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Conditional stability of diatomic molecule driven by a weak laser field*

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Using a direct perturbation method, we investigate the stability of a diatomic molecule modelled by a weakly laser-driven Morse oscillator. It is shown that stationary state solution of the system is stable in the sense of Lyapunov and the periodical one possesses conditional stability, namely its stability depends on the initial conditions and system parameters. The corresponding sufficient and necessary conditions are established that indicate the stable states associated with some discrete energies. The results reveal how a diatomic molecule can be stabilized or dissociated with a weak laser, and demonstrate that the mathematical conditional stability works in the considered physical system.

Keywords: conditional stability, Morse oscillator, diatomic molecule
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1. Introduction

With the development of high-power infrared lasers, there has been considerable interest in investigating the dynamical behaviour of a molecular system under intensive laser radiation.[1−4] Classical theories have often been used successfully to investigate the microscopic nonlinear system, such as the subjects of multiphoton ionization of the Rydberg atoms,[5−7] the properties like the chaotic photodissociation,[8,9] harmonic-generation spectrum,[10,11] and the analytic structure[12] of diatomic molecules in an external field that are moulded by a laser-driven Morse oscillator[8−12] and the multiphoton process of excitation and dissociation of diatomic molecule driven by the chirped pulses.[13−15] The classical results are in accordance with the quantum-mechanical calculations.[8,9,14,16−19] In these theories, chaos diffusion theory gave an explanation to the process of dissociation induced by an external field in the Morse oscillator model;[8,9,20,21] in other respects, the "bucket dynamics"[14,15] gave the dynamic behaviour of the diatomic molecule driven by the chirped pulses, which is in agreement with quantum-mechanical results.[14,15]

In the previous work,[8,9,13−15] Chirikov's resonance overlap criterion was applied to treat the molecule dissociation. The stability of the system plays an important role, since the dissociation is correlated with loss of the stability. In this paper, we study the stability of diatomic molecules that are moulded by a Morse oscillator driven by a weak laser field with a constant frequency. We seek the stabilities of the stationary solution and the periodical one through a direct-perturbation method.[22−24] We find that, according to Lyapunov's linear stability theorem,[25,26] the stationary solution is stable except in the resonance case, but stability of the periodical solution depends on the initial conditions and system parameters. It is well known that if the stability of a nonlinear system does not depend on the selection of initial time, it is called uniform stability.[26] Therefore, the periodical solution is non-uniformly stable. Some researchers called the stability depending on initial conditions "the conditional stability".[27] We present the conditions promising the stability[28] and show how the periodical solution's stability varies with the perturbation parameters and initial conditions. A small change of the initial condition will be amplified lin-

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early with time, which leads the perturbed solution to linearly diverge. In other words, the periodical solution is stable for some initial conditions, but unstable for other ones, so it is called the conditional stability. The conditionally stable solution corresponds to a critical case with zero Lyapunov exponent in which it is unknown to characterize the periodical solution as "stable" or "unstable" through the Lyapunov exponent method.[25,26]

In the resonance case, from the stability condition of a parameter space we find that the diatomic molecule has a series of discrete energies associated with the stable orbits, which is similar to the quantized energy-level. For a certain initial condition the diatomic molecule will be excited and transits to these stable orbits or be dissociated. The system possesses this kind of conditional stability too. We numerically calculate the time evolution of the first-order corrected solution and the track in phase space, showing that the calculations are in good agreement with the theoretical analysis. The results display that for any initial setting we can control the stability of the diatomic molecule by adjusting the weak laser field.

2. Analyses of the linear stability

Generally, a diatomic molecule driven by a laser is described by a Morse oscillator interacting with the laser field. The Hamiltonian of the system is given by[8,13–15]

$$H = H_0 + H', \tag{1}$$

$$H_0 = \frac{P^2}{2\mu} + D(1 - \mathrm{e}^{-\alpha(r-r_\mathrm{e})})^2, \tag{2}$$

$$H' = -B\alpha(r - r_\mathrm{e})\cos(\omega' t), \tag{3}$$

where D represents the dissociation energy, μ denotes the reduced mass of the molecule, B is the driving amplitude related to the intensity of the laser, α characterizes the steepness of the potential, r is the relative position between two atoms, and r_e the equilibrium position. So the Hamilton equations associated with Eq.(1) are

$$\dot{r} = \frac{P}{\mu}, \tag{4}$$

$$\dot{P} = -2D\alpha\mathrm{e}^{-\alpha(r-r_\mathrm{e})}(1 - \mathrm{e}^{-\alpha(r-r_\mathrm{e})}) + B\alpha\cos(\omega' t). \tag{5}$$

