Vol 15 No 1, January 2006
1009-1963/2006/15(01)/0077-06

Chinese Physics

© 2006 Chin. Phys. Soc.
and IOP Publishing Ltd

Quasinormal modes of a stationary axisymmetric EMDA black hole*

Pan Qi-Yuan(潘启沅) and Jing Ji-Liang(荆继良)†

Institute of Physics, Hunan Normal University, Changsha 410081, *China*

(Received 22 April 2005; revised manuscript received 11 July 2005)

The massless scalar quasinormal modes (QNMs) of a stationary axisymmetric Einstein–Maxwell dilaton–axion (EMDA) black hole are calculated numerically using the continued fraction method first proposed by Leaver. The fundamental quasinormal frequencies (slowly damped QNMs) are obtained and the peculiar behaviours of them are studied. It is shown that these frequencies depend on the dilaton parameter D, the rotational parameter a, the multiple moment l and the azimuthal number m, and have the same values with other authors at the Schwarzschild and Kerr limit.

Keywords: Quasinormal modes (QNMs), the stationary axisymmetric EMDA black hole, the continued fraction method

PACC: 0420, 9760, 0440

1. Introduction

The evolution of the external field perturbation around a black hole is dominated by three stages:[1] the initial wave burst, the damped oscillations called QNMs and the power-law tail behaviour of the waves at very late time. The QNMs of black holes have been studied widely because they give information about the main parameters of a black hole such as its mass M, charge Q and angular momentum per unit mass $a \equiv J/M$. For a long time people generally believed that the QNMs are part of the signals which will be registered by the gravitational wave detectors, and so these will be a very important tool in astronomy in the next years.[2]

Regge and Wheeler first studied the linear perturbations of static black holes in 1957.[3] This pioneering work on this topic has led to many investigations concerning the evolution of various fields in different black holes. Vishveshwara[4] and Press[5] first found the QNMs through numerical computations of the time evolution of the gravitational waves around the black hole. Using a direct numerical integration of the Regge–Wheeler equation, Chandrasekhar and Detweiler[6] proposed the first method for calculating QNMs numerically and gave the values of the first few least-damped QNMs. Meanwhile, there have been analytical attempts to obtain QNMs,[7−16] but these methods cannot provide very accurate values of the frequencies and break down for rapidly damped modes.

In 1985, Leaver[17] presented a continued fraction method for calculating the QNMs of both static and rotating black holes. Then, Onozawa *et al* improved this method to account for the extreme case in 1996.[18] Leaver's method provides extremely accurate values for the QNMs of each black hole which involves scalar, electromagnetic and gravitational perturbations.[19−21] Recently it was extended to compute the QNMs of the Dirac field.[22−24] Although there are several methods to precisely get the QNMs for static black holes, the continued fraction method is the only one which can be generalized to the rotating cases, such as the Kerr[19] and Kerr–Newman black holes,[21,24] where besides the quasinormal frequencies we must evaluate the separation constants which is the angular eigenvalues associated with the Teukolsky equations.

Nowadays, it seems that the superstring theory is the most promising candidate for a consistent quantum theory of gravitation. So there have been a

*Project supported by the National Natural Science Foundation of China (Grant No 10473004), the FADEDD (Grant No 200317), and the SRFDP (Grant No 20040542003).

†Corresponding author. E-mail: jljing@hunnu.edu.cn

lot of investigations concerning the QNMs of dilaton black holes.[25−28] For instance, Konoplya explored the QNMs of the electrically charged dilaton black hole and studied the effect of the dilaton parameter on QNMs. Obviously they only discussed the QNMs of the static, spherically symmetric black holes. Thus, it is worthwhile to extend the investigation to QNMs of the dilaton black hole to be the stationary axisymmetric solution of the so-called low-energy string theory.[29,30]

The main purpose of this paper is to calculate the scalar QNMs of a stationary axisymmetric EMDA black hole. In Section 2 we give a short description of the continued fraction equations for the angular part and radial part. In Section 3 we will compute the fundamental quasinormal frequencies (slowly damped QNMs, i.e. $a\omega$ is small) of this rotating dilaton black hole. The last section gives the summary and conclusion.

