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Variable separation solutions and new solitary wave structures to the (1+1)-dimensional Ito system

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A variable separation approach is proposed and extended to the (1+1)-dimensional physics system. The variable separation solution of (1+1)-dimensional Ito system is obtained. Some special types of solutions such as non-propagating solitary wave solution, propagating solitary wave solution and looped soliton solution are found by selecting the arbitrary function appropriately.

Keywords: (1+1)-dimensional Ito system, variable separation approach, new solitary wave structures
PACC: 0230, 0340

1. Introduction

In nonlinear science, soliton theory plays an essential role and has been applied in all the natural sciences, especially in all the physical branches such as fluid physics, condensed matter, biophysics, plasma physics, nonlinear optics, quantum field theory and particle physics, etc.[1−3] The study of integrable models attracts much attention of many mathematicians and physicists. To search for exact solutions of a nonlinear physics system, especially for new exponential structures like soliton solutions, is a very significant work. Recently, there are a wealth of approaches for finding special solutions of the nonlinear partial differential equations, some of the most important methods are the inverse scattering transformation (IST),[4] bilinear method,[5] symmetry method,[6] Bäcklund and Darboux transformations,[7] extended algebraic method[8−17] and so on.

In comparison with the linear case, it is known that IST is an extension of the Fourier transformation in the nonlinear case. In addition to the Fourier transformation, there is another powerful tool called the variable separation approach in the linear case. Since Lou and Lu first successfully applied a kind of variable separation method, now called multi-linear variable separations approach (MLVSA), in 1995 for the Davey Stewartson (DS) equation,[18] the MLVSA has been revised and well developed for various (2+1)-dimensional nonlinear evolution systems. A nearly systematic process of the MLVSA to solve (2+1)-dimensional nonlinear evolution systems has been established.[19−30] Now an important question is whether MLVSE in (1+1)-dimensional physics systems can be improved further. Recently, there is some development.

In Ref.[31], we obtained the variable separation approach solution of two classes of variable-coefficient (1+1)-dimensional nonlinear Schrödinger equation. In Ref.[32], the new exact solution of (1+1)-dimensional nonlinear models related to Schrödinger equation by the entrance of three arbitrary functions is successfully derived by using MLVSA. Some special types of soliton wave solution such as multi-soliton wave solutions, non-steady soliton solutions, oscillating soliton solutions and periodic soliton solutions are discussed by selecting the arbitrary functions appropriately. Such as the compact, peak, loop solutions are derived by Zheng *et al.*[33] Shen *et al*[34] obtained a general exact solution of the Redekopp system by using MLVSA. In Ref.[35], the variable separation solution of (1+1)-dimensional equations of long-wave-short-wave resonant interaction was obtained. Some new solitary wave solutions, non-propagating solitary wave solutions, propagating solitary wave solutions, are found by selecting the arbitrary function appropriately. In this paper, we further investigate the ex-

tension of the MLVSA to (1+1)-dimensional nonlinear models by considering a class of (1+1)-dimensional Ito systems[36−39]

$$u_t = v_x, \tag{1}$$

$$v_t = -2v_{xxx} - 6(uv)_x. \tag{2}$$

Research results of the present paper indicate that Eqs.(1) and (2) may possess the solutions of variable separation approach and some new solitary wave structures.

This paper is organized as follows. In Section 2, we apply the variable separation approach to Ito equation and obtain its new exact explicit solutions. In Section 3, we reveal some new special types of solitary wave structures. A brief summary and discussion are made in the last section.

2. Variable separation procedure for Eqs.(1) and (2)

To use the variable separation approach, we take the following transformation:

$$u = 2\ln(f)_{xx} + u_0, \tag{3}$$

$$v = 2\ln(f)_{xt} + v_0, \tag{4}$$

which can be obtained from the standard Painlevé truncated expansion. Here $f \equiv f(x,t)$, and $\{u_0, v_0\}$ is an arbitrary known seed solution of the Ito equation. It is evident that Eqs.(1) and (2) possess a trivial seed solution