Setting $x = \alpha(r - r_\mathrm{e})$, and combining Eq.(4) with Eq.(5) yields

$$\ddot{x} = -A\mathrm{e}^{-x}(1 - \mathrm{e}^{-x}) + \xi\cos\omega' t, \tag{6}$$

where $A = 2\mu^{-1}D\alpha^2$ and $\xi = \alpha^2 B\mu^{-1}$.

Under the periodical laser field the motion equation (6) is non-integrable that necessitates the perturbed solution. For a weakly external field with $|\xi| < |A|$, we can use a direct perturbation method to obtain the analytical solution. To do this we set

$$x = x_0 + x_1 + x_2 + \cdots$$

$$\text{for} \quad |\xi| < |A|, \quad |x_i| < |x_{i-1}|, \ i = 0, 1, 2, \cdots. \tag{7}$$

Here x_0 is the unperturbed solution and x_i $(i > 0)$ the ith-order corrected solution. Substituting Eq.(7) into Eq.(6) results in the zeroth equation

$$\ddot{x}_0 = -A(\mathrm{e}^{-x_0} - \mathrm{e}^{-2x_0}) \tag{8}$$

and the first-order approximation equation

$$\ddot{x}_1 = -A(2x_1\mathrm{e}^{-2x_0} - x_1\mathrm{e}^{-x_0}) + \xi\cos\omega' t. \tag{9}$$

Note that if the parameter condition $|\xi| \ll |A|$ is satisfied, the first approximation is a good one, because of the inequality $|x_i| \ll |x_{i-1}|$.

At first, we consider the unperturbed system in which there is no external field. Integrating Eq.(8) we obtain

$$\frac{1}{2}\left(\frac{\mathrm{d}x_0}{\mathrm{d}t}\right)^2 = A\left(\mathrm{e}^{-x_0} - \frac{1}{2}\mathrm{e}^{-2x_0}\right) + C_1, \tag{10}$$

where C_1 denotes the first integration, $C_1 = \frac{1}{2}\dot{x_0}(t_0) - A\left(\mathrm{e}^{x_0(t_0)} - \frac{1}{2}\mathrm{e}^{-2x_0(t_0)}\right)$, $x_0(t)$ is the unperturbed solution and t_0 is the initial time. In this case Eq.(8) may be written in the simple form

$$\dot{u} = v, \tag{11}$$

$$\dot{v} = -A\mathrm{e}^{-u}(1 - \mathrm{e}^{-u}) \tag{12}$$

with $u = x_0$. These unperturbed and first-order perturbed equations are the bases for the stability analysis.

2.1. Analysis of the stationary solution's stability

According to the nonlinear dynamic theory,[25] from Eqs.(11) and (12) we know the stationary solution of system is $(u,\ v) = (x_0, \dot{x}_0) = (0,\ 0)$, namely the two atoms lie in the equilibrium positions. Substituting the unperturbed solution $x_0 = 0$ into the first-order perturbed Eq.(9) yields

$$\ddot{x}_1 + Ax_1 = \xi\cos\omega' t. \tag{13}$$

Because $A > 0$ we can set $A = \omega_0^2$ such that Eq.(13) becomes

$$\ddot{x}_1 + \omega_0^2 x_1 = \xi \cos \omega' t. \tag{14}$$

When $\omega_0 \neq \omega'$, the solution of Eq.(14) can be easily obtained as

$$x_1 = c' \cos(\omega_0 t + \varphi) - \frac{\xi}{(\omega_0^2 - \omega'^2)} \cos \omega' t, \tag{15}$$

where c' and φ are two arbitrary constants to be determined through the initial conditions. We can observe that for any $t \geq t_0$ x_1 is a bounded function that vibrates around the unperturbed solution with finite amplitudes. Therefore, according to the Lyapunov linear stability theorem the stationary solution is stable. However, we know that for the resonance case, $\omega' = \omega_0$, the solution of Eq.(14) is

$$x_1 = c' \cos(\omega_0 t + \varphi) - \frac{\xi}{2\omega_0} t \cos \omega_0 t. \tag{16}$$

Obviously the corrected solution contains the term proportional to time, it will approach to infinity linearly with time evolution such that the system loses its stability through the resonance. This can lead the system to excite from ground state to high-excited one or even to be dissociated.

