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Few-cycle relativistic solitons in plasmas*

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A set of exact one-dimensional solutions to coupled nonlinear equations describing the propagation of a relativistic ultrashort circularly polarized laser pulse in a cold collisionless and bounded plasma where electrons have an initial velocity in the laser propagating direction is presented. The solutions investigated here are in the form of quickly moving envelop solitons at a propagation velocity comparable to the light speed. The features of solitons in both underdense and overdense plasmas with electrons having different given initial velocities in the laser propagating direction are described. It is found that the amplitude of solitons is larger and soliton width shorter in plasmas where electrons have a larger initial velocity. In overdense plasmas, soliton duration is shorter, the amplitude higher than that in underdense plasmas where electrons have the same initial velocity.

Keywords: soliton, ultrashort laser pulse, plasma, fluid model
PACC: 5235S, 5230, 5240D, 5260

1. Introduction

The technique of ultrashort, ultra-intense laser pulses has been developing rapidly in recent years. It makes it possible to produce laser pulses with a few-cycle duration and extremely high intensity. The interaction of these laser pulses with plasma has relevance to a number of applications, which include particle and photon acceleration, plasma lensing, plasma wakefield generation and fast igniter inertial confinement fusion schemes. These applications call for a thorough understanding of various nonlinear effects arising when a relativistically intense electromagnetic pulse propagates through plasma. Such a relativistic intense pulse in plasmas also finds an application in astrophysical phenomena including the interaction of pulsar radiation with its neighboring plasma environments.

Ultra-intense light pulses, like solitons, can propagate a long distance in plasmas without apparent change in feature, as have been observed in particle-in-cell(PIC) simulations.[1−6] Electromagnetic solitons with relativistic amplitudes were first investigated by Kozlov *et al.*[7] Analytical studies on relativistic solitons are usually based on one-dimensional theories,[8] and several novel solutions have been found.[9−16] In Ref.[6], by using one-dimensional particle-in-cell simulation, the formation of solitons was observed which could move at a high group velocity when ultrashort laser pulses of a few cycles propagated in plasmas near the critical density. Furthermore, Shen *et al* studied subcycle relativistic solitons that could propagate from low density to high density plasmas.[13] In previous studies, electrons in plasmas were assumed to be stationary before the laser pulse interacted with the plasma. However, in reality, in the process of producing an ultrashort laser pulse with a high intensity, a prepulse is often produced before the main pulse. The prepulse often interacts with the plasma first before the main laser pulse reaches the plasma. Thereby, electrons in plasmas may be accelerated in the laser propagating field by the prepulse. Therefore, we assume that electrons in the plasma have an initial velocity in the laser propagating direction before the laser pulse interacts with the plasma.

In this paper we use a one-dimensional relativistic

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fluid model to demonstrate one soliton solution with a precisely given field energy and group velocity for a given pulse width and plasma density in plasmas where electrons have a given initial velocity. With the help of the numerical solution to the initial boundary problem for the set of nonlinear ordinary differential equations derived from the one-dimensional relativistic fluid model we describe the features of solitons in plasmas with electrons having different given initial velocities in the laser propagating direction.

2. Basic equations and analysis

Let us consider a model of cold, collisionless plasma in one dimension, described by Maxwell's equations and the hydrodynamic equations of an electron fluid against a stationary ion background[8,17,18]

$$\left(\frac{\partial^2}{\partial z^2}-\frac{\partial^2}{\partial t^2}\right)\boldsymbol{a}=n_{\mathrm{e}}\boldsymbol{v}, \tag{1}$$

$$\frac{\partial^2\phi}{\partial z^2}=n_{\mathrm{e}}-n_{\mathrm{i}}, \tag{2}$$

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial z}\right)(\gamma u)=\frac{\partial\phi}{\partial z}-\frac{1}{2\gamma}\frac{\partial a^2}{\partial z}, \tag{3}$$

$$\frac{\partial n_{\mathrm{e}}}{\partial t}+\frac{\partial(n_{\mathrm{e}}u)}{\partial z}=0, \tag{4}$$

