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Optical bistability of a composite system induced by a broadband squeezed vacuum*

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A composite system consisting of a degenerate optical parametric oscillator (DOPO) and N two-level atoms interacting with a broadband squeezed vacuum (SV) centred at frequency $\omega_{\rm s}$ and an input monochromatic pumping field with a frequency $\omega_{\rm p}$ is analysed. The corresponding explicit analytical steady-state solutions in the central mode $\omega_{\rm p} = \omega_{\rm s}$ are derived, and the result displays optical bistability (OB). In addition, the influence of the broadband SV on the bistable behaviour is analysed in detail.

Keywords: bistability, parametric oscillator, squeezed vacuum
PACC: 4250, 4265P

1. Introduction

Since the first observation of optical bistability (OB) in 1976,[1−3] the subject of extensive theoretical and experimental investigations about this phenomenon have appeared in diverse fields due to its potential and wide applications.[4] A general fundamental research focuses on the interesting physical behaviour of simple bistable systems, particularly of the bistable system composed of two-level atoms within an optical resonator.[5−12] For example, Wu *et al* investigated the behaviours of a laser system of driven two-level atoms with incoherent injection and showed that such a system could display OB in the absence of population inversion.[6,7] In a recent research, Xiao *et al* identified bistable behaviour in a realistic system consisting of a degenerate optical parametric (DOPO) and N two-level atoms and discussed the DOPO threshold as well as the optical and atomic spectra.[8] Quite recently, Maia *et al* considered N two-level atoms in a ring cavity interacting with a broadband SV, and showed that the frequency detuning between input pump and SV fields, interacting with two-level atoms, could give rise to a multiple-mode output field.[12] In their paper the theoretical model consisting of N two-level atoms in a ring cavity is simple and is only applicable to a dilute system. Furthermore, the input driving field is thought as classical strong field, so the decay of the pump field is neglected. In this regard, here we consider a different system consisting of a DOPO and N two-level atoms, which interacts with a broadband SV. Under the condition of the quantum theory we derive the steady-state solution at exact resonance between frequencies of the pump and SV fields, $\omega_{\rm p} = \omega_{\rm s}$. We also analyse the influence of the SV parameters on the bistable behaviour.

The structure of our paper is as follows. In Section 2 using the quantum theory we describe the model and derive a set of Ito stochastic differential equations which describe the atom-field interaction for the system under consideration as well as its steady-state solutions. In Section 3, the influence of the broadband SV on the bistable behaviour is discussed and analysed via a nice Mathematica code. Finally, Section 4 is a brief summary of our results.

2. The model and steady-state bistable solutions

A composite system consisting of a DOPO and N two-level atoms in a ring cavity interacting with a broadband SV centred at frequency $\omega_{\rm s}$ is investigated. Alternatively, we consider the decay of the fields, which consist of a fundamental monochromatic

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pump field and a subharmonic field, but neglect the collision decay of the atoms. In the electric-dipole and rotating-wave approximations,[13−16] the Hamiltonian of the composite system under consideration can be written as[8]

$$\begin{aligned}
\hat{H} &= \sum_{i=1}^{7} \hat{H}_i, \\
\hat{H}_1 &= \sum_k \omega_k \hat{b}_k^+ \hat{b}_k + \sum_k \left(g_k \hat{b}_k \hat{S}_+ + g_k^* \hat{b}_k^+ \hat{S}_- \right), \\
\hat{H}_2 &= \hbar\omega_c \hat{a}^+ \hat{a} + \frac{1}{2}\hbar\omega_a \hat{S}_0 + \hbar\omega_2 \hat{a}_2^+ \hat{a}_2, \\
\hat{H}_3 &= \mathrm{i}\hbar g \left(\hat{a}^+ \hat{S}_- - \hat{a} \hat{S}_+ \right), \\
\hat{H}_4 &= \mathrm{i}\hbar \frac{\kappa}{2} \left(\hat{a}^{+2} \hat{a}_2 - \hat{a}^2 \hat{a}_2^+ \right), \\
\hat{H}_5 &= \hat{a}_2 \hat{B}_2^+ + \hat{a}_2^+ \hat{B}_2, \\
\hat{H}_6 &= \hat{a} \hat{B}_1^+ + \hat{a}^+ \hat{B}_1, \\
\hat{H}_7 &= \mathrm{i}\hbar \left(\varepsilon_2 \hat{a}_2^+ \mathrm{e}^{-\mathrm{i}\omega_\mathrm{p} t} - \varepsilon_2^* \hat{a}_2 \mathrm{e}^{\mathrm{i}\omega_\mathrm{p} t} \right),
\end{aligned} \tag{1}$$