2. Continued fraction equations

In the Boyer–Lindquist coordinates (t, r, θ, ϕ), the stationary axisymmetric EMDA black hole solution is given[29,30] by

$$\begin{aligned} \mathrm{d}s^2 =& -\frac{\Sigma - a^2\sin^2\theta}{\Delta}\mathrm{d}t^2 \\ & - \frac{2a\sin^2\theta}{\Delta}[(r^2 - 2Dr + a^2) \\ & - \Sigma]\mathrm{d}t\mathrm{d}\varphi + \frac{\Delta}{\Sigma}\mathrm{d}r^2 \\ & + \Delta\mathrm{d}\theta^2 + \frac{\sin^2\theta}{\Delta}[(r^2 - 2Dr + a^2)^2 \\ & - \Sigma a^2\sin^2\theta]\mathrm{d}\varphi^2, \end{aligned} \tag{1}$$

with $\Sigma = r^2 - 2Mr + a^2$, $\Delta = r^2 - 2Dr + a^2\cos^2\theta$, and D, a, and M represent the dilaton, rotational, and mass parameters, respectively. The ADM mass is $M_{\mathrm{ADM}} = M - D$. Throughout this paper we use $G = c = 1$.

In the background geometry of the black hole considered, a massless scalar field Φ evolves according to

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\left(\sqrt{-g}g^{\mu\nu}\frac{\partial\Phi}{\partial x^\nu}\right) = 0. \tag{2}$$

So we derive the master equation that governs the evolution of massless scalar perturbations of the EMDA spacetime,

$$\begin{aligned} & \frac{(r^2 - 2Dr + a^2)^2 - \Sigma a^2\sin^2\theta}{\Sigma}\frac{\partial^2\Phi}{\partial t^2} \\ & - \frac{\partial}{\partial r}\left(\Sigma\frac{\partial\Phi}{\partial r}\right) - \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Phi}{\partial\theta}\right) \\ & - \frac{\Sigma - a^2\sin^2\theta}{\sin^2\theta\Sigma}\frac{\partial^2\Phi}{\partial\varphi^2} \\ & + \frac{4a(M - D)r}{\Sigma}\frac{\partial^2\Phi}{\partial t\partial\varphi} = 0. \end{aligned} \tag{3}$$

In case of a t and φ dependence given by $\mathrm{e}^{-\mathrm{i}\omega t + \mathrm{i}m\varphi}$, we get the separated differential equation for an angular part of the perturbations,

$$\begin{aligned} & [(1 - u^2)Y_{lm,u}(u)]_u + (a^2\omega^2u^2 \\ & - \frac{m^2}{1 - u^2} + A_{lm})Y_{lm}(u) = 0, \end{aligned} \tag{4}$$

and that for a radial part,

$$\left[\frac{\mathrm{d}}{\mathrm{d}r}\left(\Sigma\frac{\mathrm{d}}{\mathrm{d}r}\right) + \frac{(r^2 - 2Dr + a^2)^2\omega^2 - 4a\omega m(M - D)r + a^2m^2}{\Sigma} - A_{lm} - a^2\omega^2\right]R_{lm}(r) = 0, \tag{5}$$

where $u = \cos\theta$.

For Eq.(4) A_{lm} is the angular separation constant, and it is reduced to $l(l+1)$ in the Schwarzschild case. But for the stationary axisymmetric EMDA black holes, we have to solve it numerically following Leaver.[17] Boundary conditions for Eq.(4) are that $Y_{lm}(u)$ are finite at the regular singular points $u = \pm 1$, where the indices are given by $\pm m/2$. So a solution to Eq.(4) may be expressed as

$$Y_{lm}(u) = \mathrm{e}^{a\omega u}(1 - u^2)^{|m|/2}\sum_{n=0}^{\infty}a_n^\theta(1 + u)^n, \tag{6}$$

where the superscript θ denotes the association with the angular equation. The expansion coefficients are related by a three-term recurrence relation and the boundary condition at $u = +1$ is satisfied only by its minimal solution sequence. The three-term recurrence relation is written as

$$\alpha_0^\theta a_1^\theta + \beta_0^\theta a_0^\theta = 0, \tag{7}$$

$$\alpha_n^\theta a_{n+1}^\theta + \beta_n^\theta a_n^\theta + \gamma_n^\theta a_{n-1}^\theta = 0, \quad (n \geq 1), \tag{8}$$

where

$$\begin{aligned}\alpha_n^\theta =& -2(n+1)(n+2|m|+1),\\ \beta_n^\theta =& n(n-1)+2n(|m|+1-2a\omega)-[2a\omega(|m|+1)\\ &-|m|(|m|+1)]-(a^2\omega^2+A_{lm}),\\ \gamma_n^\theta =& 2a\omega(n+|m|).\end{aligned} \tag{9}$$

