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Controlling and synchronizing spatiotemporal chaos of the coupled Bragg acousto-optical bistable map system using nonlinear feedback

Yue Li-Juan(岳立娟)[a)b)†] and Shen Ke(沈 柯)[a)]

[a)] *Department of Physics, Changchun University of Science and Technology, Changchun 130022, China*

[b)] *Department of Physics, Northeast Normal University, Changchun 130024, China*

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In this paper we present the control and synchronization of a coupled Bragg acousto-optic bistable map system using nonlinear feedback technology. This nonlinear feedback technology is useful to control a temporally chaotic system as well as a spatiotemporally chaotic system. It can be extended to synchronize the spatiotemporal chaos. It can work in a wide range of the controlled and synchronized signals, so it can decrease the sensitivity down to a noise level. The synchronization can be obtained by the analysis of the largest conditional Lyapunov exponent spectrum, and easily implemented in practical systems just by adjusting the coupled strength without any pre-knowledge of the dynamic system required.

Keywords: synchronization and control, spatiotemporal chaos, nonlinear feedback, acousto-optic bistable map system

PACC: 0545, 4265T

1. Introduction

Controlling spatiotemporal chaos is an important and difficult subject that has attracted much attention. The coupled map lattice (CML)[1,2] is a prototypical model to explore methods of controlling spatiotemporal chaos. Recently, many techniques have been suggested to control the spatiotemporal chaos.[3−12] The feedback pinning technique for controlling CML was discussed.[3−5] Parmananda *et al* have studied several global and local feedback techniques and obtained some encouraging numerical results.[6] Gade has given some analytic results.[7] Kwon *et al* have given the improved technique for linear pinning feedback.[8] Parekh *et al* proposed a constant pinning feedback technique.[9] Sinha and Gupta discussed the adaptive controlling technique.[10] Zhang and Shen gave the controlling method of phase space compression.[11] De Sousa Vieira and Lichtenberg have presented the nonlinear feedback control.[12] In the recent decade, chaos synchronization has aroused a great deal of interest in potential applications. Many synchronization techniques have been very successful in low-dimensional systems.[13−20] Currently, synchronizing the spatiotemporal chaos has been investigated.[21−28] There are some great advantages of synchronizing spatiotemporal chaos in comparison with controlling temporal chaos. The information operations can be performed simultaneously and in parallel by many subunits. The applications of spatiotemporal chaos control and synchronization are extremely great and extensive. In the present paper we use a nonlinear feedback method that was early used in a low-dimensional temporally chaotic system.[12] We control and synchronize temporal chaos and spatiotemporal chaos in the Bragg acousto-optic bistable system. Our results show that this method works successfully in not only a temporally chaotic system and but also a spatiotemporally chaotic system. This method is easy to implement in experiment since the controlling term contains only an adjustable weight. No pre-knowledge of the dynamic system is required in implementation.

2. Controlling chaos in a Bragg acousto-optic bistable system

The Bragg acousto-optic bistable device consists

†Corresponding author, E-mail: ljyue@nenu.edu.cn

of a laser, acousto-optic modulator, driving source, photodiode, electronic amplifier, and electronic delay line. In theory, the evolution of the Bragg acousto-optic bistable system is described by the following equation:[18]

$$\tau_0 \frac{\mathrm{d}x(t)}{\mathrm{d}t} + x(t) = \pi\{A - \mu \sin^2[x(t-\tau_\mathrm{d}) - x_\mathrm{b}]\}, \quad (1)$$

where $x(t)$ is the state variable of the bistable system; μ is the bifurcation parameter (controlling parameter) that determines the evolution of the bistable system, and it is proportional to the input light intensity and the gain of the electronic amplifier; A and x_b are constants related to the voltage offsets of the amplifier and the driver; τ_0 and τ_d are the respective response time and the delay time. If the response time becomes much faster than the delay time ($\tau_0 << \tau_\mathrm{d}$), the dynamical equation (1) can be written as

$$x_{n+1} = f(x_n) = \pi[A - \mu \sin^2(x_n - x_\mathrm{b})], \quad (2)$$

where the parameters are $x_\mathrm{b} = \dfrac{\pi}{4}$, A= 0.5, and μ= 0.9. The Lyapunov exponent in Eq.(2) is 0.72581. The Bragg acousto-optic bistable system is chaotic. The nonlinear feedback method[12] is applied to the Bragg acousto-optic bistable system as follows:

$$x_{n+1} = (1-\gamma) f(x_n) + \gamma x_n, \quad (3)$$

where γ is the feedback strength. The schematic diagram of nonlinear feedback control is showed in Fig.1. The Bragg acousto-optic bistable system is steered to different periodic orbits by varying the feedback strength γ in Fig.2. Figures 2(a) and 2(b) show the controlled period 8 and period 4 as γ=0.125 and γ=0.15 respectively. Figures 2(c) and 2(d) show the controlled period 2 and period 1 as γ=0.20 and γ=0.40 respectively.

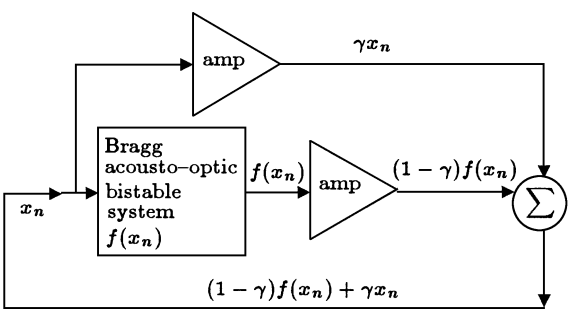


Fig.1. The schematic diagram of nonlinear feedback control, where Amp represents amplifier and $\sum$ addition.