where the first term of the right-hand side of $\hat{H}_1$ is the SV modes and the second term stands for the interaction between atoms and SV field. $\hat{H}_2$ is the free energy of the fields and atoms, $\hat{H}_3$ is the interaction between the atoms and subharmonic field, $\hat{H}_4$ describes the DOPO process, $\hat{H}_5$ is the decay of the field at the pumping frequency, $\hat{H}_6$ is the decay of the subharmonic field, $\hat{H}_7$ is the pumping terms. $\hat{a}_2^+(\hat{a}_2)$ denotes the creation (annihilation) operator of the fundamental monochromatic field mode. $\hat{a}^+(\hat{a})$ denotes the creation (annihilation) operator of the subharmonic field mode. $\hat{b}_k^+(\hat{b}_k)$ denotes the creation (annihilation) operator of the squeezed vacuum field at frequency ω_k. ω_a is the atomic transition frequency. ω_2 and ω_c are the frequencies of the fundamental field and the cavity subharmonic field respectively. ω_p is the pump frequency which is twice the atomic transition frequency ω_a. g is the coupling strength between atoms and the intracavity field, and g_k is the coupling strength between atoms and the SV field. κ is the coupling coefficient of the nonlinear process of the crystal. $\hat{B}_2^+$, $\hat{B}_2$ are the noise operators for the pump-field mode, and $\hat{B}_1^+$, $\hat{B}_1$ for the subharmonic field mode. ε_2 is the amplitude of the external driving field. $\hat{S}_0$ and $\hat{S}_\pm$ are the atomic collective operators, given by

$$\hat{S}_0 = \sum_{i=1}^{N} \hat{s}_0(i) \quad \text{and} \quad \hat{S}_\pm = \sum_{i=1}^{N} \hat{s}_\pm(i), \tag{2}$$

where $\hat{s}_0(i)$ and $\hat{s}_\pm(i)$ are single-particle operators satisfying the commutation relations $[\hat{s}_0(i), \hat{s}_\pm(i)] = \pm 2\delta_{i,j}\hat{s}_\pm(i)$ and $[\hat{s}_+(i), \hat{s}_-(i)] = \delta_{i,j}\hat{s}_0(i)$. In what follows, we deal with the atomic collective operators throughout the remainder of this paper.

Using the electric dipole, rotating-wave, and Markovian approximations,[17] the quantum master equation for the density operator $\hat{\rho}(t)$ of the composite system can be expressed as

$$\begin{aligned}
\frac{\mathrm{d}\hat{\rho}(t)}{\mathrm{d}t} =& \frac{1}{\mathrm{i}\hbar} \left[\hat{H}_2 + \hat{H}_3 + \hat{H}_4 + \hat{H}_7, \hat{\rho}(t) \right] \\
& - \{ \xi_{11}(t) [\hat{S}_+, \hat{S}_+ \hat{\rho}(t)] + \xi_{12} [\hat{S}_-, \hat{S}_+ \hat{\rho}(t)] \\
& + \xi_{21} [\hat{S}_+, \hat{S}_- \hat{\rho}(t)] + \mathrm{H.c.} \} \\
& + \gamma_1 \left\{ [\hat{a}\hat{\rho}(t), \hat{a}^+] + [\hat{a}, \hat{\rho}(t)\hat{a}^+] \right\} \\
& + \gamma_2 \left\{ [\hat{a}_2, \hat{\rho}(t)\hat{a}_2^+] + [\hat{a}_2 \hat{\rho}(t), \hat{a}_2^+] \right\},
\end{aligned} \tag{3}$$

where the symbol "H.c." means Hermitian conjugation. The terms in the first circle braces represent the phase-sensitive damping due to the SV and the terms of the 2nd and the 3rd circle braces represent the decay of the pump-field and the subharmonic field respectively. The coefficients ξ_{ij} are characterized by the kind of reservoir and are separately defined as [12]