We will obtain the minimal solution if the angular separation constant A_{lm} is a root of the continued fraction equation

$$0=\beta_0^\theta-\frac{\alpha_0^\theta\gamma_1^\theta}{\beta_1^\theta-}\frac{\alpha_1^\theta\gamma_2^\theta}{\beta_2^\theta-}\frac{\alpha_2^\theta\gamma_3^\theta}{\beta_3^\theta-}\frac{\alpha_3^\theta\gamma_4^\theta}{\beta_4^\theta-}\cdots. \tag{10}$$

A solution to the radial Eq.(5) is obtained similarly to the solution for Eq.(4). Obviously, the quasinormal boundary conditions for Eq.(5) are

$$\begin{aligned}&R_{lm}(r)\sim(r-r_+)^{-\mathrm{i}\sigma_+}, \qquad as\ r\to r_+,\\ &R_{lm}(r)\sim r^{-1+2\mathrm{i}(M-D)\omega}\mathrm{e}^{\mathrm{i}\omega r},\ as\ r\to\infty,\end{aligned} \tag{11}$$

where

$$\sigma_\pm=\frac{2(M-D)\omega r_\pm-am}{b}$$

and

$$b=2\sqrt{M^2-a^2}.$$

Thus, our solution should be

$$\begin{aligned}R_{lm}(r)=&\mathrm{e}^{\mathrm{i}\omega r}(r-r_-)^{-1+2\mathrm{i}(M-D)\omega+\mathrm{i}\sigma_+}\\ &\times(r-r_+)^{-\mathrm{i}\sigma_+}\sum_{n=0}^{\infty}a_n^r\left(\frac{r-r_+}{r-r_-}\right)^n.\end{aligned} \tag{12}$$

The expansion coefficients here are again defined by a three-term recurrence relation

$$\begin{aligned}&\alpha_0^r a_1^r+\beta_0^r a_0^r=0,\\ &\alpha_n^r a_{n+1}^r+\beta_n^r a_n^r+\gamma_n^r a_{n-1}^r=0, \qquad (n\geq 1).\end{aligned} \tag{13}$$

The recursion coefficients are

$$\begin{aligned}\alpha_n^r=&n^2+(c_0+1)n+c_0,\\ \beta_n^r=&-2n^2+(c_1+2)n+c_3,\\ \gamma_n^r=&n^2+(c_2-3)n+c_4-c_2+2,\end{aligned} \tag{14}$$

where

$$\begin{aligned}c_0=&1-2\mathrm{i}(M-D)\omega\\ &-\frac{2\mathrm{i}}{b}\left[2M(M-D)\omega-am\right],\\ c_1=&-4+2\mathrm{i}\omega[4(M-D)+b]\\ &+\frac{4\mathrm{i}}{b}\left[2M(M-D)\omega-am\right],\\ c_2=&3-6\mathrm{i}(M-D)\omega\\ &-\frac{2\mathrm{i}}{b}\left[2M(M-D)\omega-am\right],\\ c_3=&\frac{8(M-D)\omega+2\mathrm{i}}{b}\left[2M(M-D)\omega-am\right]\\ &-1+\mathrm{i}\omega[4(M-D)+b]-2am\omega\\ &+\omega^2[8(M-D)^2+4(M-D)b\\ &+8M(M-D)-a^2]-A_{lm},\\ c_4=&1-8(M-D)^2\omega^2\\ &-6\mathrm{i}(M-D)\omega-\frac{8(M-D)\omega+2\mathrm{i}}{b}\\ &\times\left[2M(M-D)\omega-am\right].\end{aligned} \tag{15}$$