2.2. Analyses of periodic solution's stability

After analysing the stationary solution, we now begin to investigate the periodic one. According to the nonlinear dynamics theory, the asymptotic stationary solution of Eqs.(11) and (12) require

$$\lim_{t\to\infty} \dot{u} = 0, \quad \text{and} \quad \lim_{t\to\infty} \dot{v} = 0. \tag{17}$$

Setting $C_1 = -Ac/2$, Eq.(10) becomes

$$\dot{x}_0^2 = -A(\mathrm{e}^{-2x_0} - 2\mathrm{e}^{-x_0} + c). \tag{18}$$

The diatomic system is a conservative Hamiltonian one when there exists no external laser action. Consequently, the system energy is $E = H_0$, from Eqs.(2), (4) and (18) we know the constant c being

$$c = 1 - \frac{E}{D}. \tag{19}$$

Setting $y = \mathrm{e}^{-x_0}$ here, then $\dot{x}_0 = -\dot{y}/y$, and instituting them into Eq.(18) we have

$$\begin{aligned} \frac{\dot{y}}{y} &= \pm \sqrt{A}(-y^2 + 2y - c)^{1/2} \\ &= \pm \sqrt{A}[1 - c - (y-1)^2]^{1/2}. \end{aligned} \tag{20}$$

From Eq.(19), if the diatomic molecule system is in bounded states, the system energy must be lower than the dissociation energy, i.e. $0 < c < 1$. Integrating Eq.(20) yields

$$x_0 = \ln\left[\frac{1 + f_1(E)\cos\omega(t - C_0)}{f_2(E)}\right], \tag{21}$$

where

$$\omega = \sqrt{Ac} = \sqrt{A(1 - E/D)} = \alpha\sqrt{2(D-E)/\mu},$$

$$f_1(E) = -\sqrt{1-c} = -\sqrt{E/D}, \quad f_2(E) = c = 1 - E/D,$$

$$C_0 = t_0 - \frac{1}{\sqrt{Ac}} \arcsin \frac{\mathrm{e}^{-x_0(t_0)} - c}{\mathrm{e}^{-x_0(t_0)}\sqrt{1-c}}. \tag{22}$$

Equation (21) is just the periodical solution of the system.

We now use the direct perturbation method to investigate the molecule system driven by the weak laser field. At $\xi = 0$, the non-homogeneous equation (9) becomes a homogeneous one with the two linearly independent solutions[22–24]

$$x_1' = \dot{x}_0 = -\frac{\omega f_1(E) f_2(E) \sin\omega(t - C_0)}{1 + f_1(E)\cos\omega(t - C_0)}, \tag{23}$$

$$\begin{aligned} x_1'' =& x_1' \int (x_1')^{-2} \mathrm{d}t \\ =& \frac{1}{1 + f_1(E)\cos\omega(t - C_0)} \\ & \times \left[\frac{\cos\omega(t - C_0)}{\omega^2 f_1(E) f_2(E)}\right. \\ & + \frac{2 + f_1(E)\cos\omega(t - C_0)}{\omega^2 f_2(E)} \\ & \left. + \frac{f_1(E)}{\omega f_2(E)} t \sin\omega(t - C_0)\right], \end{aligned} \tag{24}$$

therefore the general solution of Eq.(9) can be written in the integral form

$$\begin{aligned} x_1 =& c_1 x_1'' + c_2 x_1' + x_1'' \int_0^t x_1' \xi \cos\omega' t \mathrm{d}t \\ & - x_1' \int_0^t x_1'' \xi \cos\omega' t \mathrm{d}t \end{aligned} \tag{25}$$

