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Solitons and intrinsic localized modes in a one-dimensional antiferromagnetic chain*

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By use of the Hartree approximation and the method of multiple scales, we investigate quantum solitons and intrinsic localized modes in a one-dimensional antiferromagnetic chain. It is shown that there exist solitons of two different quantum frequency bands: i.e., magnetic optical solitons and acoustic solitons. At the boundary of the Brillouin zone, these solitons become quantum intrinsic localized modes: their quantum eigenfrequencies are below the bottom of the harmonic optical frequency band and above the top of the harmonic acoustic frequency band.

Keywords: antiferromagnetic chain, Hartree approximation, quantum soliton, quantum intrinsic localized mode

PACC: 0340K, 0365, 0547

1. Introduction

Looking for solitary wave solutions in various physical systems is a main subject in the nonlinear physical field, and studies on this subject have proved to be quite fruitful.[1−20] In one-dimensional ferromagnetic and antiferromagnetic chains, the nonlinear solitary wave excitations have been also intensively investigated by researchers.[7−20] It was shown that, similar to many other nonlinear physical systems, there exist not only moving solitons but also immobile solitons (i.e., intrinsic localized modes).[12−14] Recently, by employing the Riemann–Hilbert method, Liu and co-workers examined the dynamics and interaction of magnetic solitons in a ferromagnet,[19,20] in which many interesting results were obtained. Their elegant work has made the study of magnetic solitons reach a new level. However, in one-dimensional antiferromagnetic chains, no analytical solitary wave solutions are obtained, and the theoretical methods employed are classical or semiclassical.

In this paper, by using the Hartree approximation[21] and the method of multiple scales, quantum solitons and intrinsic localized modes in a one-dimensional antiferromagnetic chain are investigated. Solitons of two different quantum frequency bands are reported.

2. The two-sublattice model

The Hamiltonian of the two-sublattice model of a one-dimensional antiferromagnetic chain is[22]

$$\begin{aligned} H =& J\sum_{i,\delta}\Big\{\hat{S}^z_{a,i}\hat{S}^z_{b,i+\delta} \\ &+\frac{1}{2}(\hat{S}^+_{a,i}\hat{S}^-_{b,i+\delta}+\hat{S}^-_{a,i}\hat{S}^+_{b,i+\delta})\Big\} \\ &+J\sum_{j,\delta}\Big\{\hat{S}^z_{b,j}\hat{S}^z_{a,j+\delta} \\ &+\frac{1}{2}(\hat{S}^+_{b,j}\hat{S}^-_{a,j+\delta}+\hat{S}^-_{b,j}\hat{S}^+_{a,j+\delta})\Big\}, \end{aligned} \tag{1}$$

where $J>0$, a and b represent the sublattice a and the sublattice b, respectively. In the following treatment, we introduce the Holstein–Primakoff transformation for the spin operators

$$\begin{aligned} \hat{S}^+_{a,i} &= \sqrt{2S-a_i^+a_i}\,a_i, \\ \hat{S}^-_{a,i} &= a_i^+\sqrt{2S-a_i^+a_i}, \\ \hat{S}^z_{a,i} &= S-a_i^+a_i; \end{aligned} \tag{2}$$

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$$\hat{S}^{+}_{b,j} = b^{+}_{j}\sqrt{2S - b^{+}_{j}b_{j}},$$
$$\hat{S}^{-}_{b,j} = \sqrt{2S - b^{+}_{j}b_{j}}\,b_{j},$$
$$\hat{S}^{z}_{b,j} = b^{+}_{j}b_{j} - S, \tag{3}$$

where $a_i(a_i^+)$ and $b_i(b_i^+)$ are boson operators and satisfy boson commutation relations. At low-temperatures, $a_i^+a_i \ll 2S$, $b_j^+b_j \ll 2S$, we can write Eqs.(2) and (3) as the Taylor series; in this case including quartic terms of field operators, Eq.(1) becomes