where t, z, u, $\boldsymbol{v}=\boldsymbol{a}/\gamma$, n_{e}, $\boldsymbol{a}$ and ϕ are the normalized time, space in the laser propagating direction, longitudinal electron velocity, transverse electron velocity and electron density, transverse vector and scalar potential, respectively, and $\gamma=[(1+a^2)/(1-u^2)]^{1/2}$ is the relativistic factor of electrons. In previous studies the time is often normalized by the plasma frequency. Under this ansatz, soliton solution was given in Ref.[8]. However in order to make the theoretical results easily comparable with experimental results, here we normalize the time using the laser frequency ω_{L}. And the other normalizations $\omega_{\mathrm{L}}z/c$, u/c, $n_{\mathrm{e}}/n_{\mathrm{c}}$, $n_{\mathrm{i}}/n_{\mathrm{c}}$, $\boldsymbol{a}=e\boldsymbol{A}/mc^2$ and $e\phi/mc^2$ are used, and n_{c} is the critical density for the laser with a frequency ω_{L}.

In the quasistatic approximation,[8,17,18] it proves convenient to perform an algebraic transformation from the independent space and time variables (z,t) in a laboratory frame to the independent variables (ξ,τ), where $\xi=z-\nu_{\mathrm{g}}t$ and $\tau=t$, and ν_{g} is the normalized group velocity of the soliton. To investigate the propagation of the soliton in plasma where electrons have an initial velocity in the laser propagating direction, using these transformations, we integrate Eqs.(3) and (4) to obtain

$$\gamma(1-u\nu_{\mathrm{g}})-\phi=\gamma_0(1-u_0\nu_{\mathrm{g}}), \tag{5}$$

$$n_{\mathrm{e}}(\nu_{\mathrm{g}}-u)=n_{\mathrm{i}}(\nu_{\mathrm{g}}-u_0). \tag{6}$$

Here u_0 and $\gamma_0=(1-u_0^2)^{-1/2}$ are respectively the initial velocity of electrons and relativistic factor when $\xi\le 0$. And the Poisson's equation (2) can be written as

$$\phi''=n_{\mathrm{e}}-n_{\mathrm{i}}=\frac{u-u_0}{\nu_{\mathrm{g}}-u}n_{\mathrm{i}}, \tag{7}$$

with u given by

$$u=\frac{\nu_{\mathrm{g}}(1+a^2)-[\gamma_0(1-u_0\nu_{\mathrm{g}})+\phi]\{[\gamma_0(1-u_0\nu_{\mathrm{g}})+\phi]^2-(1-\nu_{\mathrm{g}}^2)(1+a^2)\}^{1/2}}{[\gamma_0(1-u_0\nu_{\mathrm{g}})+\phi]^2+\nu_{\mathrm{g}}^2(1+a^2)}, \tag{8}$$

where the prime denotes the derivative with respect to the variable ξ.

For a circularly polarized laser pulse, the vector potential of the electromagnetic field can be represented by $\boldsymbol{a}=a(\xi)\exp(\mathrm{i}[\omega\tau+\theta(\xi)])(\hat{x}+\mathrm{i}\hat{y})$. Here $a(\xi)$ is the envelop of laser pulse and $\exp(\mathrm{i}[\omega\tau+\theta(\xi)])$ is the phase. Therefore $\boldsymbol{a}$ is the complete description for the laser pulse. After substituting the above expression of the vector potential into the wave equation (1), we obtain two integration constants

$$M=a^2[(1-\nu_{\mathrm{g}}^2)\theta'+\omega\nu_{\mathrm{g}}], \tag{9}$$

$$W=\frac{a'^2}{2}-\frac{\phi'^2}{2(1-\nu_{\mathrm{g}}^2)}+V(a,\phi)), \tag{10}$$

with $V(a,\phi)$ given by

$$\begin{aligned}V(a,\phi)=&\frac{a^2}{2}+\frac{M^2}{2a^2(1-\nu_{\mathrm{g}}^2)}-\frac{\phi n_{\mathrm{i}}}{1-\nu_{\mathrm{g}}^2}\\&-\frac{\nu_{\mathrm{g}}n_{\mathrm{i}}}{(1-\nu_{\mathrm{g}}^2)^2}\{\nu_{\mathrm{g}}[\gamma_0(1-u_0\nu_{\mathrm{g}})+\phi]\\&-\{[\gamma_0(1-u_0\nu_{\mathrm{g}})+\phi]^2\\&-(1-\nu_{\mathrm{g}}^2)(1+a^2)\}^{1/2}\},\end{aligned} \tag{11}$$

where the prime denotes the derivative with respect to the variable ξ. For an infinite plasma, we have $M=W=0$. Then from Eq.(9) we obtain

$$\theta=-\frac{\omega\nu_{\mathrm{g}}}{1-\nu_{\mathrm{g}}^2}\xi+\theta_0, \tag{12}$$

where θ_0 is a constant. Thus the laser frequency is $\omega_L = \omega/(1-\nu_g^2)$.