$$u_0 = u_0(x), \quad v_0 = 0 \tag{5}$$

with $u_0(x)$ being an arbitrary function of the indicated variables. Substituting Eqs.(3), (4) and (5) into Eqs.(1) and (2) , we obtain the bilinear forms of Eq.(2), while Eq.(1) is satisfied identically under the transformation using Eqs.(3), (4) and (5)

$$(D_t^2 + 2D_x^3 D_t + 6u_0 D_x D_t) f \cdot f = 0, \tag{6}$$

where D_x, D_t are defined as

$$D_x^m D_t^n g \cdot f \equiv (\partial_x - \partial_{x'})^m (\partial_t - \partial_{t'})^n g\ f|_{(x=x',t=t')}, \tag{7}$$

which are the usual bilinear operators introduced first by Hirota.[5]

To find some interesting solution of Eq.(6), we can use the variable separation ansatz

$$f = a_0 + a_1\tau(x) + a_2\chi(t) + a_3\tau(x)\chi(t), \tag{8}$$

where the variable separation functions $\chi \equiv \chi(t)$ and $\tau \equiv \tau(x)$ are only functions of $\{t\}$ and $\{x\}$ respectively. Substituting Eq.(8) into Eq.(6), we have

$$u_0 = -(6\tau_x)^{-1}\{2\tau_{xxx} + \lambda(a_0a_3 - a_1a_2)^{-1} \times [a_0a_2 + (a_0a_3 + a_1a_2)\tau + a_1a_3\tau^2]\}, \tag{9}$$

$$\chi_t = \lambda\chi, \tag{10}$$

where a_0, a_1, a_2, a_3 and λ is arbitrary real constants. Finally, we obtain the general solutions of systems (1) and (2)

$$v(x,t) = \frac{2(a_0a_3 - a_1a_2)\chi_t\tau_x}{(a_0 + a_1\tau + a_2\chi + a_3\tau\chi)^2}, \tag{11}$$

$$u(x,t) = 2\frac{(a_0 + a_1\tau + a_2\chi + a_3\tau\chi)(a_1 + a_3\chi)\tau_{xx} - (a_1 + a_3\chi)^2\tau_x^2}{(a_0 + a_1\tau + a_2\chi + a_3\tau\chi)^2} + u_0, \tag{12}$$

where $\tau(x)$ is an arbitrary function of $\{x\}$, $\chi(t)$ can be expressed by Eq.(10), u_0 can be expressed by Eq.(9). In the following discussion, we study the structure of the field $v(x,t)$. Due to the arbitrariness of the function of $\tau(x)$, the field $v(x,t)$ shown by Eq.(11) possess abundant solitary wave structures. Here we list some examples.

3. Some special types of solitary wave structures

3.1. Non-propagating solitary wave solution

When $\tau(x)$ and $\chi(t)$ are chosen as

$$\tau(x) = \exp(\tanh(x) + \tanh(x-5) + \tanh(x+5)),$$

$$\chi(t) = \exp(t+1), \tag{13}$$

we can obtain a kind of non-propagating triple solitary wave for Eqs.(1) and (2). Figure 1 is the evolution plots of $v(x,t)$ expressed by Eq.(11) with Eq.(13) and $a_0 = 3$, $a_1 = a_2 = 0$, $a_3 = 1$, at time $t = -2$, -1, 0, 1, 2. From Figs.1(a)–1(e), we find that the non-propagating solitary wave is fixed in the x-axis but its amplitude changes with time t.