with c_1 and c_2 being two arbitrary constants that depend on initial perturbation. By setting $C_0 = t_0 = 0$ as a simplest example, from Eqs.(23), (24) and (25) we find $c_1 = \omega^2 f_1(E) f_2(E) x_1(0)/(1 + f_1(E))$, and $c_2 = -(1 + f_1(E))\dot{x}_1(0)/[\omega^2 f_1(E) f_2(E)]$, where $x_1(0)$, and $\dot{x}_1(0)$ are the initial perturbations. For the reason that x_1'' has the term that is proportional to the time, to study whether x_1 is a bounded function or not, we insert Eqs.(23) and (24) into Eq.(25), giving the first-order corrected solution

$$
\begin{aligned}
x_1 =& \frac{x_1(0)}{[1+f_1(E)][1+f_1(E)\cos\omega t]}[\cos\omega t + f_1(E)(2+f_1(E)\cos\omega t) + \omega f_1^2(E) t\sin\omega t] \\
& - \frac{[1+f_1(E)]\dot{x}_1(0)}{\omega} \times \frac{\sin\omega t}{1+f_1(E)\cos\omega t} \\
& + \frac{1}{\omega} \times \frac{\sin\omega t}{[1+f_1(E)\cos\omega t]} \int_0^t \frac{\cos\omega t}{[1+f_1(E)\cos\omega t]} \xi\cos\omega' t \mathrm{d}t \\
& - \frac{1}{\omega} \times \frac{\cos\omega t}{[1+f_1(E)\cos\omega t]} \int_0^t \frac{\sin\omega t}{[1+f_1(E)\cos\omega t]} \xi\cos\omega' t \mathrm{d}t \\
& + \frac{f_1(E)}{\omega} \times \frac{\sin\omega t}{[1+f_1(E)\cos\omega t]} \int_0^t \frac{[2+f_1(E)\cos\omega t]}{[1+f_1(E)\cos\omega t]} \xi\cos\omega' t \mathrm{d}t \\
& - \frac{f_1(E)}{\omega} \times \frac{[2+f_1(E)\cos\omega t]}{[1+f_1(E)\cos\omega t]} \int_0^t \frac{\sin\omega t}{[1+f_1(E)\cos\omega t]} \xi\cos\omega' t \mathrm{d}t \\
& + f_1^2(E) \frac{\sin\omega t}{[1+f_1(E)\cos\omega t]} \int_0^t \frac{t\sin\omega t}{[1+f_1(E)\cos\omega t]} \xi\cos\omega' t \mathrm{d}t \\
& - f_1^2(E) \frac{t\sin\omega t}{[1+f_1(E)\cos\omega t]} \int_0^t \frac{\sin\omega t}{[1+f_1(E)\cos\omega t]} \xi\cos\omega' t \mathrm{d}t. \qquad (26)
\end{aligned}
$$

In order to calculate the integration of $\int_0^t \frac{t\sin\omega t\xi\cos\omega' t}{1+f_1(E)\cos\omega t}\mathrm{d}t$, we make the Fourier expansion to $\frac{\sin\omega t}{1+f_1(E)\cos\omega t} = \sum_{n=1}^{\infty} b_n \sin(n\omega t)$, where $b_n = \frac{2\omega}{\pi}\int_0^{\pi/\omega} \frac{\sin\omega t\sin(n\omega t)}{1+f_1(E)\cos\omega t}\mathrm{d}t$. Inserting these into Eq.(26) and simplifying it, we have

$$
x_1 = G(t) + \frac{\omega f_1^2(E) x_1(0)}{[1+f_1(E)]} \times \frac{t\sin\omega t}{[1+f_1(E)\cos\omega t]}, \qquad (27)
$$