$$\begin{aligned}\hat{H} =& -2ZNJS^2 + JS\Big(\sum_{i,\delta} a_i^+a_i + \sum_{j,\delta} b_j^+b_j\Big) + JS\Big(\sum_{i,\delta} b_{i+\delta}^+b_{i+\delta} + \sum_{j,\delta} a_{j+\delta}^+a_{j+\delta}\Big)\\ &+ JS\Big(\sum_{i.\delta}(a_ib_{i+\delta} + a_i^+b_{i+\delta}^+) + \sum_{j,\delta}(b_ja_{j+\delta} + b_j^+a_{j+\delta}^+)\Big)\\ &- J\Big(\sum_{i,\delta} a_i^+a_ib_{i+\delta}^+b_{i+\delta} + \sum_{j,\delta} b_j^+b_ja_{j+\delta}^+a_{j+\delta}\Big)\\ &- \frac{1}{4}J\sum_{i.\delta}(b_{i+\delta}^+b_{i+\delta}b_{i+\delta}a_i + a_i^+b_{i+\delta}^+b_{i+\delta}^+b_{i+\delta} + a_i^+a_i^+a_ib_{i+\delta}^+ + a_i^+a_ia_ib_{i+\delta})\\ &- \frac{1}{4}J\sum_{j.\delta}(a_{j+\delta}^+a_{j+\delta}a_{j+\delta}b_j + b_j^+a_{j+\delta}^+a_{j+\delta}^+a_{j+\delta} + b_j^+b_j^+b_ja_{j+\delta}^+ + b_j^+b_jb_ja_{j+\delta}).\end{aligned} \tag{4}$$

From the Hamiltonian in Eq.(4), we can conclude that the status of operators a_i and b_j is completely tantamount and symmetrical; therefore, we can consider the following two cases:
1) $a_i = b_i^+$, in this case, Eq.(4) becomes

$$\begin{aligned}\hat{H} =& -2ZNJS(S-1) + 2J(2S-1)\sum_{i.\delta} a_i^+a_i + J(2S-1)\sum_{i,\delta}\big(a_ia_{i+\delta}^+ + a_i^+a_{i+\delta}\big)\\ &- 2J\sum_{i.\delta} a_i^+a_ia_{i+\delta}^+a_{i+\delta} - \frac{1}{2}J\sum_{i.\delta}(a_{i+\delta}^+a_{i+\delta}^+a_{i+\delta}a_i + a_{i+\delta}^+a_i^+a_{i+\delta}a_{i+\delta}\\ &+ a_i^+a_i^+a_ia_{i+\delta} + a_{i+\delta}^+a_i^+a_ia_i).\end{aligned} \tag{5}$$

2) $a_i = -b_i^+$, combining this and Eq.(4) gives

$$\begin{aligned}\hat{H} =& -2ZNJS(S-1) + 2J(2S-1)\sum_{i.\delta} a_i^+a_i - J(2S-1)\sum_{i,\delta}(a_ia_{i+\delta}^+ + a_i^+a_{i+\delta})\\ &- 2J\sum_{i.\delta} a_i^+a_ia_{i+\delta}^+a_{i+\delta} + \frac{1}{2}J\sum_{i.\delta}(a_{i+\delta}^+a_{i+\delta}^+a_{i+\delta}a_i + a_{i+\delta}^+a_i^+a_{i+\delta}a_{i+\delta}\\ &+a_i^+a_i^+a_ia_{i+\delta} + a_{i+\delta}^+a_i^+a_ia_i).\end{aligned} \tag{6}$$

3. The nonlinear equation of motion for the Hartree wavefunction

According to the quantum theory, the state of a physical system is determined by the state vector $|\Psi\rangle$, $|\Psi\rangle$ carries all information about measurable physical variables. The state vector $|\Psi\rangle$ satisfies the Schrödinger equation

