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Teleportation of atomic states with a weak coherent cavity field*

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A scheme is proposed for the teleportation of an unknown atomic state. The scheme is based on the resonant interaction of atoms with a coherent cavity field. The mean photon-number of the cavity field is much smaller than one and thus the cavity decay can be effectively suppressed. Another advantage of the scheme is that only one cavity is required.

Keywords: teleportation, atomic state, coherent cavity field
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In recent years, much attention has been paid to quantum entanglement. A measurement on one of two entangled systems not only gives information of the other system, but also provides possibilities of manipulating it. This striking feature is useful for realizing quantum cryptography[1] and computers.[2] On the other hand, Bennett *et al* have shown that the quantum entanglement can also be used to teleport an unknown quantum state.[3] In the first step of the teleportation process, two spin-1/2 particles are prepared in the maximally entangled state. Then, a joint measurement is performed on the particle to be teleported and one of the correlated pair. Finally, the information of the joint measurement is transmitted to an other observer through a classical channel and thus he can reconstruct the initial state of the teleported particle on the second particle of the correlated pair. In recent years, quantum teleportation has become a central topic in quantum information.[4,5]

In cavity QED, Davidovich *et al* have presented a scheme for the teleportation of an unknown atomic state between two high-Q cavities initially prepared in an entangled photon-number states.[6] Cirac *et al* have made another cavity QED proposal for the realization of quantum teleportation of an atomic state by using two additional atomic levels of one of the correlated pair.[7] We have also presented a scheme for teleporting an unknown atomic state by using resonant atom–cavity interaction.[8] All of the above mentioned schemes require the cavity be prepared in the superposition of the vacuum state and one-photon state with the same weights. In this paper, we propose an alternative scheme for the teleportation of an unknown atomic state. This time the cavity is first in a weak coherent state with the mean photon-number much smaller than one and thus the cavity decay can be effectively suppressed. Furthermore, only one cavity is required to complete the procedure.

We consider a ladder-type three-level atoms, whose states are denoted by $|i\rangle$, $|g\rangle$, and $|e\rangle$, with the energy level of the state $|i\rangle$ being the lowest and the energy level of the state $|e\rangle$ being the highest, as shown in Fig.1. Let the atom interact with a single-mode cavity field. Suppose the atomic transition frequency between the states $|g\rangle$ and $|e\rangle$ is resonant with the cavity field. On the other hand, the transition frequency between the states $|i\rangle$ and $|g\rangle$ is highly detuned from the cavity frequency and thus the state $|i\rangle$ is not affected during the atom–cavity interaction. Then, in the interaction picture, the atom–cavity interaction is described by the Jaynes–Cummings Hamiltonian

$$H_{\rm I} = g(a^{+}|g\rangle\langle e| + a|e\rangle\langle g|), \qquad (1)$$

where a^{+} and a are the creation and annihilation operators for the cavity mode, and g is the atom–cavity coupling strength.

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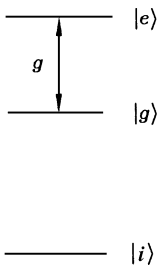


Fig.1. The level configuration of the atoms. The cavity mode resonantly couples the state $|g\rangle$ to $|e\rangle$. The state $|i\rangle$ is not affected by the cavity mode.

Assume the atom (Atom a) to be teleported is of the above mentioned type and in the state

$$|\phi\rangle_{\rm a} = c_i\,|i\rangle_{\rm a} + c_g\,|g\rangle_{\rm a}\,, \tag{2}$$

where c_i and c_g are unknown coefficients. The atom (Atom b) to receive the teleported state is initially prepared in the state $\frac{1}{\sqrt{2}}(|i\rangle_{\rm b} + |g\rangle_{\rm b})$ by a classical field. The cavity is initially in the vacuum state. Then a small coherent field is injected into the cavity. The coherent state can be expanded in terms of Fock states

$$|\alpha\rangle = {\rm e}^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}\,|n\rangle\,. \tag{3}$$

We first send Atom b through the cavity. Assume that α is small enough so that the terms with $n > 1$ are negligible. In this case the coherent state $|\alpha\rangle$ is approximately given by the superposition of the vacuum state and one-photon state,

$$|\alpha\rangle \simeq {\rm e}^{-|\alpha|^2/2}(|0\rangle + \alpha\,|1\rangle). \tag{4}$$

