_1x_3 \\ -0.5x_3 + 0.05x_4 \end{pmatrix}. \tag{34}$$

Numerical evidence indicates that the attractor has two positive Lyapunov exponents. The Rössler hyperchaotic system (34) can be decomposed as

$$\dot{x} = Ax + Bf(x) + C,$$

where

$$A = \begin{bmatrix} 0 & -1 & -1 & 0 \\ 1 & 0.25 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -0.5 & 0.05 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 \\ 0 \\ 3 \\ 0 \end{bmatrix} \text{ and } f(x) = x_1x_3.$$

Apparently, (A, B) is a controllable pair. Note that the synchronization methods in Refs.[5, 6] cannot be used for synchronizing Rössler hyperchaotic systems because the linear part matrix A and nonlinear function $f(\boldsymbol{x})$ of the Rössler hyperchaotic system have not the special structure needed by Refs.[5, 6]. To illustrate the applicability of our approach, we show now that synchronization of Rössler hyperchaotic systems can be attained using our proposed approach. As derived early, we take a linear combination of the original system (34) state variables $y(t)$ as a scale-driving signal

$$y(t) = \boldsymbol{H}^{\mathrm{T}}\boldsymbol{x}(t),$$

where $\boldsymbol{H}^{\mathrm{T}} = [0.6715, \quad 0.0224, \quad 0, \quad -1.3430]$ from Eq.(6). The response system is given by

$$\begin{pmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \\ \dot{\hat{x}}_3 \\ \dot{\hat{x}}_4 \end{pmatrix} = \begin{pmatrix} -\hat{x}_2 - \hat{x}_3 \\ \hat{x}_1 + 0.25\hat{x}_2 + \hat{x}_4 \\ 3.0 + \hat{x}_1\hat{x}_3 \\ -0.5\hat{x}_3 + 0.05\hat{x}_4 \end{pmatrix} + K(\hat{y} - y), \tag{35}$$

where from Eq.(7)

$$\boldsymbol{K} = \begin{bmatrix} 0.4875 & 0.3 & -1.0 & 0 \\ 0.05 & -1.5 & 0 & 0 \\ -0.05 & 1.0125 & -0.3 & 1 \\ -0.5 & 0.125 & -0.5 & 0 \end{bmatrix} \begin{bmatrix} 4\theta \\ 6\theta^2 \\ 4\theta^3 \\ \theta^4 \end{bmatrix}.$$

In simulation, we take $\theta = 1$ and then $\boldsymbol{K} = [-0.25, \quad -8.8, \quad 5.675, \quad -3.25]^{\mathrm{T}}$. The evolution of the synchronization error with time is shown in Fig.1 where we can see that the synchronization error tends to zero asymptotically, which implies that synchronization between the systems (34) and (35) is achieved.

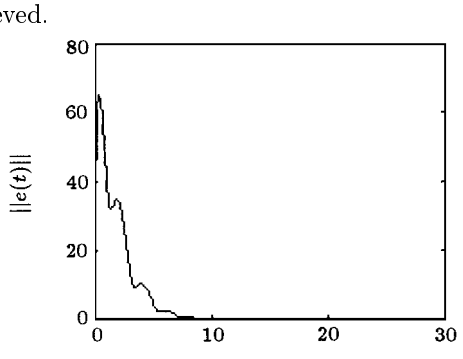


Fig.1. Synchronization error of two Rössler hyperchaotic systems performed with $\theta = 1$.

Example 2 Synchronization of the Chua's chaotic oscillators

The state equation of the drive Chua oscillator[12] in a dimensionless form is given by

$$\begin{cases} \dot{x}_1 = \alpha(x_2 - x_1 - g(x_1)), \\ \dot{x}_2 = x_1 - x_2 + x_3, \\ \dot{x}_3 = -\beta x_2 - \gamma x_3, \end{cases} \tag{36}$$

with the nonlinear characteristic

$$g(x_1) = bx_1 + \frac{1}{2}(a-b)\left(|x_1+1| - |x_1-1|\right).$$

Chua's oscillator can exhibit a large variety of steady-state behaviours (including chaos) in the region of its parameter space defined by