where

$$
\begin{aligned}
G(t) =& \frac{x_1(0)}{[1+f_1(E)][1+f_1(E)\cos\omega t]}[\cos\omega t + f_1(E)(2+f_1(E)\cos\omega t)] \\
& - \frac{[1+f_1(E)]\dot{x}_1(0)}{\omega} \times \frac{\sin\omega t}{1+f_1(E)\cos\omega t} \\
& + \frac{1}{\omega} \times \frac{\sin\omega t}{[1+f_1(E)\cos\omega t]} \int_0^t \frac{\cos\omega t}{[1+f_1(E)\cos\omega t]} \xi\cos\omega' t \mathrm{d}t \\
& - \frac{1}{\omega} \times \frac{\cos\omega t}{[1+f_1(E)\cos\omega t]} \int_0^t \frac{\sin\omega t}{[1+f_1(E)\cos\omega t]} \xi\cos\omega' t \mathrm{d}t \\
& + \frac{f_1(E)}{\omega} \times \frac{\sin\omega t}{[1+f_1(E)\cos\omega t]} \int_0^t \frac{[2+f_1(E)\cos\omega t]}{[1+f_1(E)\cos\omega t]} \xi\cos\omega' t \mathrm{d}t \\
& - \frac{f_1(E)}{\omega} \times \frac{[2+f_1(E)\cos\omega t]}{[1+f_1(E)\cos\omega t]} \int_0^t \frac{\sin\omega t}{[1+f_1(E)\cos\omega t]} \xi\cos\omega' t \mathrm{d}t \\
& + f_1^2(E) \frac{\sin\omega t}{[1+f_1(E)\cos\omega t]} \frac{\xi}{2} \sum_{n=1}^{\infty} b_n \left[\frac{\sin(n\omega+\omega')t}{(n\omega+\omega')^2} + \frac{\sin(n\omega-\omega')t}{(n\omega-\omega')^2}\right]
\end{aligned}
$$

is a bounded periodical function under the non-resonance circumstances.

Looking at Eq.(27) we find that it includes a term proportional to the time and this term is related to the initial perturbation $x_1(0)$. If there exists no resonance, $G(t)$ is a periodical function, therefore, only when $x_1(0) = 0$ can the corrected solution maintain bounded. Under the condition the periodical solution is called the conditionally stable for the deterministic perturbation.[27] If there appears a very small change of the initial perturbation, this change will grow up linearly with time evolution. Consequently, the corrected solution will spread apart from the unperturbed one and the system will become unstable. The system remains bounded only for a particular initial perturbation but loses its stability for other initial conditions.

Although the stability condition $x_1(0) = 0$ has been found, we do not know whether the motion is stable, for the initial perturbation $x_1(0)$ cannot be determined experimentally. In order to control stability of the system, we need to seek connection between the stability and the controllable parameters. From Eq.(27) we know that the function $G(t)$ includes some linearly increasing terms for the resonance case with parameter $\omega' = m\omega(m = 1, 2, 3, \cdots)$. If these terms just balance the final term of Eq.(27), the corrected solution becomes bounded and the system is stable. The laser frequency ω' is just a controllable parameter so we can adjust it to arrive at the resonance. To clear this, we start by making a Fourier expansion of the term $1/[1 + f_1(E)\cos\omega t]$,

$$\frac{1}{[1 + f_1(E)\cos\omega t]} = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega t) \tag{28}$$

with

$$a_0 = \frac{\omega}{\pi}\int_0^{\pi/\omega} \frac{1}{[1 + f_1(E)\cos\omega t]}\mathrm{d}t,$$
$$a_n = \frac{2\omega}{\pi}\int_0^{\pi/\omega} \frac{\cos(n\omega t)}{[1 + f_1(E)\cos\omega t]}\mathrm{d}t \tag{29}$$

so the integrations included in Eq.(27) become

$$\begin{aligned}
&\int_0^t \frac{\cos\omega t\cos\omega' t}{[1 + f_1(E)\cos\omega t]}\mathrm{d}t \\
=&\frac{a_0}{4}\int_0^t [\cos(\omega - \omega')t + \cos(\omega + \omega')t]\mathrm{d}t \\
&+ \sum_{n=1}^{\infty}\frac{a_n}{4}\int_0^t [\cos(\omega + n\omega + \omega')t \\
&+ \cos(n\omega - \omega - \omega')t + \cos(n\omega + \omega - \omega')t \\
&+ \cos(n\omega - \omega + \omega')t]\mathrm{d}t,
\end{aligned} \tag{30}$$