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}|\Psi(t)\rangle = \hat{H}|\Psi(t)\rangle, \tag{7}$$

where $\hbar$ is assumed to be unity. We can prove that $\hat{H}$ in Eq.(5) or Eq.(6) commutes with the number operator $\hat{N} = \sum_{i=1}^{N} a_i^+a_i$, therefore, in the Fock space, a general n-boson state vector can be expanded as[21]

$$\begin{aligned}|\Psi_n(t)\rangle =& \frac{1}{\sqrt{n!}}\\ &\times\sum_{j_1=1}^{N}\sum_{j_2=1}^{N}\cdots\sum_{j_n=1}^{N}(\theta_n(j_1,j_2\cdots j_n,t)\\ &\times a_{j_1}^+a_{j_2}^+\cdots a_{j_n}^+|0\rangle)\end{aligned} \tag{8}$$

with

$$\sum_{j_1=1}^{N}\sum_{j_2=1}^{N}\cdots\sum_{j_n=1}^{N}\theta_n(j_1,j_2\cdots j_n,t)|^2=1, \qquad (9)$$

where $|0\rangle = |0\rangle_1 |0\rangle_2 \cdots |0\rangle_N$ is the vacuum state, and $\theta_n(j_1, j_2, \cdots j_n, t)$ is the n-boson wavefunction in the Fock space. Inserting the state vector in Eq.(8) into the Schrödinger equation in Eq.(7) and noting the boson commutation relations $\left[a_i, a_j^+\right] = \delta_{i,j}$, for Eqs.(5) and (6) we obtain respectively the following Schrödinger equations for the n-boson wavefunction

$$\begin{aligned}
&\left(\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}-n\omega_0\right)\theta_n(j_1,\cdots j_n,t)\\
=&2J(2S-1)\sum_{k=1}^{n}\sum_{\delta}\theta_n(j_1,\cdots j_k-\delta,\cdots j_n,t)\\
&-2J\sum_{k=1}^{n}\sum_{\ell\neq k}\sum_{\delta}\delta_{j_k+\delta,j_\ell}\theta_n(j_1,\cdots j_k\cdots j_\ell\cdots j_n,t)\\
&-\frac{1}{2}J\sum_{\delta}\sum_{k=1}^{n}\sum_{\ell\neq k}^{n}[\delta_{j_k,j_\ell}\theta_n(j_1,\cdots j_k-\delta,\cdots j_\ell\cdots j_n,t)\\
&+\delta_{j_k-\delta,j_\ell}\theta_n(j_1,\cdots j_k-\delta,\cdots j_\ell\cdots j_n,t)\\
&+\delta_{j_k+\delta,j_\ell}\theta_n(j_1,\cdots j_k+\delta,\cdots j_\ell\cdots j_n,t)\\
&+\delta_{j_k+\delta,j_\ell+\delta}\theta_n(j_1,\cdots j_k+\delta,\cdots j_\ell\cdots j_n,t)], \qquad (10)
\end{aligned}$$

$$\begin{aligned}
&\left(\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}-n\omega_0\right)\theta_n(j_1,\cdots j_n,t)\\
=&-2J(2S-1)\sum_{k=1}^{n}\sum_{\delta}\theta_n(j_1,\cdots j_k-\delta,\cdots j_n,t)\\
&-2J\sum_{k=1}^{n}\sum_{\ell\neq k}\sum_{\delta}\delta_{j_k+\delta,j_\ell}\theta_n(j_1,\cdots j_k\cdots j_\ell\cdots j_n,t)\\
&+\frac{1}{2}J\sum_{\delta}\sum_{k=1}^{n}\sum_{\ell\neq k}^{n}[\delta_{j_k,j_\ell}\theta_n(j_1,\cdots j_k-\delta,\cdots j_\ell\cdots j_n,t)\\
&+\delta_{j_k-\delta,j_\ell}\theta_n(j_1,\cdots j_k-\delta,\cdots j_\ell\cdots j_n,t)\\
&+\delta_{j_k+\delta,j_\ell}\theta_n(j_1,\cdots j_k+\delta,\cdots j_\ell\cdots j_n,t)\\
&+\delta_{j_k+\delta,j_\ell+\delta}\theta_n(j_1,\cdots j_k+\delta,\cdots j_\ell\cdots j_n,t)], \qquad (11)
\end{aligned}$$