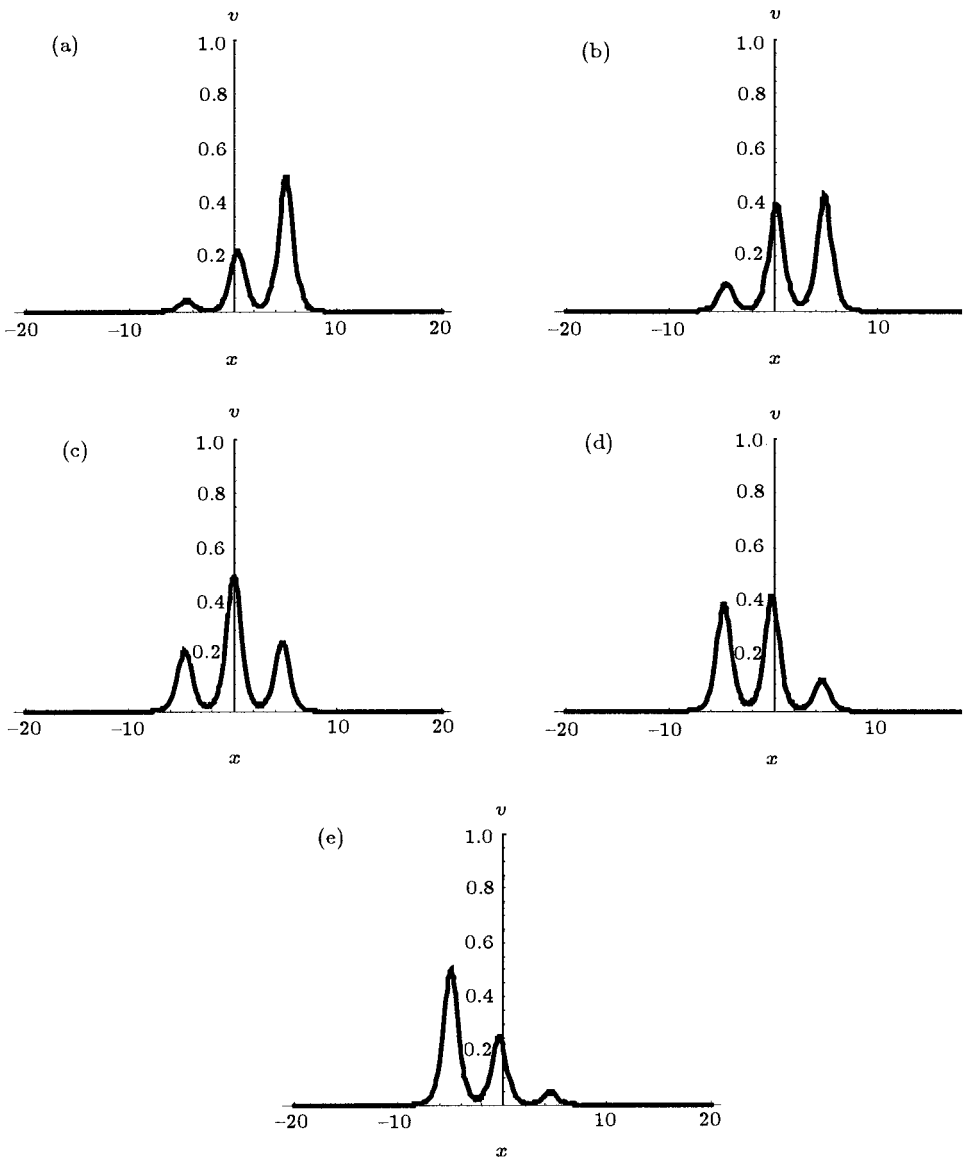


Fig.1. Evolution plots of non-propagating triple solitary wave for the $v(x,t)$ expressed by Eq.(11) with Eq.(13) and $\{a_0, a_1, a_2, a_3\} = \{3, 0, 0, 1\}$ at (a) $t = -2$, (b) $t = -1$, (c) $t = 0$, (d) $t = 1$, (e) $t = 2$.

3.2. Propagating solitary wave solution

When $\tau(x)$ and $\chi(t)$ are chosen as

$$\tau(x) = \exp\left(-\frac{x^3}{3} + 2x\right), \quad \chi(t) = \exp(\lambda t), \quad (14)$$

we can obtain a kind of propagating solitary waves for Eqs.(1) and (2). Figure 2 shows the evolution plots of $v(x,t)$ expressed by Eq.(11) with Eq.(14) and $a_0 = 1$, $a_1 = a_2 = 0$, $a_3 = 1$, $\lambda = 1$, at $t = -5, -2.5, 0, 2.5, 5$. From Figs.2(a)–2(e), we can see that the amplitude of propagating solitary wave will change with time t. This solitary wave can be either left-travelling or right-travelling.