$$\xi_{11}(t) = -\mathrm{e}^{\mathrm{i}\theta} \sinh r \cosh r \mathrm{e}^{2\mathrm{i}\varepsilon t} \left(\frac{\gamma}{2} - \mathrm{i}\gamma \right), \tag{4}$$

$$\xi_{12} = \left(\frac{\gamma}{2} - \mathrm{i}\nu \right) \sinh^2 r, \quad \text{and}$$

$$\xi_{21} = \left(\frac{\gamma}{2} - \mathrm{i}\nu \right) \cosh^2 r, \tag{5}$$

where r is the squeeze parameter, $\varepsilon = 2(\omega_\mathrm{p} - \omega_\mathrm{s})$ is twice the detuning between input pumping and SV fields. γ is the damping constant of the atoms due to the SV field. θ is a relative phase between the input pumping and SV fields. ν is the dynamic frequency shift , being much smaller than γ, and it can be neglected.[18] γ_1 is the cavity decay rate of the subharmonic field and γ_2 is the cavity decay rate of the fundamental field mode.

There is a standard technique developed for laser theory to transform the operator master equation into a c-number Fokker–Planck equation. The complex phase-space representation is introduced by defining a correspondence between complex c-numbers and quantum operators as follows:[19]

$$\begin{aligned}
& (\hat{a}, \hat{a}^+, \hat{a}_2, \hat{a}_2^+, \hat{S}_-, \hat{S}_0, \hat{S}_+) \rightarrow \\
& (\alpha, \alpha^*, \alpha_2, \alpha_2^*, S_-, S_0, S_+) \equiv u.
\end{aligned} \tag{6}$$

The corresponding normal ordering is defined by introducing the characteristic function,

$$\begin{aligned}
\tilde{\chi}(\vec{\lambda}) =& \exp(\mathrm{i}\lambda_7 \hat{S}_+) \exp(\mathrm{i}\lambda_6 \hat{S}_0) \\
& \times \exp(\mathrm{i}\lambda_5 \hat{S}_-) \exp(\mathrm{i}\lambda_4 \hat{a}_2^+) \exp(\mathrm{i}\lambda_3 \hat{a}_2) \\
& \times \exp(\mathrm{i}\lambda_2 \hat{a}^+) \exp(\mathrm{i}\lambda_1 \hat{a}).
\end{aligned} \tag{7}$$

And a probability distribution in a seven-dimensional complex phase space is determined by

$$\mathrm{Tr}\left[\hat{\rho}\tilde{\chi}(\vec{\lambda})\right] = \int P(\vec{\alpha})\mathrm{e}^{\mathrm{i}\vec{\lambda}\cdot\vec{\alpha}}\mathrm{d}^2\alpha_1\cdots\mathrm{d}^2\alpha_7, \qquad (8)$$

where

$$\vec{\lambda} = (\lambda_1, \lambda_2, \cdots, \lambda_7),$$
$$\bar{\alpha} = (\alpha_1, \alpha_2, \cdots, \alpha_7) \equiv (\alpha, \alpha^*, \alpha_2, \alpha_2^*, S_-, S_0, S_+).$$

Following a procedure similar to Ref.[8], we can arrive at a set of Ito stochastic differential equations without fluctuations in the following forms:[20]

$$\begin{aligned}
\frac{\partial S_+}{\partial t} &= \left[\mathrm{i}\left(\frac{\delta_a}{2} - \nu\cosh 2r\right) - \frac{\gamma}{2}\cosh 2r\right] S_+ \\
&\quad + g\alpha^* S_0 - \mathrm{e}^{-\mathrm{i}\theta}\sinh 2r\mathrm{e}^{-\mathrm{i}\varepsilon t}\left(\frac{\gamma}{2} + \mathrm{i}\nu\right) S_-, \\
\frac{\partial S_-}{\partial t} &= \left[-\mathrm{i}\left(\frac{\delta_a}{2} - \nu\cosh 2r\right) - \frac{\gamma}{2}\cosh 2r\right] S_- \\
&\quad + g\alpha S_0 - \mathrm{e}^{\mathrm{i}\theta}\sinh 2r\mathrm{e}^{\mathrm{i}\varepsilon t}\left(\frac{\gamma}{2} - \mathrm{i}\nu\right) S_+,
\end{aligned}$$