where $\omega_0 = 2ZJ(2S-1) - \dfrac{2ZNJS(S-1)}{n}$, Z is the number of the nearest-neighbour lattice sites. Equations (10) and (11) are the equations for a system of bosons in a discrete antiferromagnetic chain. In the Hartree approximation, the n-boson wavefunction $\theta_n(j_1, \ldots, j_n, t)$ can be written in the following form:

$$\theta_n^{(H)}(j_1,\ldots,j_n,t)=\prod_{k=1}^{n}\phi_{n,j_k}, \qquad (12)$$

where $\phi_{n,j_k}(t)$ is the Hartree approximation wavefunction for the single boson. The basic idea is that each boson is subjected to the same mean field potential of all the other bosons, since these single-boson wavefunctions are independent of k. We write them simply as $\phi_{n,j}(t)$, where $j = 1, 2 \ldots, N$, then the n-boson state vector can be written as

$$|\Psi_n(t)\rangle^H=\frac{1}{\sqrt{n!}}\left(\sum_{j=1}^{N}\phi_{n,j}(t)a_j^+\right)^n|0\rangle \qquad (13)$$

with

$$\sum_{j=1}^{N}|\phi_{n,j}|^2=1. \qquad (14)$$

In the following sections, for the convenience of writing, we omit the subscript n and write $\phi_{n,j}(t)$ as ϕ_j. From the variational principle,[21] under the nearest-neighbour approximation, the Hartree approximation wavefunction ϕ_j satisfies the following nonlinear equations:

$$\begin{aligned}
&\mathrm{i}\frac{\mathrm{d}\phi_j}{\mathrm{d}t}-\omega_0\phi_j-2J(2S-1)(\phi_{j+1}+\phi_{j-1})\\
&+2J(n-1)[|\phi_{j+1}|^2+|\phi_{j-1}|^2]\phi_j\\
&+J(n-1)[2(\phi_{j+1}+\phi_{j-1})|\phi_j|^2\\
&+|\phi_{j+1}|^2\phi_{j+1}+|\phi_{j-1}|^2\phi_{j-1}]=0, \qquad (15)
\end{aligned}$$

$$\begin{aligned}
&\mathrm{i}\frac{\mathrm{d}\phi_j}{\mathrm{d}t}-\omega_0\phi_j+2J(2S-1)(\phi_{j+1}+\phi_{j-1})\\
&+2J(n-1)[|\phi_{j+1}|^2+|\phi_{j-1}|^2]\phi_j\\
&-J(n-1)[2(\phi_{j+1}+\phi_{j-1})|\phi_j|^2\\
&+|\phi_{j+1}|^2\phi_{j+1}+|\phi_{j-1}|^2\phi_{j-1}]=0. \qquad (16)
\end{aligned}$$

4. Quantum solitons and intrinsic localized modes

Equations (15) and (16) are two complicated discrete nonlinear equations satisfied by the effective single-boson wavefunction ϕ_j, in which the exact solution is impossible to be obtained. In the following, we look for approximation solutions of Eqs.(15) and (16) by using the multiple-scale method or derivative expansion method.[23] Firstly, we solve Eq.(15), for this purpose we perform the following replacement:

$$\phi_j \to \varepsilon\phi_j,$$

where ε is a small parameter, and Eq.(15) becomes

$$\begin{aligned}
&\mathrm{i}\frac{\mathrm{d}\phi_j}{\mathrm{d}t}-\omega_0\phi_j-2J(2S-1)(\phi_{j+1}+\phi_{j-1})\\
&+\varepsilon^2 2J(n-1)[|\phi_{j+1}|^2+|\phi_{j-1}|^2]\phi_j\\
&+\varepsilon^2 J(n-1)[2(\phi_{j+1}+\phi_{j-1})|\phi_j|^2\\
&+|\phi_{j+1}|^2\phi_{j+1}+|\phi_{j-1}|^2\phi_{j-1}]=0. \qquad (17)
\end{aligned}$$

We now introduce the multiple-scale variables

$$t_1 = \varepsilon t, t_2 = \varepsilon^2 t, \xi_j = \varepsilon j a. \tag{18}$$