The radial series solution (12) converges and the boundary condition (11) is satisfied as the frequency ω is a root of the three-term continued fraction equation

$$0=\beta_0^r-\frac{\alpha_0^r\gamma_1^r}{\beta_1^r-}\frac{\alpha_1^r\gamma_2^r}{\beta_2^r-}\frac{\alpha_2^r\gamma_3^r}{\beta_3^r-}\frac{\alpha_3^r\gamma_4^r}{\beta_4^r-}\cdots. \tag{16}$$

To find these QNMs using Leaver's continued fraction method, we will seek the roots which satisfy both Eqs.(10) and (16) for a given set of D, a, l and m.

3. Numerical results

In order to compare with other authors' results, we set $2M_{\mathrm{ADM}}=2(M-D)=1$. Because $r_\pm=M_{\mathrm{ADM}}+D\pm\sqrt{(M_{\mathrm{ADM}}+D)^2-a^2}$, we have $(M_{\mathrm{ADM}}+D)^2>a^2$. Notice that the dilaton parameter D is negative[29,30] but the rotational parameter a is positive. Thus, as $D\to 0$ and $a\to 0$ our results are reduced to the static Schwarzschild case, and as $D\to 0$ reduced to the rotating Kerr case. The continued fractions Eq.(10) can be evaluated by using an excellent method known as the modified Lentz algorithm.[31]

Because we are interested in the fundamental QNMs (slowly damped QNMs) of the black hole considered, the angular separation constant A_{lm} can be approximated for small $a\omega$ by perturbation theory:[32]

$$A_{lm}=A_0+a\omega A_1+a^2\omega^2A_2+O((a\omega)^3), \tag{17}$$

where the coefficients are

$$A_0 = l(l+1) - s(s+1),$$
$$A_1 = -\frac{2ms^2}{l(l+1)},$$
$$A_2 = H(l+1) - H(l) - 1,$$

with

$$H(l) = \frac{2(l^2-m^2)(l^2-s^2)^2}{(2l-1)l^3(2l+1)}.$$

This approximation still gives us the extremely accurate values for these fundamental QNMs, and it makes our computing easy. In this paper we only study the case $s = 0$, i.e., the scalar field perturbation.

In Table 1 we give numerical results for the fundamental scalar QNMs with $l = m = 0$ as a function of dilaton and angular momentum. The left value in each bracket is the real part of the fundamental quasinormal frequency $\omega_{\rm R}$, and the right one is the imaginary part $\omega_{\rm I}$. Dashed entries in this and the following tables correspond to combinations of D and a for which $(M_{\rm ADM} + D)^2 \leq a^2$. Tables 2 and 3 give numerical results for the fundamental scalar QNMs with $l = 2, m = 0$ and $l = m = 2$, respectively.

Table 1. Fundamental scalar QNMs of the EMDA metric with $l = m = 0$ for selected values of the dilaton and angular momentum. The left value in each bracket is the real part of the fundamental quasinormal frequency $\omega_{\rm R}$, and the right one is the imaginary part $\omega_{\rm I}$. Dashed entries in this and the following tables correspond to combinations of D and a for which $(M_{\rm ADM} + D)^2 \leq a^2$.

a	$D = 0$	$D = -0.1$	$D = -0.2$
0	(0.220910, –0.209791)	(0.238212, –0.213368)	(0.260585, –0.217266)
0.1	(0.221535, –0.209025)	(0.239078, –0.212226)	(0.261789, –0.215056)
0.2	(0.223398, –0.206506)	(0.241573, –0.208257)	(0.264577, –0.206624)
0.3	(0.226342, –0.201397)	(0.244600, –0.199110)	–
0.4	(0.229074, –0.191402)	–	–
0.45	(0.227696, –0.183138)	–	–
a	$D = -0.3$	$D = -0.4$	$D = -0.45$
0	(0.291943, –0.221038)	(0.343595, –0.221904)	(0.390392, –0.215480)
0.1	(0.293461, –0.215597)	–	–
≥ 0.2	–	–	–

Table 2. Fundamental scalar QNMs of the EMDA metric with $l = 2, m = 0$.