$$\begin{aligned}
\frac{\partial S_0}{\partial t} &= -2g\alpha^* S_- - 2g\alpha S_+ + \gamma N - \gamma\cosh 2r S_0, \\
\frac{\partial \alpha_2}{\partial t} &= -(\gamma_2 + \mathrm{i}\delta_2)\alpha_2 - \frac{\kappa\alpha^2}{2} + \varepsilon_2, \\
\frac{\partial \alpha_2^*}{\partial t} &= -(\gamma_2 - \mathrm{i}\delta_2)\alpha_2^* - \frac{\kappa\alpha^{*2}}{2} + \varepsilon_2^*, \\
\frac{\partial \alpha}{\partial t} &= -(\gamma_1 + \mathrm{i}\delta_c)\alpha + gS_- + \kappa\alpha^*\alpha_2, \\
\frac{\partial \alpha^*}{\partial t} &= -(\gamma_1 - \mathrm{i}\delta_c)\alpha^* + gS_+ + \kappa\alpha\alpha_2^*, \qquad (9)
\end{aligned}$$

where we have defined the detuning $\delta_2 = \omega_\mathrm{p} - \omega_2$, $\delta_\mathrm{a} = \omega_\mathrm{p}/2 - \omega_\mathrm{a}$ and $\delta_\mathrm{c} = \omega_\mathrm{p}/2 - \omega_\mathrm{c}$. In the following discussion, owing to $\nu \ll \gamma$ we will neglect ν.

For no detuning between SV and pump fields, i.e. $\varepsilon = 0$, there is no explicit time dependence in Eq.(9). For the sake of simplicity, we consider a double resonant cavity such that $\delta_c = 0$, $\delta_2 = 0$, and now $\delta_a = \omega_c - \omega_a$ can be regarded as the detuning between the frequency of the subharmonic mode of the DOPO and the atomic transition frequency which determines the coupling strength of the atoms to the field. Therefore, the steady-state solutions can be calculated by setting $\mathrm{d}u_i/\mathrm{d}t = 0$, $(i = 1, \cdots, 7)$. After some algebra we can obtain the steady-state equations

$$\begin{aligned}
&\left[1 + \beta X + \frac{C(\sinh 2r\cos\theta - \cosh 2r)}{2X(\cosh 2r - \sinh 2r\cos\theta) + \cosh 2r(1 + \Delta^2)}\right]^2 \\
&+ \left[\frac{C(\Delta - \sinh 2r\sin\theta)}{2X(\cosh 2r - \sinh 2r\cos\theta) + \cosh 2r(1 + \Delta^2)}\right]^2 = Y, \qquad (10)
\end{aligned}$$

and

$$X = 0, \qquad (11)$$

where several new parameters have been defined by

$$\begin{aligned}
&\Delta \equiv \frac{\delta_a}{\gamma},\ C \equiv \frac{2Ng^2}{\gamma_1\gamma},\ N_\mathrm{s} \equiv \frac{\gamma^2}{4g^2}, \\
&n_0 \equiv \frac{2\gamma_1\gamma_2}{\kappa^2},\ \beta \equiv \frac{N_\mathrm{s}}{n_0},\ |\varepsilon_0| \equiv \frac{\gamma_1\gamma_2}{\kappa}, \qquad (12)
\end{aligned}$$

where C is the atomic-cooperative parameter which is defined as the ratio of atomic absorption due to the broadband SV to the cavity-decay rate of the subharmonic mode. N_s is the single atom saturation photon number. n_0 is the scaling photon number for the DOPO system. β is a measure of the relative strength of the atomic system over the DOPO. ε_0 is the threshold value of the pumping amplitude for this DOPO system when the atoms are not present. The normalized field variables are

$$x \equiv \frac{\alpha}{\sqrt{N_\mathrm{s}}}, \quad X \equiv xx^*, \quad y \equiv \frac{\varepsilon_2}{\varepsilon_0}, \quad Y \equiv yy^*. \qquad (13)$$