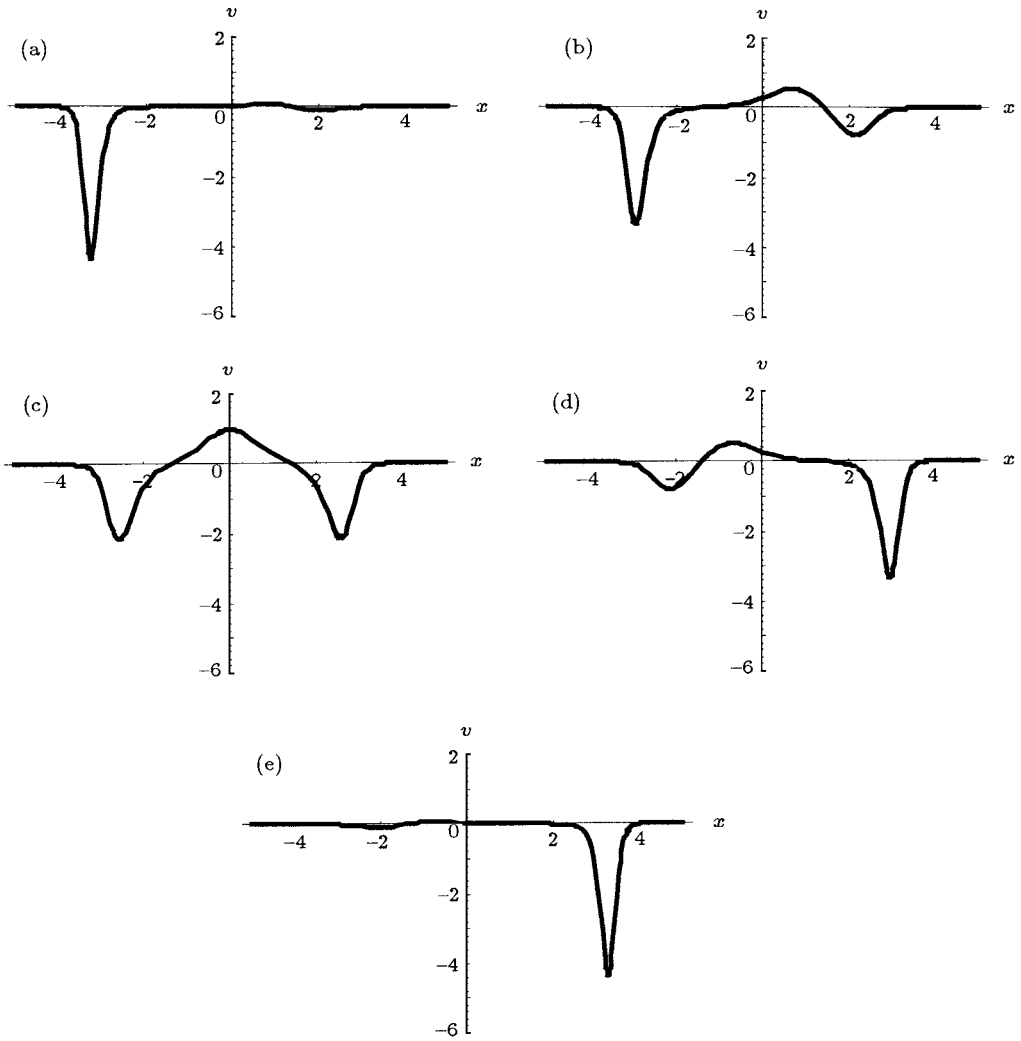


Fig.2. Evolution plots of propagating solitary wave for the field $v(x,t)$ expressed by Eq.(11) with Eq.(14) and $\{a_0, a_1, a_2, a_3, \lambda\} = \{1, 0, 0, 1, 1\}$, at (a)$t = -5$, (b)$t = -2.5$, (c)$t = 0$, (d)$t = 2.5$, (e)$t = 5$.

When $\tau(x)$ and $\chi(t)$ are chosen to be

$$\tau(x) = \cosh(x + \sin x), \quad \chi(t) = \exp(t + 10), \quad (15)$$

we can obtain a kind of propagating solitary waves for Eqs.(1) and (2). Figure 3 illustrates the evolution plots of $v(x,t)$ expressed by Eq.(11) with Eq.(15) and $a_0 = 3$, $a_1 = a_2 = 1$, $a_3 = 0$, at time $t = -8, -5, -2, 1, 4, 8$.