a	$D = 0$	$D = -0.1$	$D = -0.2$
0	(0.967288, –0.193518)	(1.04035, –0.198010)	(1.13626, –0.203100)
0.1	(0.969828, –0.192911)	(1.04410, –0.197018)	(1.14239, –0.201252)
0.2	(0.977719, –0.190929)	(1.05593, –0.193641)	(1.16235, –0.194391)
0.3	(0.991872, –0.186957)	(1.07790, –0.186160)	–
0.4	(1.01423, –0.179323)	–	–
0.45	(1.02953, –0.172788)	–	–
a	$D = -0.3$	$D = -0.4$	$D = -0.45$
0	(1.27266, –0.208572)	(1.50083, –0.212329)	(1.71088, –0.208579)
0.1	(1.28473, –0.204176)	–	–
≥ 0.2	–	–	–

Table 3. Fundamental scalar QNMs of the EMDA metric with $l = m = 2$.

a	$D = 0$	$D = -0.1$	$D = -0.2$
0	(0.967288, –0.193518)	(1.04035, –0.198010)	(1.13626, –0.203100)
0.1	(1.03424, –0.192764)	(1.12587, –0.196022)	(1.25400, –0.198451)
0.2	(1.11929, –0.189862)	(1.24210, –0.189782)	(1.43728, –0.182529)
0.3	(1.23473, –0.182492)	(1.42258, –0.171754)	–
0.4	(1.41365, –0.163040)	–	–
0.45	(1.56328, –0.138578)	–	–
a	$D = -0.3$	$D = -0.4$	$D = -0.45$
0	(1.27266, –0.208572)	(1.50083, –0.212329)	(1.71088, –0.208579)
0.1	(1.46098, –0.195988)	–	–
≥ 0.2	–	–	–