3. Discussion and analysis for the bistable behaviour

From the steady-state equation (10), the threshold value of the pumping amplitude for subharmonic generation in this coupling system may be modified as

$$\begin{aligned}
|\bar{\varepsilon}^c| = |\varepsilon_0|\Bigg\{&\left[1 + \frac{C(\sinh 2r\cos\theta - \cosh 2r)}{\cosh 2r(1 + \Delta^2)}\right]^2 \\
&+ \left[\frac{C(\Delta - \sinh 2r\sin\theta)}{\cosh 2r(1 + \Delta^2)}\right]^2\Bigg\}^{1/2}, \qquad (14)
\end{aligned}$$

where the DOPO threshold is different from that of Ref.[12], and is not only determined by parameters C and Δ, but also explicitly dependent on both the squeeze parameter r and the relative phase θ due to

the broadband SV. To this end, we can control the threshold value only by changing the SV parameters.

According to Eqs.(10) and (11), due to the DOPO process the solution $X = 0$ is yet one steady-state solution for any pumping power and is independent on the SV parameters, but the other solution is not. For the cases $r = \theta = 0$ (without VC field) and $r \neq 0, \theta \neq 0$ (with VC field), we plot X as a function of Y based on Eq.(10) via a nice Mathematica code, as shown in Fig.1. It is clear that there are multiple values of intracavity subharmonic intensity X for the given pumping intensity Y just below the lower "tuning point", which obviously is also the DOPO threshold value. We can understand this instability in the following way. When the input intensity is small, the subharmonic field photons produced in the parametric down-conversion process can be brought out by the broadband SV field. So the intracavity subharmonic field can not build up with the increase of the input intensity until the atoms reach saturation at the lower turning point, where the intracavity field starts to oscillate. In such a case, the intracavity intensity jumps from the lower branch to the upper branch. Then when the input intensity decreases from far above threshold, the intracavity subharmonic intensity X decreases stably until it reaches the upper turning point, where the intracavity field becomes unstable and jumps back to $X = 0$. This is the typical bistability. Comparing the two curves, we can easily find that both the upper turning point and the lower branch of the curves are quite sensitive to the SV parameters, r and θ. As a consequence, we can separately analyse the influence of the two SV parameters on the bistable behaviour.

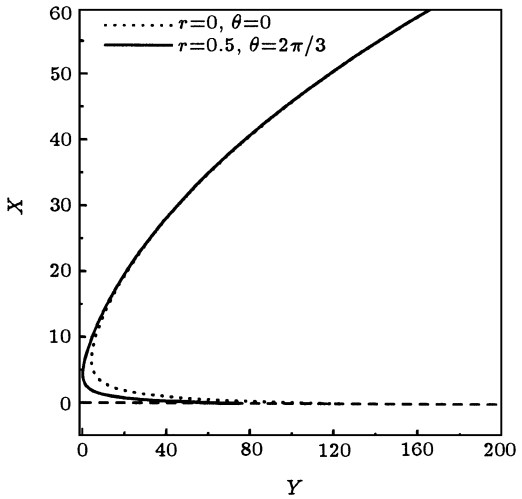


Fig.1. Intracavity subharmonic intensity X vs input intensity Y. The fitting parameters are: $\Delta = 1.4$, $C = 20$ and $\beta = 0.2$.

The general results of Eq.(10) are shown in Figs.2(a) and 2(b) with different values of the phase θ, while r is keeping constant. From Fig.2(a), when $0 \leq \theta \leq \pi/2$, as θ increases, the DOPO threshold value decreases, which means that though the bistable region decreases, the bistability is much easier to be achieved with a lower input intensity. On the other hand, when θ increases, the upper turning point moves backwards, so that the bistable region is enlarged. When θ reaches $\pi/2$, the bistable behaviour changes greatly. The output intensity X can stably decrease until the input intensity Y reduces to 0. Furthermore, the DOPO threshold value increases sharply, which enlarges the bistable region greatly. In Fig.2(b), when θ increases from $\pi/2$ to π, the shape of curve is almost unchanged, whereas the DOPO threshold value increases sharply, which enlarges the bistable region.