$$\begin{aligned}
&\int_0^t \frac{\sin\omega t\cos\omega' t}{1 + f_1(E)\cos\omega t}\mathrm{d}t \\
=&\frac{a_0}{4}\int_0^t [\sin(\omega + \omega')t + \sin(\omega - \omega')t]\mathrm{d}t \\
&+ \sum_{n=1}^{\infty}\frac{a_n}{4}\int_0^t [\sin(n\omega + \omega + \omega')t \\
&+ \sin(n\omega + \omega - \omega')t + \sin(\omega + \omega' - n\omega)t \\
&+ \sin(\omega - \omega' - n\omega)t]\mathrm{d}t,
\end{aligned} \tag{31}$$

$$\int_0^t \frac{2\cos\omega' t}{[1 + f_1(E)\cos\omega t]}\mathrm{d}t = a_0\int_0^t \cos\omega' t + \sum_{n=1}^{\infty} a_n \int_0^t [\cos(\omega' + n\omega)t + \cos(\omega' - n\omega)t]\mathrm{d}t. \tag{32}$$

Obviously, for $\omega' = m\omega(m = 1, 2, 3, \cdots)$ there exist some terms proportional to time in Eqs.(30) and (32). We collect all of such terms in Eq.(27) and obtain the coefficient of t as

$$\begin{aligned}
\Gamma =& \left(\frac{\xi}{4\omega} + \frac{\xi f_1^2(E)}{4\omega}\right)(a_{m+1} + a_{m-1}) \\
&+ \frac{\xi f_1(E)}{\omega}a_m + \frac{\omega f_1^2(E)}{1 + f_1(E)}x_1(0).
\end{aligned} \tag{33}$$

If the system is in the stable states, the first-order corrected solution should be bounded, so the coefficient in Eq.(33) must equal zero, i.e.

$$\Gamma = 0 \quad \text{for} \quad \omega' = m\omega, \quad m = 1, 2, 3, \cdots. \tag{34}$$

This is the sufficient and necessary condition of stability in parameter space for the resonance case. Given Eqs.(33), (22) and (29), for any fixed initial value $x_1(0)$ the stability condition (34) implies that the energy must take a series of discrete values, $E = E(m)$ for $m = 1, 2, 3, \cdots$ and only those orbits with these discrete energies can maintain bounded. Those molecules that are not in these energy states will be excited to these states or be dissociated. Combining this with Eq.(33) shows that boundedness of the corrected solution and stability of the unperturbed periodical solution depend on not only the initial condition but also the system parameters. Therefore, the system has this conditional stability under deterministic perturbation too.

Now, we analyse the stability using the Lyapunov exponent method. Employing Eq.(27), the Lyapunov exponent reads $\sigma = \lim_{t\to\infty}\frac{1}{t}\ln\frac{|x_1(t)|}{|x_1(0)|} = 0$. According to the linear stability theory, for $\sigma < 0$ the system is stable and the system will lose its stability at $\sigma > 0$. Unfortunately, in the present case the Lyapunov exponent is just the critical value $\sigma = 0$ such that the function $x_1(t)$ may be a bounded one, indicating the stability, or may be the sum of a bounded function and a linear function of time t, implying the instability. The stability cannot be determined and the Lyapunov exponent method becomes invalid thereby. But using the direct perturbation method we can study the stability of the critical case analytically and obtain the condition for controlling the instability, namely Eq.(34).