Then the state $|g\rangle_{\rm b}\,|\alpha\rangle$ undergoes the transition

$$\begin{aligned}|g\rangle_{\rm b}\,|\alpha\rangle \rightarrow & \frac{1}{\sqrt{2}}{\rm e}^{-|\alpha|^2/2}\{|g\rangle_{\rm b}\,|0\rangle \\ & + \alpha[\cos(gt)\,|g\rangle_{\rm b}\,|1\rangle \\ & - {\rm i}\sin(gt)\,|e\rangle_{\rm b}\,|0\rangle]\},\end{aligned} \tag{5}$$

where t is the interaction time. We choose the atomic velocity carefully so that $gt = \pi$ is fulfilled. This leads to

$$\begin{aligned}|g\rangle_{\rm b}\,|\alpha\rangle \rightarrow & \frac{1}{\sqrt{2}}{\rm e}^{-|\alpha|^2/2}(|g\rangle_{\rm b}\,|0\rangle - \alpha\,|g\rangle_{\rm b}\,|1\rangle) \\ & \simeq |g\rangle_{\rm b}\,|-\alpha\rangle).\end{aligned} \tag{6}$$

On the other hand, the state $|i\rangle_{\rm b}\,|\alpha\rangle$ undergoes no changes since the state $|i\rangle_{\rm b}$ does not couple with the cavity. When the atom exits in the cavity the state of the system combined by Atom b and the cavity is given by

$$|\psi\rangle_1 \simeq \frac{1}{\sqrt{2}}(|i\rangle_{\rm b}\,|\alpha\rangle + |g\rangle_{\rm b}\,|-\alpha\rangle). \tag{7}$$

Now the state of the whole system combined by two atoms and one cavity can be expanded as

$$\begin{aligned}|\psi\rangle \simeq & \frac{1}{2}[|\,\Psi^+\rangle\,(c_g\,|g\rangle_{\rm b} + c_i\,|i\rangle_{\rm b}) \\ & + |\,\Psi^-\rangle\,(c_g\,|g\rangle_{\rm b} - c_i\,|i\rangle_{\rm b}) \\ & + |\,\Phi^+\rangle\,(c_g\,|i\rangle_{\rm b} + c_i\,|g\rangle_{\rm b}) \\ & + |\,\Phi^-\rangle\,(c_g\,|i\rangle_{\rm b} - c_i\,|g\rangle_{\rm b})],\end{aligned} \tag{8}$$

where $|\,\Psi^{\pm}\rangle$ and $|\,\Phi^{\pm}\rangle$ are the Bell states

$$|\,\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|g\rangle_{\rm a}\,|-\alpha\rangle \pm |i\rangle_{\rm a}\,|\alpha\rangle), \tag{9}$$

$$|\,\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|g\rangle_{\rm a}\,|\alpha\rangle \pm |i\rangle_{\rm a}\,|-\alpha\rangle). \tag{10}$$

It should be noted that $|\,\Psi^{\pm}\rangle$ are not orthogonal to $|\,\Phi^{\pm}\rangle$ since $\langle\alpha|\,-\alpha\rangle \neq 0$.

We now let Atom a cross the cavity and choose the interaction time t' appropriately so that $gt' = \pi$. Then we obtain

$$\begin{aligned}|g\rangle_{\rm a}\,|\alpha\rangle \rightarrow & \frac{1}{\sqrt{2}}{\rm e}^{-|\alpha|^2/2}(|g\rangle_{\rm a}\,|0\rangle - \alpha\,|g\rangle_{\rm a}\,|1\rangle) \\ & \simeq |g\rangle_{\rm a}\,|-\alpha\rangle),\end{aligned} \tag{11}$$

and

$$\begin{aligned}|g\rangle_{\rm a}\,|-\alpha\rangle \rightarrow & \frac{1}{\sqrt{2}}{\rm e}^{-|\alpha|^2/2}(|g\rangle_{\rm a}\,|0\rangle + \alpha\,|g\rangle_{\rm a}\,|1\rangle) \\ & \simeq |g\rangle_{\rm a}\,|\alpha\rangle).\end{aligned} \tag{12}$$

Thus, the evolutions of $|\,\Psi^{\pm}\rangle$ and $|\,\Phi^{\pm}\rangle$ are

$$|\,\Psi^{\pm}\rangle \rightarrow \frac{1}{\sqrt{2}}(|g\rangle_{\rm a} \pm |i\rangle_{\rm a})\,|\alpha\rangle\,, \tag{13}$$

$$|\,\Phi^{\pm}\rangle \rightarrow \frac{1}{\sqrt{2}}(|g\rangle_{\rm a} \pm |i\rangle_{\rm a})\,|-\alpha\rangle\,. \tag{14}$$