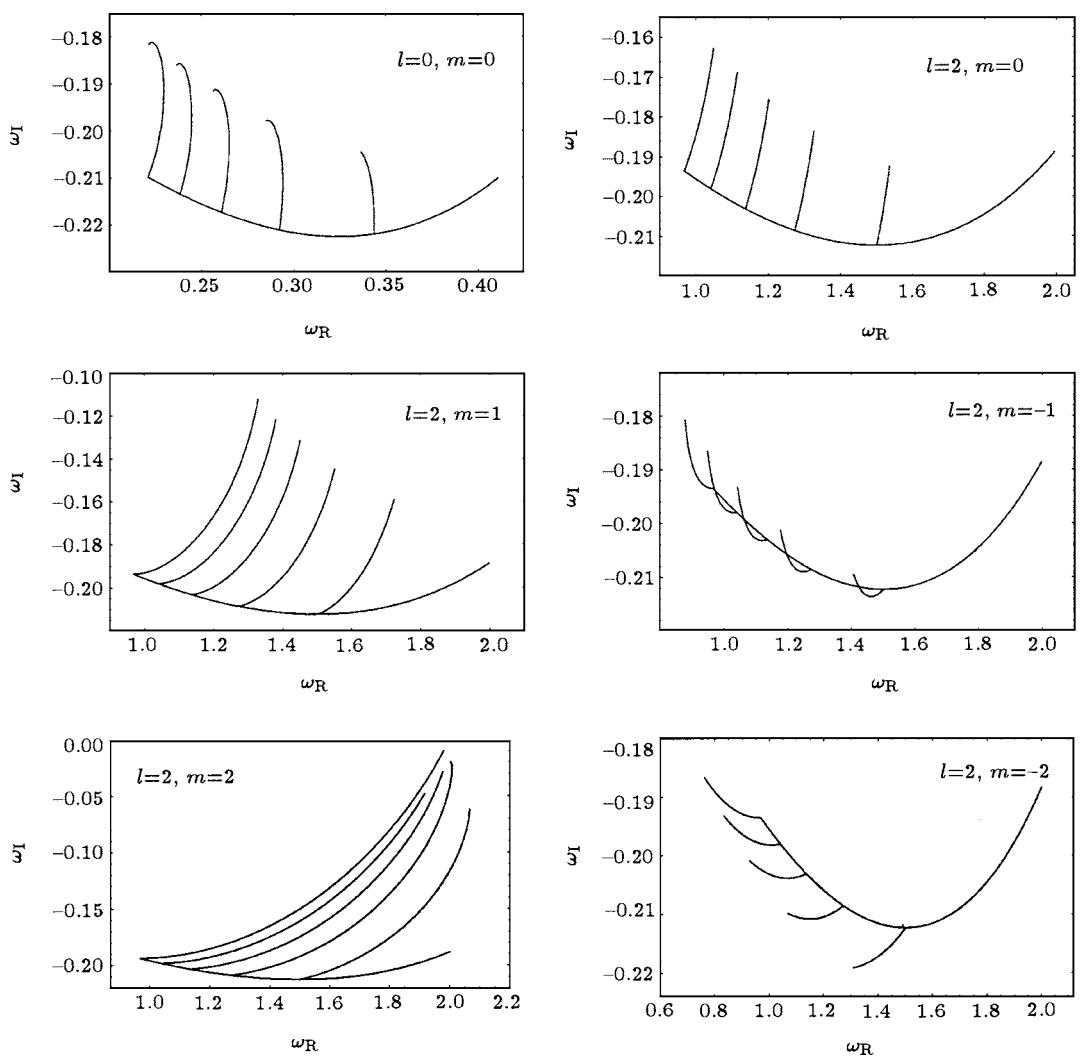


Fig.1. Graphs of selected fundamental scalar QNMs in the complex plane. In each panel, the bottom-lying curved line corresponds to modes of non-rotating black hole, i.e. $a = 0$ and $D = 0 \to -0.5$. As D decreases the static modes move counterclockwise in the complex frequency plane. The other five lines from left to right on this bottom-lying line correspond to modes of rotating black hole, i.e., $D = 0$ and $a = 0 \to 0.5$, $D = -0.1$ and $a = 0 \to 0.4$, $D = -0.2$ and $a = 0 \to 0.3$, $D = -0.3$ and $a = 0 \to 0.2$ and $D = -0.4$ and $a = 0 \to 0.1$ respectively. These six panels show the trajectories of the fundamental scalar modes with $l = m = 0$, $l = 2$ and $m = 0$, $l = 2$ and $m = \pm 1$ and $l = \pm m = 2$, respectively.

Figure 1 shows trajectories described by selected fundamental scalar QNMs in the complex plane. In each panel, the bottom-lying curved line corresponds to modes of non-rotating black hole, i.e., $a = 0$ and $D = 0 \to -0.5$. As D decreases the static modes move counterclockwise in the complex frequency plane. The other five lines from left to right on this bottom-lying line correspond to modes of rotating black hole, i.e., $D = 0$ and $a = 0 \to 0.5$, $D = -0.1$ and $a = 0 \to 0.4$, $D = -0.2$ and $a = 0 \to 0.3$, $D = -0.3$ and $a = 0 \to 0.2$ and $D = -0.4$ and $a = 0 \to 0.1$ respectively. These six panels show the trajectories of the fundamental scalar modes with $l = m = 0$, $l = 2$ and $m = 0$, $l = 2$ and $m = \pm 1$ and $l = \pm m = 2$ respectively. The observed behaviour (as a function of both dilaton parameter D and rotational parameter a) is very interesting and confirms the results presented in Refs.[19] and [21].

4. Conclusions

We have calculated numerically the massless scalar quasinormal modes (QNMs) of the EMDA black hole using Leaver's continued fraction method. We obtained the fundamental QNMs (slowly damped QNMs) and studied the peculiar behaviours of them. The main conclusions and their physical implications are the following: 1) The massless scalar fundamental quasinormal frequencies of the EMDA black hole are affected by the dilaton parameter D, the rotational parameter a, the multiple moment l and the azimuthal number m. 2) If a, l and m are fixed, the real part of the fundamental quasinormal frequency $\omega_{\rm R}$ increases as D decreases, but the imaginary part $\omega_{\rm I}$ decreases first and then increases. It is shown that the intermediate decay of the massless scalar perturbation around the EMDA black hole depends on the dilaton parameter D. The oscillating frequency of this decay which corresponds to $\omega_{\rm R}$ increases as D decreases, and the amplitude per unit time $\sim {\rm e}^{\omega_{\rm I}}$ which depends on $\omega_{\rm I}$ decreases first and then increases. 3) If D and l are fixed, $\omega_{\rm R}$ and $\omega_{\rm I}$ generally increases as a increases for the case $m \geq 0$, but for the case $m < 0$, $\omega_{\rm R}$ decreases while $\omega_{\rm I}$ decreases first and then increases. (4) If D and a are fixed, $\omega_{\rm R}$ and $\omega_{\rm I}$ generally increases as l increases except for some special case. (5) If D and l are fixed, these QNMs are independent of m for the static black hole $a = 0$, but $\omega_{\rm R}$ and $\omega_{\rm I}$ increase as m increases for the stationary black hole $a \neq 0$. We also noted that these QNMs have the same values as given by other authors at the Schwarzschild and Kerr limit.