The derivative operators $\dfrac{\mathrm{d}}{\mathrm{d}t}$ and $\dfrac{\mathrm{d}}{\mathrm{d}x}$ are expanded as

$$\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + \varepsilon\frac{\partial}{\partial t_1} + \varepsilon^2\frac{\partial}{\partial t_2}, \frac{\mathrm{d}}{\mathrm{d}x} = \varepsilon\frac{\mathrm{d}}{\mathrm{d}\xi_j}. \tag{19}$$

A solution of Eq.(17) is assumed as

$$\phi_j = \alpha_j(t_1, t_2, \xi_j)\mathrm{e}^{\mathrm{i}(kja-\omega^{(1)}t)}. \tag{20}$$

Using Eqs.(17)–(20), we obtain

$$\begin{aligned}&[\omega^{(1)} - \omega_0 - 4J(2S-1)\cos(ka)]\alpha_j \\ &+ \varepsilon\left[\mathrm{i}\frac{\partial\alpha_j}{\partial t_1} - \mathrm{i}4J(2S-1)a\sin(ka)\frac{\partial\alpha_j}{\partial\xi_j}\right] \\ &+ \varepsilon^2\left[\mathrm{i}\frac{\partial\alpha_j}{\partial t_2} - 2J(2S-1)a^2\cos(ka)\frac{\partial^2\alpha_j}{\partial\xi_j^2}\right. \\ &\left.+ 4J(n-1)\left(1+\frac{3}{2}\cos(ka)\right)|\alpha_j|^2\alpha_j\right] \\ &+ 0(\varepsilon^3) = 0.\end{aligned} \tag{21}$$

Comparing the powers of ε in Eq.(21), we have

$$\omega^{(1)} = \omega_0 + 4J(2S-1)\cos(ka), \tag{22}$$

$$\frac{\partial\alpha_j}{\partial t_1} - 4J(2S-1)a\sin(ka)\frac{\partial\alpha_j}{\partial\xi_j} = 0, \tag{23}$$

$$\begin{aligned}&\mathrm{i}\frac{\partial\alpha_j}{\partial t_2} - 2J(2S-1)a^2\cos(ka)\frac{\partial^2\alpha_j}{\partial\xi_j^2} \\ &+ 4J(n-1)\left(1+\frac{3}{2}\cos(ka)\right)|\alpha_j|^2\alpha_j = 0.\end{aligned} \tag{24}$$

From Eq.(22), for k=0, $\omega^{(1)} = \omega^{(1)}_{\max} = \omega_0 + 4J(2S+1)$, therefore, we describe $\omega^{(1)}$ as the dispersion relation of the harmonic optical frequency band. According to Eq.(23), we can conclude

$$\alpha_j = \alpha_j(t_2, z) \tag{25}$$

with

$$z = \xi_j + 4J(2S-1)a\sin(ka)t_1. \tag{26}$$

Combining Eqs.(25), (26) and (24), we have

$$\begin{aligned}&\mathrm{i}\frac{\partial\alpha_j}{\partial t_2} - 2J(2S-1)a^2\cos(ka)\frac{\partial^2\alpha_j}{\partial z^2} \\ &+ 4J(n-1)\left(1+\frac{3}{2}\cos(ka)\right)|\alpha_j|^2\alpha_j = 0.\end{aligned} \tag{27}$$

Let $\alpha_j = \dfrac{u}{\varepsilon}$, $z_j = ja + 4J(2S-1)a\sin(ka)t$, then Eq.(27) becomes

$$\mathrm{i}\frac{\partial u}{\partial t} + P_1\frac{\partial^2 u}{\partial z_j^2} + Q_1|u|^2u = 0 \tag{28}$$

with

$$\begin{aligned}P_1 &= -2J(2S-1)a^2\cos(ka), \\ Q_1 &= 4J(n-1)\left(1+\frac{3}{2}\cos(ka)\right).\end{aligned} \tag{29}$$