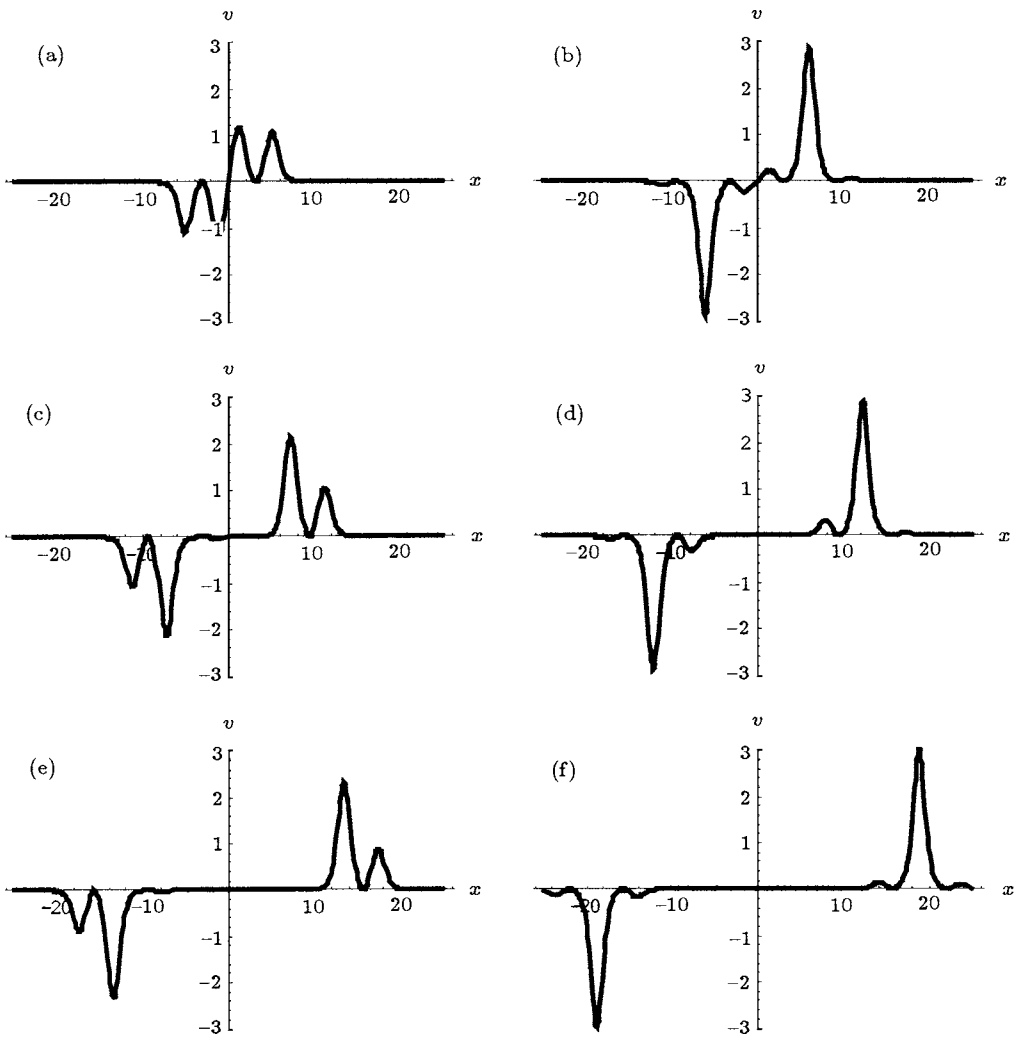


Fig.3. Evolution plots of propagating solitary wave for the field $v(x,t)$ expressed by Eq.(11) with Eq.(15) and $\{a_0, a_1, a_2, a_3\} = \{3, 1, 1, 0\}$, at (a)$t = -8$, (b)$t = -5$, (c)$t = -2$, (c)$t = 1$, (d)$t = 4$, (e)$t = 8$.