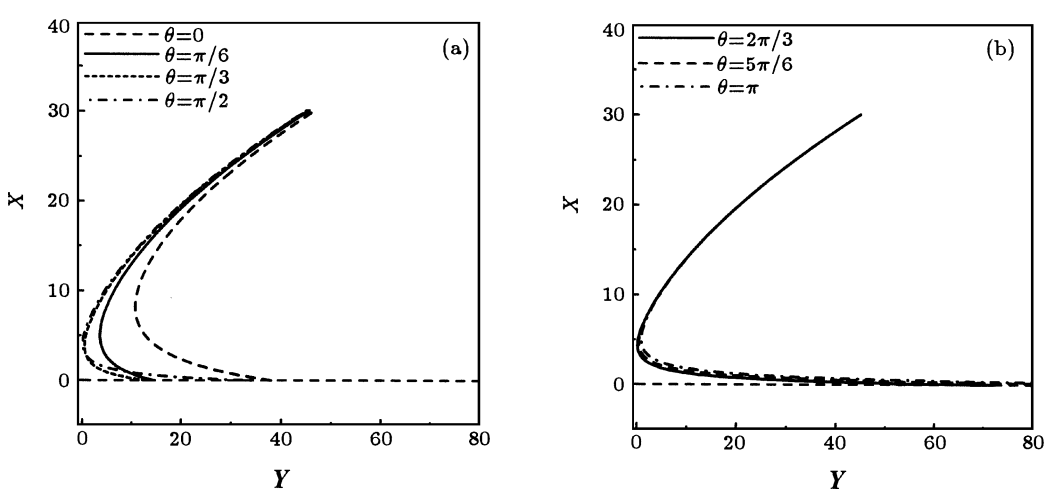


Fig.2. (a) and (b) Intracavity subharmonic intensity X vs input intensity Y with different values of θ. The fitting parameters are: $r = 0.5$, $\Delta = 1.4$, $C = 20$, and $\beta = 0.2$.

In Fig.3, we also plot intracavity X as a function of pumping intensity Y for different values of squeeze parameter r, while θ is keeping fixed. We can find that the upper turning point changes with the different values of r, which suggests that the SV has a main influence on the bistable behaviour when the input intensity decreases. Moreover, the influence of the SV on the bistable behaviour occurs only when the value of r is quite small.

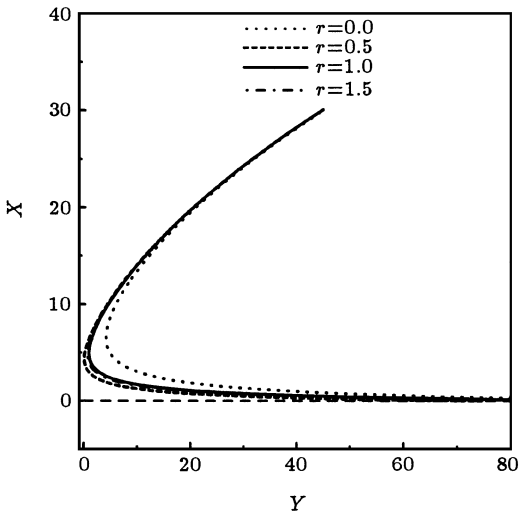


Fig.3. Intracavity subharmonic intensity X vs input intensity Y with different values of r. The fitting parameters are: $\theta = 2\pi/3$, $\Delta = 1.4$, $C = 20$ and $\beta = 0.2$.

4. Conclusions

In summary, in the present work, we have discussed the system consisting of a DOPO and N two-level atoms interacting with a broadband squeezed vacuum, and derived the steady-state solutions in the central mode, which displays OB. Then we place emphasis on analysing the effect of OB parameters on the optical behaviour. From the above discussion, we can see that the two OB parameters make great influence on the optical behaviour. As a result, we can operate the optical behaviour by changing the OB parameters, which is much easier to be realized in experiment, when we compare them with those in Ref.[12]. Nevertheless, here we only discuss the condition of the exact resonance between pump and squeezed field frequencies, $\omega_{\rm p} = \omega_{\rm s}$. Consideration of detuning, $\omega_{\rm p} \neq \omega_{\rm s}$ will be a subject of our future investigation.

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