3. Numerical solutions and discussion

In the limit of weak density response, Eqs.(7), (8), (10) and (11) can be reduced into one-dimensional problem about envelop a by the substitutions $\phi \approx (1+a^2)^{1/2} - 1$ and $n_e \approx (1+a^2)^{1/2''}$. Under this condition, the analytical envelop soliton solution can be obtained.[3] However, for arbitrary amplitudes we need to fully analyse a two-dimensional problem with respect to a, ϕ described by Eqs.(7), (8), (10) and (11) where the laser amplitude is strongly coupled to the scalar potential. In other words, for a given ion density n_i and an initial velocity of electrons in plasma u_0 finding soliton solutions turns out to be an eigenvalue problem in ν_g. To investigate soliton solutions in plasmas with electrons having different given initial velocities, here we have solved Eqs.(7), (8), (10) and (11) numerically and found solutions of soliton propagating at a special group velocity in plasmas with electrons having different initial velocities. The group velocity ν_g of the shortest two solitons ($p = 1, 2$, where p is the number of zeros of the a profile) in plasmas with a given ion density n_i and an initial velocity of electrons u_0 is presented in Table 1.

Table 1. Value of ν_g of the shortest two solitons (p is the number of zeros of the a profile) in a plasma with ion density n_i and initial velocity of electrons u_0.

p	n_i/n_c	u_0/c	ν_g/c	p	n_i/n_c	u_0/c	ν_g/c
1	0.5	0.0	0.76403	2	0.5	0.0	0.78236
1	0.5	0.5	0.8298953	2	0.5	0.5	0.8465869
1	0.5	0.8	0.9053382	2	0.5	0.8	0.9168614
1	1.2	0.0	0.46126	2	1.2	0.0	0.526165
1	1.2	0.5	0.6802811	2	1.2	0.5	0.7266507
1	1.2	0.8	0.8421328	2	1.2	0.8	0.86651332

For an infinite underdense plasma with a density $n_i = 0.5n_c$, the shortest soliton ($p = 1$, where p is the number of zeros of the a profile) is illustrated in Fig.1. When electrons in plasmas have different initial velocities the laser pulse will propagate at a special group velocity as a soliton. In the previous analytical studies, quasi-neutrality with $n_e \approx n_i$ is assumed due to the slow group velocity of the laser pulse. In this paper, we find that the laser pulse propagates at a high speed as a soliton in plasmas where electrons have a given particular velocity. The solitons propagating at different given group velocities ν_g=0.76403, 0.8298953, and 0.9053382 in a plasma where electrons correspondingly have the particular initial velocities u_0=0, 0.5, and 0.8 are respectively shown in Fig.1(a), 1(b) and 1(c). The laser amplitude, electron density, electron velocity and relativistic factor are also given in Fig.1. Clearly, the electron density structure shows significant perturbations about 10%, 60% and 100% that respectively correspond to the initial velocities of electrons in plasmas u_0=0, 0.5, and 0.8. Therefore it is difficult to find the analytical solutions of such solitons due to the invalidation of quasi-neutral of plasmas. From Fig.1 we can conclude that with the initial velocity of electrons in plasmas increasing the laser amplitude must be increased in order to make laser pulses propagate as a soliton for the amplitude of soliton is larger in plasmas with electrons possessing higher initial velocity. Furthermore, from Fig. 1, the durations of peak and valley of solitons are about 0.365, 0.303, 0.275 times laser wavelength respectively for u_0=0, 0.5, and 0.8. This means that the soliton duration becomes short when the initial velocity of electrons in plasmas increases. Electrons in plasmas are expelled by the relativistic ponderomotive force. The expelled electrons, being balanced by the space-charge and ponderomotive fields, pile up just outside the intense field region, resulting in a pit in the electron density profile, as shown in Fig.1(a), 1(b) and 1(c). The depth of the pit increases with the initial velocity of electrons increasing.