$$\alpha > 0, \quad \beta > 0, \quad \gamma > 0, \quad a < b < 0.$$

Here, the system parameters of Chua's oscillator are chosen as

$$\alpha = 10, \quad \beta = 15, \quad \gamma = 0.0385,$$

$$a = -1.27, \quad b = -0.68.$$

Chua's circuit (36) can be decomposed as

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + Bf(\boldsymbol{x}) + C,$$

where

$$A = \begin{bmatrix} -\alpha & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & -\gamma \end{bmatrix},$$

$$B = \begin{bmatrix} -\alpha \\ 0 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$$

$$f(x) = g(x_1) = bx_1 + \tfrac{1}{2}(a-b)(|x_1+1| - |x_1-1|).$$

Apparently, (A, B) is a controllable pair. As derived early, we take a linear combination of the original system (36) state variables $y(t)$ as a scale-driving signal

$$y(t) = \boldsymbol{H}^{\mathrm{T}} x(t),$$

where $\boldsymbol{H}^{\mathrm{T}} = [0, \quad 0, \quad 0.0067]$. The response system is described by

$$\begin{pmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \\ \dot{\hat{x}}_3 \end{pmatrix} = \begin{pmatrix} \alpha(\hat{x}_2 - \hat{x}_1 - g(\hat{x}_1)) \\ \hat{x}_1 - \hat{x}_2 + \hat{x}_3 \\ -\beta\hat{x}_2 - \gamma\hat{x}_3 \end{pmatrix} + \boldsymbol{K}(\hat{y} - y), \tag{37}$$

where

$$K = \begin{bmatrix} -150.385 & -10.385 & -10 \\ -0.385 & -10 & 0 \\ 150 & 0 & 0 \end{bmatrix} \begin{bmatrix} 3\theta \\ 3\theta^2 \\ \theta^3 \end{bmatrix}.$$

We find that synchronization between the systems (36) and (37) occurs when $\theta \geq 0.4$. When $\theta = 1$, the evolution of the synchronization error with time is shown in Fig.2 where we can see that the synchronization error between the states of (34) and (35) tends to zero asymptotically. The time taken to achieve the synchronization ($\|x - \hat{x}\| \leq 10^{-4}$) varying with θ is shown in Fig.3, and it is obvious that the synchronization time reduces as θ increases.

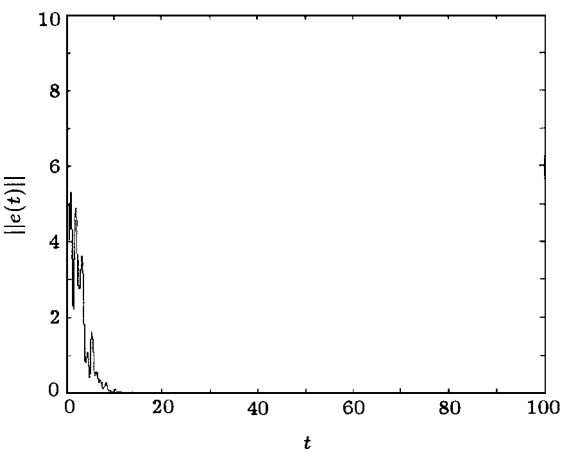


Fig.2. Synchronization error of two Chua oscillators performed with $\theta = 1$.

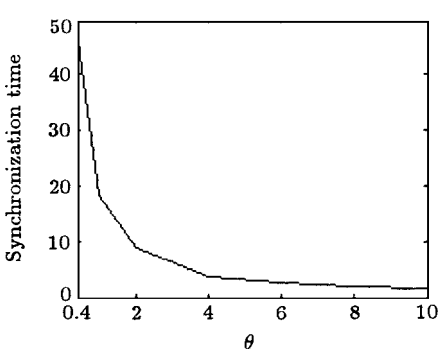


Fig.3. Synchronization time of two Chua oscillators as a function of θ.

4. Conclusions

In this paper, we have proposed a new linear output feedback control method of synchronizing a large class of chaotic systems which is more general than that considered in Refs.[5, 6]. We take the driving signal as a linear combination of the drive system state variables. The feedback gain vector is chosen as a

function of a free parameter. We have proved that the asymptotic synchronization between the drive and the response systems can be attained when the parameter is large enough. The simulation results have shown that our theorem has a wider application than that proposed in Refs.[5, 6]. Our proposed results may be useful for developing the practical synchronized chaotic systems.

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