Our study shows that the qualitatively different properties between the dilaton spacetimes in string theory and those appearing in general relativity are because of the appearance of dilaton. And an interesting extension of our work concerns the study of highly damped black hole QNMs which removes the limit of small $a\omega$. We hope to return to this problem in the future.

References

[1] Frolov V P and Novikov I D 1998 *Black Hole Physics: Basic Concepts and New Developments* (Dordrecht: Kluwer)
[2] Kokkotas K D and Schmidt B 1999 *Liv. Rev. Rel.* **2** 2
[3] Regge T and Wheeler J 1957 *Phys. Rev.* **108** 1063
[4] Vishveshwara C 1970 *Nature* (London) **227** 936
[5] Press W 1971 *Astrophys. J.* **170** L105
[6] Chandrasekhar S and Detweiler S 1975 *Proc. R. Soc. London* A **344** 441
[7] Ferrari V and Mashhoon B 1984 *Phys. Rev.* D **30** 295
[8] Schutz B and Will C 1985 *Astrophys. J.* **291** L33
[9] Iyer S and Will C 1987 *Phys. Rev.* D **35** 3621
[10] Iyer S 1987 *Phys. Rev.* D **35** 3632
[11] Seidel E and Iyer S 1990 *Phys. Rev.* D **41** 374
[12] Kokkotas K and Schutz B 1988 *Phys. Rev.* D **37** 3378
[13] Jing J L 2004 *Phys. Rev.* D **69** 084009
[14] Giammatteo M and Jing J L 2005 *Phys. Rev.* D **71** 024007
[15] Jing J L and Pan Q Y 2005 *Phys. Rev.* D **71** 124011
[16] Jing J L and Chen S B 2005 *Chin. Phys.* **14** 683
[17] Leaver E 1985 *Proc. R. Soc. London* A **402** 285
[18] Onozawa H, Mishima T, Okamura T and Ishihara H 1996 *Phys. Rev.* D **53** 7033
[19] Berti E, Cardoso V and Yoshida S 2004 *Phys. Rev.* D **69** 124018
[20] Giammatteo M and Moss I G 2005 *Class. Quant. Grav.* **22** 1803
[21] Berti E and Kokkotas K D 2005 *Phys. Rev.* D **71** 124008
[22] Castello-Branco K H C, Konoplya R A and Zhidenko A 2005 *Phys. Rev.* D **71** 047502
[23] Jing J L 2005 *Phys. Rev.* D **71** 124006
[24] Jing J L and Pan Q Y 2005 gr-qc/0506098
[25] Konoplya R A 2002 *Gen. Rel. Grav.* **34** 329
[26] Konoplya R A 2002 *Phys. Rev.* D **66** 084007
[27] Chen S B and Jing J L 2005 *Class. Quant. Grav.* **22** 533
[28] Shu F W and Shen Y G 2004 *Phys. Rev.* D **70** 084046
[29] Garcia A, Galtsov K and Kechkin O 1995 *Phys. Rev. Lett.* **74** 1276
[30] Jing J L and Wang S L 2002 *Phys. Rev.* D **65** 064001
[31] Press W H, Teukolsky S A, Flannery B P and Vetterling W T 1986 *Numerical Recipes, The Art of Scientific Computing* (New York: Cambridge University Press)
[32] Mino Y, Sasaki M, Shibata M, Tagoshi H and Tanaka T 1997 *Prog. Theor. Phys. Suppl.* **128** 1