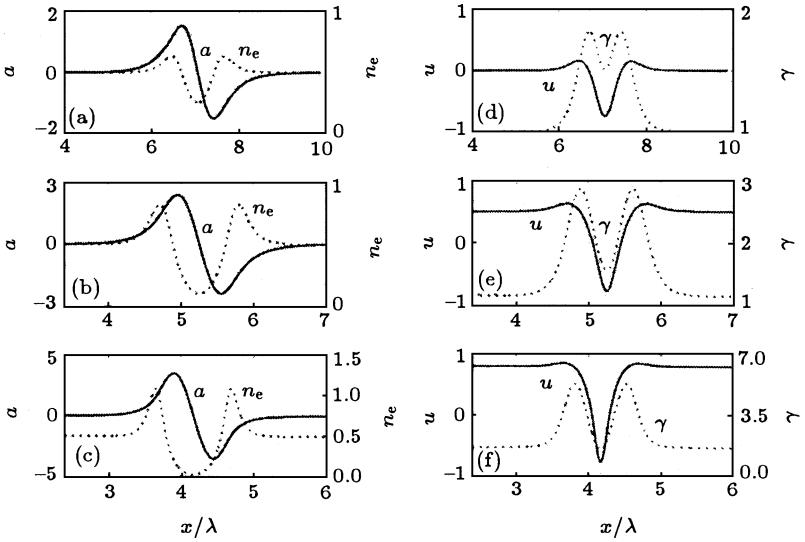


Fig.1. The shortest solitons ($p = 1$) in an underdense plasma with background density $0.5n_c$. (a), (b) and (c) are for normalized laser amplitude a and electron density n_e, and (d), (e), and (f) for the electron velocity u and relativistic factor γ. The initial electron velocity and the group velocity of the solitons are $u_0 = 0$, $\nu_g = 0.76403$ for (a) and (b); $u_0 = 0.5$, $\nu_g = 0.8298953$ for (c) and (d); $u_0 = 0.8$, $\nu_g = 0.9053382$ for (e) and (f).

In an overdense plasma with a density $n_i = 1.2n_c$, the characteristics of the solitons are similar to those in an underdense plasma with the increase of the electrons initial velocity, as shown in Fig.2. In comparison with solitons in underdense plasmas, those in ovendense plasmas have the shorter duration, the higher amplitude when electrons in both underdense and overdense plasmas have the same initial velocity. The pit depth of the density profile of electrons is larger in ovendense plasmas than in underdense plasmas. Furthermore, from Figs.1, 2 and Table 1, we know that the laser pulses can propagate as solitons at a group velocity higher than the initial velocity of electrons in plasmas, that is to say, From Figs.1 and 2, in overdense plasmas the electron density profile changes steeply with respect to the space coordinate ξ, and the maximum of electron density is larger than that in underdense plasmas. With the intensity and duration of solitons increasing and shortening, the ponderomotive force increases, which expels most electrons in plasmas

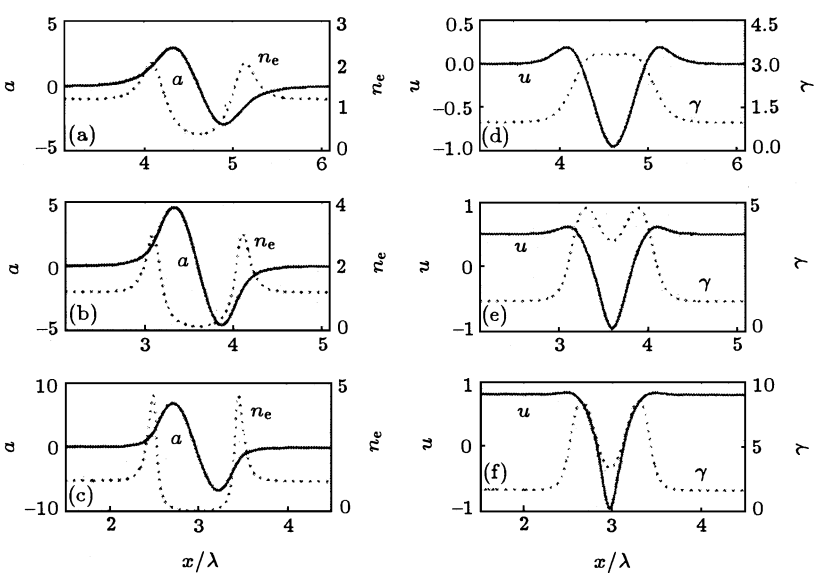


Fig.2. The shortest solitons($p = 1$) in an overdense plasma with background density $1.2n_c$. (a), (b) and (c) are for normalized laser amplitude a and electron density n_e, and (d), (e), and (f) for the electron velocity u and relativistic factor γ. The initial electron velocity and the group velocity of the solitons are $u_0 = 0$, $\nu_g = 0.46126$ for (a) and (b); $u_0 = 0.5$, $\nu_g = 0.6802811$ for (c) and (d); $u_0 = 0.8$, $\nu_g = 0.8421328$ for (e) and (f).