Equation (28) is the well-known nonlinear Schrödinger (NLS) equation, its single soliton solution is given by

$$U = \left[\frac{2P_1}{Q_1}\right]^{\frac{1}{2}} k_0\mathrm{sech}\{k_0[(j-j_0)a - v_{\mathrm{g}}^{(1)}t]\}\mathrm{e}^{\mathrm{i}k_0P_1t-\mathrm{i}\phi_0} \tag{30}$$

with

$$v_{\mathrm{g}}^{(1)} = -4J(2S-1)a\sin(ka) = \frac{\mathrm{d}\omega^{(1)}}{\mathrm{d}k}, \tag{31}$$

where $v_{\mathrm{g}}^{(1)}$ is the group velocity, k_0 and ϕ_0 are integral constants, j_0 is an arbitrary integer. Using Eqs.(20) and (30), we obtain finally the Hartree wavefunction

$$\begin{aligned}\phi_j(t) =& \left[\frac{2P_1}{Q_1}\right]^{\frac{1}{2}} k_0\mathrm{sech}\{k_0[(j-j_0)a - v_{\mathrm{g}}^{(1)}t]\} \\ &\times \mathrm{e}^{\mathrm{i}k_0^2P_1t-\mathrm{i}\phi_0}\mathrm{e}^{\mathrm{i}(kja-\omega^{(1)}t)}.\end{aligned} \tag{32}$$

Based on the same reasons, the solution of Eq.(16) can be written as

$$\begin{aligned}\phi'_j(t) =& \left[\frac{2P_2}{Q_2}\right]^{\frac{1}{2}} G_0\mathrm{sech}\{G_0[(j-j_0)a - v_{\mathrm{g}}^{(2)}t]\} \\ &\times \mathrm{e}^{\mathrm{i}G_0^2P_2t-\mathrm{i}\theta_0}\mathrm{e}^{\mathrm{i}(kja-\omega^{(2)}t)}\end{aligned} \tag{33}$$

with

$$\begin{aligned}\omega^{(2)} =& \omega_0 - 4J(2S-1)\cos(ka), \\ v_{\mathrm{g}}^{(2)} =& \frac{\mathrm{d}\omega^{(2)}}{\mathrm{d}k} = 4J(2S-1)a\sin(ka), \\ P_2 =& 2J(2S-1)a^2\cos(ka), \\ Q_2 =& 4J(n-1)\left(1-\frac{3}{2}\cos(ka)\right),\end{aligned} \tag{34}$$

where G_0 and θ_0 are integral constants, $\omega^{(2)}$ is the dispersion relation of the harmonic acoustic frequency band, $v_{\mathrm{g}}^{(2)}$ is the group velocity.

Equations (32) and (33) stand for the quantum solitons of two different frequency bands, Eq.(32) represents the magnetic optical solitons and Eq.(33) the magnetic acoustic solitons. Using Eq.(13), the mean number of quantum magnons on the jth freedom is[21]

$$\begin{aligned}\langle n_j^1(t)\rangle^H =& n|\phi_j|^2 \\ =& n\left[\frac{2P_1}{Q_1}\right]k_0^2\mathrm{sech}^2\{k_0[(j-j_0)a - v_{\mathrm{g}}^{(1)}t]\},\end{aligned} \tag{35}$$

$$\begin{aligned}\langle n_j^2(t)\rangle^H =& n|\phi'_j|^2 \\ =& n\left[\frac{2P_2}{Q_2}\right]G_0^2\mathrm{sech}^2\{G_0[(j-j_0)a - v_{\mathrm{g}}^{(2)}t]\}.\end{aligned} \tag{36}$$

Equations (35) and (36) show that quantum magnons form a quantum wave packet which moves at a certain group velocity $v_{\rm g}^{(1)}$ or $v_{\rm g}^{(2)}$ in a one-dimensional antiferromagnetic chain. At the boundary of the Brillouin zone, i.e. at $k = \pm\dfrac{\pi}{a}$, we have