We then let Atom a cross a classical field, undergoing the transition

$$|g\rangle_{\rm a} \rightarrow \frac{1}{\sqrt{2}}(|g\rangle_{\rm a} + |i\rangle_{\rm a}), \tag{15}$$

$$|i\rangle_{\rm a} \rightarrow \frac{1}{\sqrt{2}}(|i\rangle_{\rm a} - |g\rangle_{\rm a}). \tag{16}$$

Thus we obtain

$$\frac{1}{\sqrt{2}}(|g\rangle_{\rm a} \pm |i\rangle_{\rm a}) \rightarrow \begin{cases} |i\rangle_{\rm a}\,, \\ |g\rangle_{\rm a}\,. \end{cases} \tag{17}$$

Now the remaining part of the problem is how one can detect the cavity field. In order to do so we inject a field with the amplitude α into the cavity.[7] This leads

to $|\alpha\rangle \to |2\alpha\rangle$ and $|-\alpha\rangle \to |0\rangle$. Then we send an auxiliary atom a′, initially in the state $|g\rangle_{a'}$, across the cavity. The state $|g\rangle_{a'}|0\rangle$ undergoes no transitions. On the other hand, the state $|g\rangle_{a'}|2\alpha\rangle$ undergoes the transition

$$\begin{aligned}|g\rangle_{a'}|2\alpha\rangle \to & e^{-2|\alpha|^2}\{|g\rangle_{a'}|0\rangle \\ & + \sum_{n=1}^{\infty}\frac{(2\alpha)^n}{\sqrt{n!}}[\cos(\sqrt{n}g\tau)|g\rangle_{a'}|n\rangle \\ & - i\sin(\sqrt{n}g\tau)|e\rangle_{a'}|n-1\rangle]\}, \end{aligned} \quad (18)$$

where τ is the interaction time. Thus, the outcome $|i\rangle_a|e\rangle_{a'}$ corresponds to the Bell state $|\Psi^+\rangle$, while the outcome $|g\rangle_a|e\rangle_{a'}$ corresponds to $|\Psi^-\rangle$. If Alice obtains the outcome $|i\rangle_a|e\rangle_{a'}$ she can tell the receiver (Bob) that Atom b has been prepared in the initial state of Atom a. On the other hand, if Alice obtains $|g\rangle_a|e_{a'}\rangle$ she tells Bob to perform the following transformation on Atom b

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (19)$$

In this way Bob can convert Atom b into a replica of the original state of Atom a. On the other hand, if Alice detects Atom a′ in the state $|g\rangle_{a'}$ the teleportation procedure fails. The probability of success is

$$P = e^{-4|\alpha|^2}\sum_{n=1}^{\infty}\frac{|2\alpha|^{2n}}{n!}\sin^2(\sqrt{n}g\tau). \quad (20)$$

It is necessary to give a comparison of our scheme with previous schemes. Unlike the schemes in Refs.[6] and [7], our scheme only requires one cavity. In the scheme in Ref.[8], the cavity and Atom a should first be prepared in the maximally entangled state and the mean photon number is $\bar{N}= 0.5$, while in the present scheme the mean photon number is $\bar{N}= |\alpha|^2$. The live time of the coherence of the cavity field is $T_r/\bar{N}$, with T_r being the cavity decay time.[9] We have assumed that $|\alpha|^2 \ll 1$. Thus, the coherence in present scheme can be kept in a better way than that in previous schemes.

Finally we briefly address the experimental feasibility of the proposed scheme. For the Rydberg atoms with principal quantum numbers 49, 50 and 51, the radiative time is $T_r = 3 \times 10^{-2}$s, and the coupling constant is $g = 2\pi \times 25$kHz.[10] Thus the atom–cavity interaction time is on the order of $\pi/g = 2 \times 10^{-5}$s. Therefore, the time required to complete the whole procedure is on the order of 10^{-4}s, much shorter than T_r. A cavity with a decay time $T_c \simeq 10^{-3}$s has been reported.[10] Assume that $|\alpha|^2 = 0.2$. Then the live time of coherence of the cavity field is about 5×10^{-3}s, much longer than the required time. Therefore, based on cavity QED techniques presently or soon to be available the proposed scheme might be realizable.

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