to the edge of the solitons and results in an abrupt and steep jump in the density profile. This means that it is rational to assume the electron density profile discontinuous. A subcycle soliton solution based on this assumption can be found in Ref.[13].

Since solitons are associated with precisely defined group velocity and field energy, there are no neighbouring solitons at all. In underdense and overdense plasmas where electrons have a given initial velocity, when the laser pulse is given a particular group velocity larger than that of solitons with $p = 1$, it can propagate as solitons with a large p, as shown in Fig.3. Solitons with $p = 2$ (p is the number of zeros of the a profile) in underdense and overdense plasmas where the given initial velocity of electrons is different are illustrated in Fig.3. From Figs.1–3 we can see that the laser pulses with the lowest group velocity correspond to the shortest solitons. Furthermore, we can conclude that the laser amplitude must be increased to make the laser pulse propagate as solitons with a large p in plasmas.

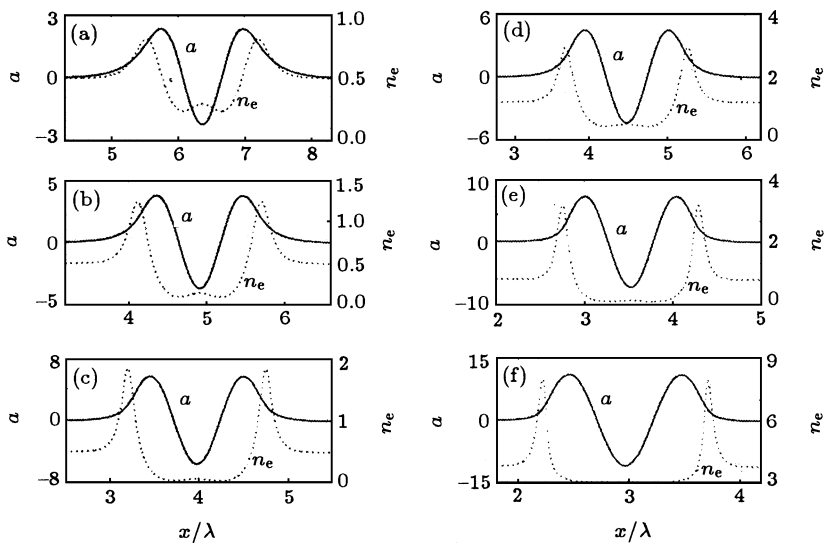


Fig.3. The normalized laser amplitude a and electron density n_e of solitons($p = 2$). (a), (b) and (c) are for underdense plasmas with background density $0.5n_c$, and (d), (e), and (f) are for overdense plasmas with background$1.2n_c$. The initial electron velocity and the group velocity of the solitons are $u_0 = 0$, $\nu_g = 0.78236$ for (a); and $u_0 = 0.5$, $\nu_g = 0.8465869$ for (b); $u_0 = 0.8$, $\nu_g = 0.9168614$ for (c); $u_0 = 0$, $\nu_g = 0.526165$ for (d); $u_0 = 0.5$, $\nu_g = 0.7266507$ for (e); $u_0 = 0.8$, $\nu_g = 0.86651332$ for (f).

4. Conclusions

In conclusion, using one-dimensional relativistic fluid model, we have shown few-cycle relativistic solitons with a high intensity in underdense and overdense plasmas where electrons have different initial velocities in the laser propagating direction. With the help of the numerical solution to the boundary problem for the set of nonlinear ordinary differential equations we have demonstrated the characteristics of solitons in different plasmas. We expect that the results of the present investigation should be useful in understanding the nonlinear propagation of localized laser pulse in plasmas such as those in inertial confinement fusion and astrophysical environments. On the other hand, since these solitons involve strong fields and high intensities, on slightly longer time scales trapped-particle and thermalization effects[14] become important, and ion dynamics also plays a role.[15,16]. In this case, the propagation properties of the solitons may be different.

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