$$\begin{aligned} v_{\rm g}^{(1)} =& \frac{{\rm d}\omega^{(1)}}{{\rm d}k} = 0, \\ v_{\rm g}^{(2)} =& \frac{{\rm d}\omega^{(2)}}{{\rm d}k} = 0, \\ \omega_{\rm min}^{(1)} =& \omega_0 - 4J(2S-1), \\ P_1 =& \frac{\omega_0 - \omega_{\rm min}^{(1)}}{2}a^2, \\ Q_1 =& -\frac{\omega_0 - \omega_{\rm min}^{(1)}}{2}\frac{(n-1)}{(2S-1)}, \\ \omega_{\rm max}^{(2)} =& \omega_0 + 4J(2S-1), \\ P_2 =& -\frac{\omega_{\rm max}^{(2)} - \omega_0}{2}a^2, \\ Q_2 =& \frac{5(\omega_{\rm max}^{(2)} - \omega_0)}{2}\frac{(n-1)}{(2S-1)}, \end{aligned} \tag{37}$$

where $\omega_{\rm min}^{(1)}$ is the bottom frequency of the harmonic optical frequency band, $\omega_{\rm max}^{(2)}$ is the top frequency of the harmonic acoustic frequency band. Then the expressions (32) and (33) become

$$\phi_j(t) = \left[\frac{2P_1}{Q_1}\right]^{\frac{1}{2}} k_0 {\rm sech}\{k_0[(j-j_0)a]\}{\rm e}^{{\rm i}(j\pi - \Omega_1 t - \phi_0)}, \tag{38}$$

$$\phi_j'(t) = \left[\frac{2P_2}{Q_2}\right]^{\frac{1}{2}} G_0 {\rm sech}\{G_0[(j-j_0)a]\}{\rm e}^{{\rm i}(j\pi - \Omega_2 t - \theta_0)} \tag{39}$$

with

$$\Omega_1 = \left[1 - \frac{K_0^2 a^2}{2}\left(\frac{\omega_0}{\omega_{\rm min}^{(1)}} - 1\right)\right]\omega_{\rm min}^{(1)}, \tag{40}$$

$$\Omega_2 = \left[1 + \frac{G_0^2 a^2}{2}\left(1 - \frac{\omega_0}{\omega_{\rm max}^{(2)}}\right)\right]\omega_{\rm max}^{(2)}. \tag{41}$$

Equations (38) and (39) represent quantum intrinsic localized modes in a one-dimensional antiferromagnetic chain, their quantum eigenfrequencies are respectively Ω_1 and Ω_2, i.e. below the bottom of the harmonic optical frequency band and above the top of the harmonic acoustic frequency band. Using Eqs.(38) and (39), Eqs.(35) and (36) become

$$\begin{aligned} \langle n_j^{(1)}(t)\rangle^H =& n|\phi_j|^2 \\ =& n\left[\frac{2P_1}{Q_1}\right]k_0^2 {\rm sech}^2\{k_0[(j-j_0)a]\}, \end{aligned} \tag{42}$$

$$\begin{aligned} \langle n_j^{(2)}(t)\rangle^H =& n|\phi_j'|^2 \\ =& n\left[\frac{2P_2}{Q_2}\right]G_0^2 {\rm sech}^2\{k_0[(j-j_0)a]\}. \end{aligned} \tag{43}$$

Therefore, the quantum magnons converge mainly on the vicinity of the central position $j = j_0$, so does the energy.

5. Conclusions

From the viewpoint of the pure quantum theory, we have investigated quantum solitons and intrinsic localized modes in a one-dimensional antiferromagnetic chain; the wavefunction of the quantum magnon is described by the quantum envelope soliton, and at the boundary of the Brillouin zone, quantum envelope solitons become quantum intrinsic localized modes. Obviously, the quantum features of the wavefunction of magnons give us a new understanding for solitary wave excitations in low-dimensional magnetic systems. If our results are compared with former ones, we can arrive at some different conclusions. For example, Lai and co-workers have demonstrated in an isotropic antiferromagnetic chain without anisotropy that intrinsic localized modes did not appear outside the plane-wave spectrum,[13] however, from the Hamiltonian employed and the results obtained in this paper, we can conclude that, even if there is no anisotropy in the Hamiltonian, the quantum intrinsic localized modes can exist in a one-dimensional antiferromagnetic chain, perhaps the reason of this difference is that we have employed different theoretical methods.

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