3.3. Loop soliton solution

If we select the functions as

$$\tau = \frac{1}{2} - \tanh(1.2\xi)\left(\frac{1}{6} - \frac{1}{3}\mathrm{sech}^2(1.2\xi)\right),$$
$$x = \xi - \tanh(1.2\xi),$$
$$\chi = \exp(-t+1), \qquad (16)$$

we can obtain loop soliton for Eqs.(1) and (2). Figure 4 are the evolution plots of $v(x,t)$ expressed by Eq.(11) with Eq. (16) and $a_0 = 2, a_1 = a_2 = 0, a_3 = 1$, at time $t = -2, 0, 2$.

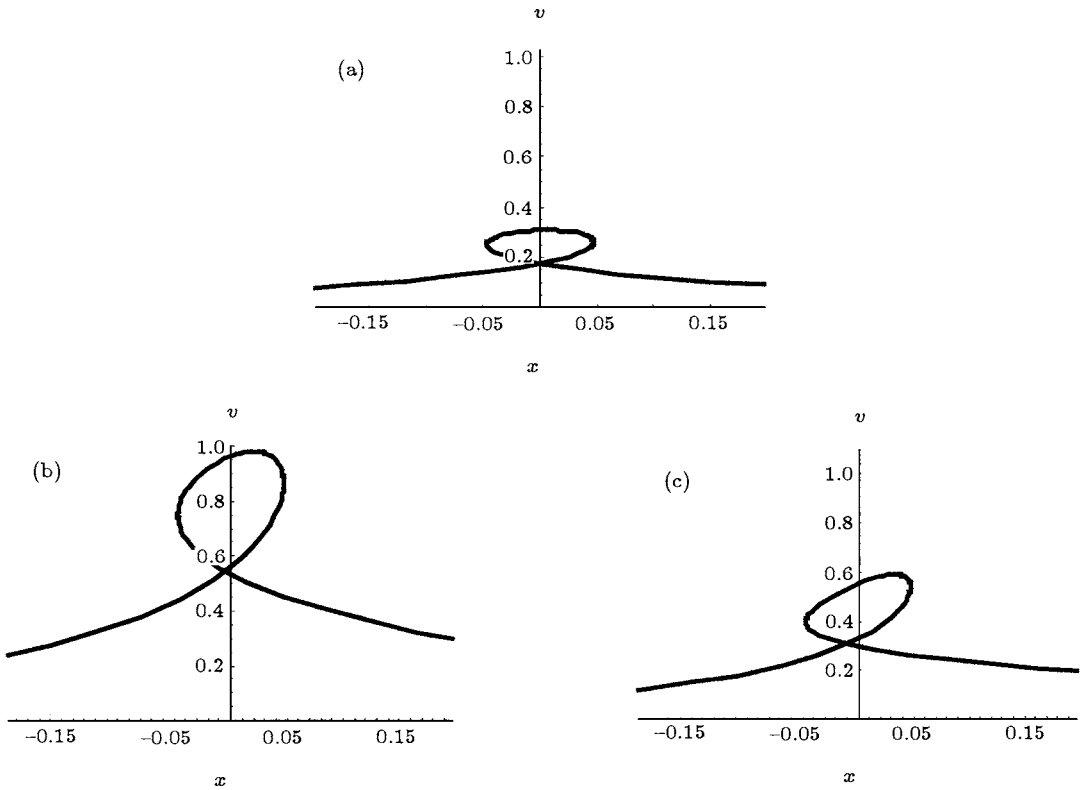


Fig.4. Evolution plots of loop soliton solution for the field $v(x,t)$ expressed by Eq.(11) with Eq.(16) and $\{a_0, a_1, a_2, a_3, \lambda\} = \{2, 0, 0, 1, 1\}$ at (a) $t = -2$, (b) $t = 0$, (c) $t = 2$.

4. Summary and discussion

In summary, with the help of the variable separation approach, the new variable separation solution of (1+1)-dimensional Ito equation is obtained. By selecting the arbitrary function appropriately, the non-propagating solitary wave solution, the propagating solitary wave solution and the loop soliton solution can be constructed. To our knowledge, the variable separation solution and these solitary wave structures of (1+1)-dimensional Ito equations found here have not been reported in previous papers published. To learn more about the non-propagating soliton solution, the propagating soliton solution and more of applications about the MLVSA to other (1+1)-dimensional physical models is worthy of further studying.

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