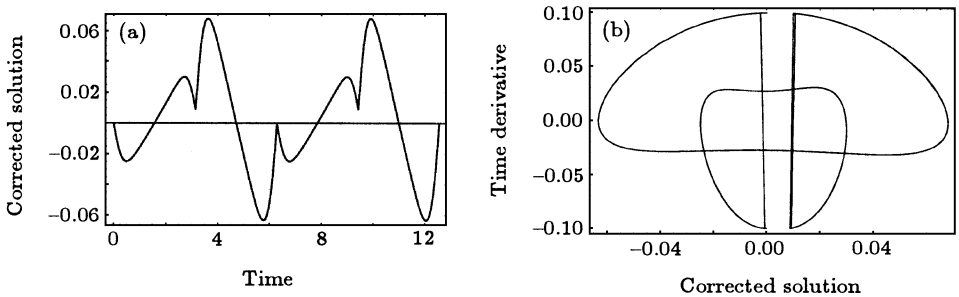


Fig.1. The time evolution of the non-resonant solution, x_1 vs t (a) and the orbit in phase space, $\dot{x}_1$ vs x_1 (b), from Eq.(27) with the initial conditions $x_1(0) = 0, \dot{x}_1(0) = 0.1$. The spatial coordinate is normalized by α^{-1} and the time is in units of $\alpha^{-1}\sqrt{\mu/2D}$. The plots show that the corrected solution vibrates with finite amplitude as in Fig.1(a) and the orbit is a closed curve as in Fig.1(b), indicating periodicity of the motion.

It is necessary to give the numerical results of the corrected solutions' time evolution and the orbit curves in phase space. For the non-resonance case, we substitute $\omega = 2, \omega' = 3, f_1(E) = -0.5, \xi = 0.01$ into Eq.(27) and choose the appropriate initial values $x_1(0) = 0, \dot{x}_1(0) = 0.1$ to satisfy the stability condition. Setting $A = 2\alpha^2 D/\mu = 1$ implies $\omega = \sqrt{A(1 - E/D)}$ being in units of $\alpha\sqrt{2D/\mu}$. Correspondingly, the time evolution curve and the orbit in phase space are given in Fig.1. Figure 1(a) shows x_1 oscillating with finite amplitude and Fig.1(b) indicates the phase orbit to be a closed curve, which means that the corrected solution vibrates around the unperturbed solution and does not stray away from it. So the solution is conditionally stable according to stability theorem. If we select another initial condition $x_1(0) = 0.1$ which does not fit the stability condition and other parameters to be the same as Fig.1, the numerical results are as shown in Fig.2. We can see from Fig.2 that the corrected solution vibrates with amplitude proportional to time, until infinity as time tending to infinity. The track in phase space is not a closed curve, it diffuses to an infinite region in phase space. So the corrected orbit will depart from the unperturbed one, and the system becomes unstable. In Fig.3 we give the results under resonant circumstances with $\omega = \omega' = 2$, the initial condition and others parameters $f_1(E), \xi$ are the same as those in Fig.1. There exists a divergent phenomenon similar to that in Fig.2, indicating the periodical solution loses its stability for the reason of resonance. On the other hand, we consider the resonant circumstances with parameters obeying the stability condition (34), that is $\omega' = 2\omega$, i.e. $m = 2$, $x_1(0) = -0.1, A = 1, \xi = 0.01$, so from Eq.(34) and Eq.(22) we have $\sqrt{E/D} = 0.02440442360913077$, and $\omega = 0.99970216770211433$. The corresponding time evolution curve and phase track are shown in Fig.4. Since the condition (34) is satisfied, periodicity of the solution is similar to that in Fig.1, the corrected solution remains bounded and the unperturbed periodical solution is stable. The numerical results are in good agreement with the theoretical analyses mentioned above.

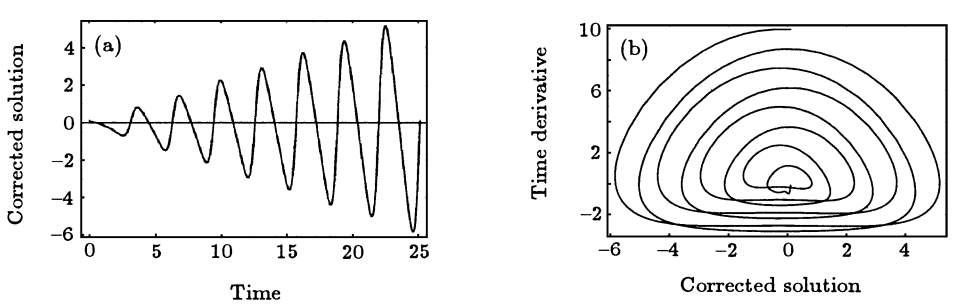


Fig.2. Plots of x_1 as a function of t and the phase orbit $\dot{x}_1$ vs x_1 for the initial perturbation $x_1(0) = 0.1, \dot{x}_1(0) = 0.1$ with the same units as Fig.1. The curve in Fig.2(a) shows that x_1 oscillates with a linearly increasing amplitude. The area of orbit in Fig.2(b) diffuses in phase space, indicating instability of the motion.

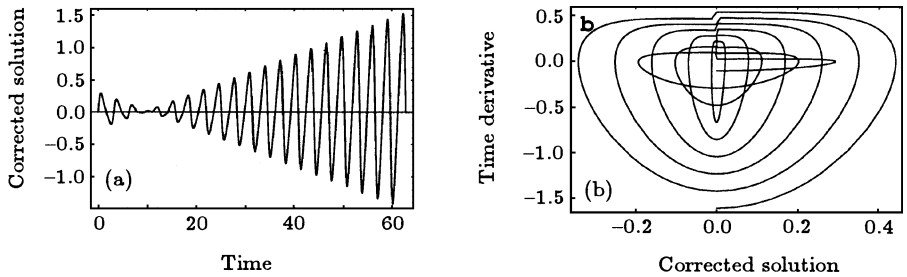


Fig.3. Plots of the time evolution $x_1(t)$ with the same units as those in Fig.2 and the track of phase space from Eq.(27) under resonant circumstances for the initial perturbation $x_1(0) = 0, \dot{x}_1(0) = 0.1$ and the forcing frequency $\omega' = \omega = 2$, which show the instability. In Fig.3(a) the perturbed solution vibrates around the unperturbed one with a linearly increasing amplitude, and Fig.3(b) shows the divergence trait in phase space.

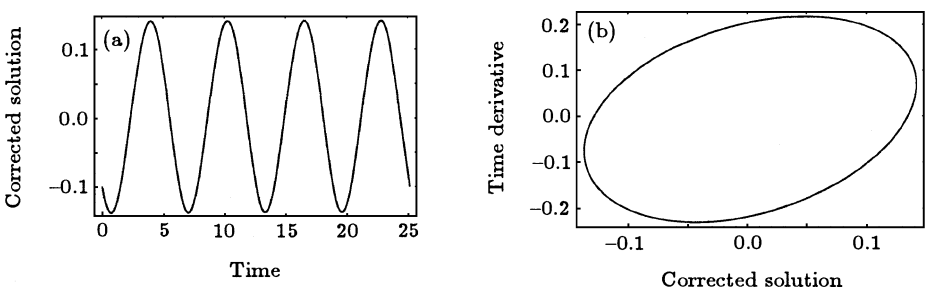


Fig.4. Plots of the time evolution of the corrected solution x_1 vs t (with the same units as those in Fig.1) and the track of phase space with parameters $\omega' = 2\omega, x_1(0) = -0.1, A = 1, \xi = 0.01, \sqrt{E/D} = 0.02440442360913077, \omega = 0.99970216770211433$ such that the stability condition (34) is satisfied. The plot in Fig.4(a) shows that x_1 vibrates in a finite region, and the closed orbit in Fig.4(b) shows its periodicity.

3. Conclusions

In conclusion, we have solved a diatomic molecular system described by a Morse oscillator in an external laser field. The Lyapunov stability of stationary solution and the conditional stability of the periodical one are investigated by applying the direct perturbation method. The sufficient and necessary conditions that cause the system to be bounded have been given for the periodical solution under resonance and non-resonance circumstances respectively. We found that once the actions of laser field satisfy the conditions, the molecular system would maintain bounded, otherwise, would lose its stability. Because the conditions depend on the initial perturbation, the stability of the system is dependent on the initial conditions. This property is called the conditional stability, which is similar to the non-uniform one. The analytical and numerical results reveal that the unperturbed periodical solution has different stabilities for different initial conditions and system parameters. Under non-resonance circumstances, a small change of the initial perturbation will be amplified linearly with the increase of time and lead a bounded solution to an unbounded one. For a given energy value and a suitable initial condition the system can also lose its stability, because of the resonance. However, in the resonance case, there exist some stable states associated with discrete energy values for any initial perturbation and small laser field strength. In order to experimentally control the stability, we must carefully trim the initial conditions for the non-resonance case or adjust the laser frequency to obeying the resonance condition and let the laser amplitude keep smaller.

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