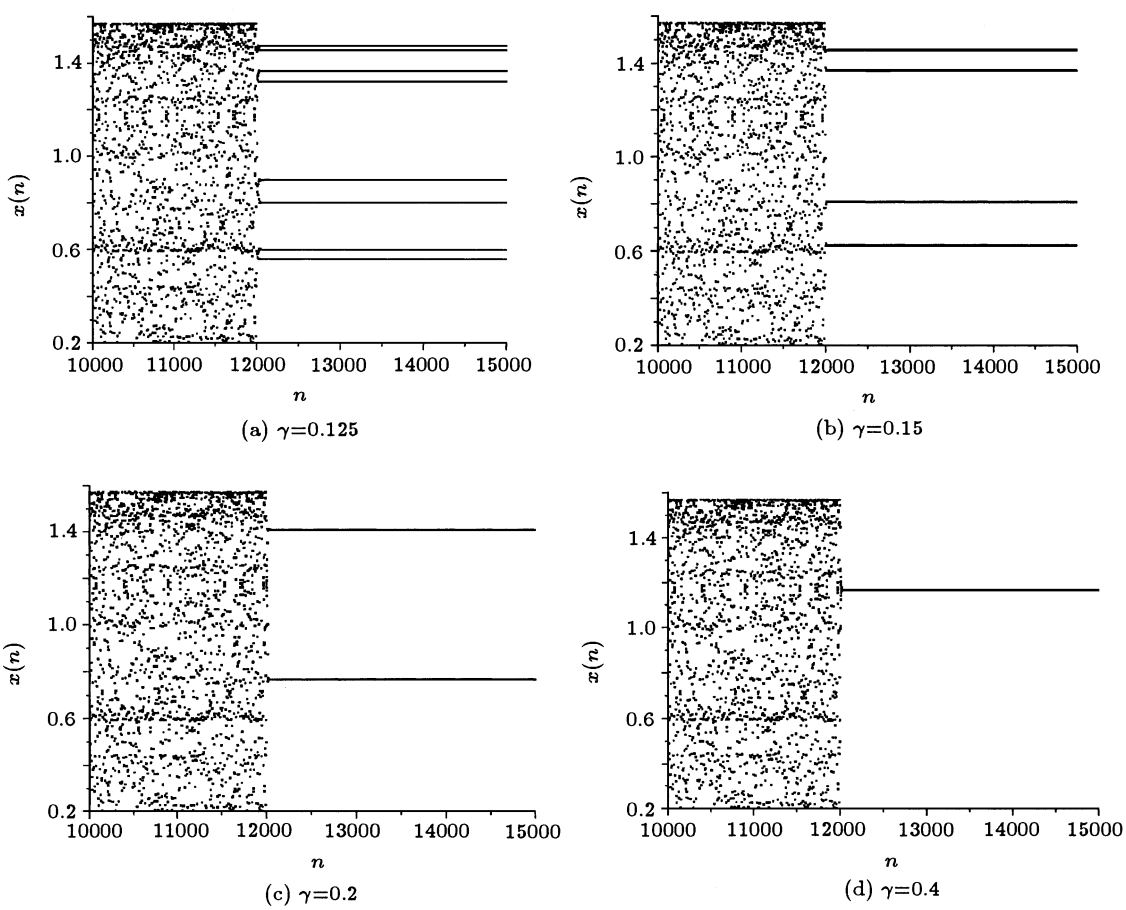


Fig.2. The control results of the Bragg acousto-optic bistable system.

3. Controlling turbulence in a coupled acousto-optic bistable map lattice system

Much attention has been paid to the relatively simple coupled map lattice (CML) model that is taken as a convenient tool to study the characteristics of spatiotemporally chaotic systems. The CML model is in the following general form:[1,2]

$$x_{n+1}(i) = (1-\varepsilon)f(x_n(i)) + \frac{\varepsilon}{2}(f(x_n(i+1)) + f(x_n(i-1))), \quad (4)$$

where n= 1,2, ..., N are the discrete time steps, i= 1,2,...,L are the lattice sites and L is the system size, ε is the coupling strength to the nearest neighbour sites, and $f(x_n)$ is the local dynamical function. Here $f(x_n)$ is chosen to be the Bragg acousto-optic bistable map system of Eq.(2), and x represents the state variable in the acousto-optic bistable system. Equation (4) is referred to as the coupled bistable map lattice (CBML) system. The largest Lyapunov exponent llyp is a very important characteristic parameter for studying the complex behaviours of a CML system. If the largest Lyapunov exponent is positive, the lattice state is temporally chaotic. If the largest Lyapunov exponent is negative, the lattice state is temporally periodic and spatially uniform. When the system parameters for the CBML system are A=0.5, μ=0.9, $x_b = \frac{\pi}{4}$, $\varepsilon = 0.2$, and L=100, the largest Lyapunov exponent spectrum λ_M is plotted in Fig.3. The CBML

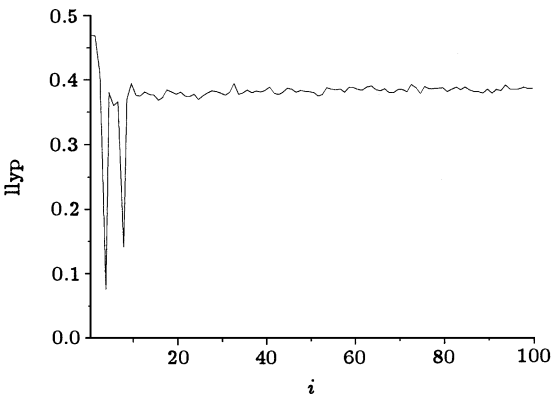


Fig.3. The largest Lyapunov exponent λ_M spectrum of the coupled Bragg acousto-optic bistable map lattice system at different lattice sites.

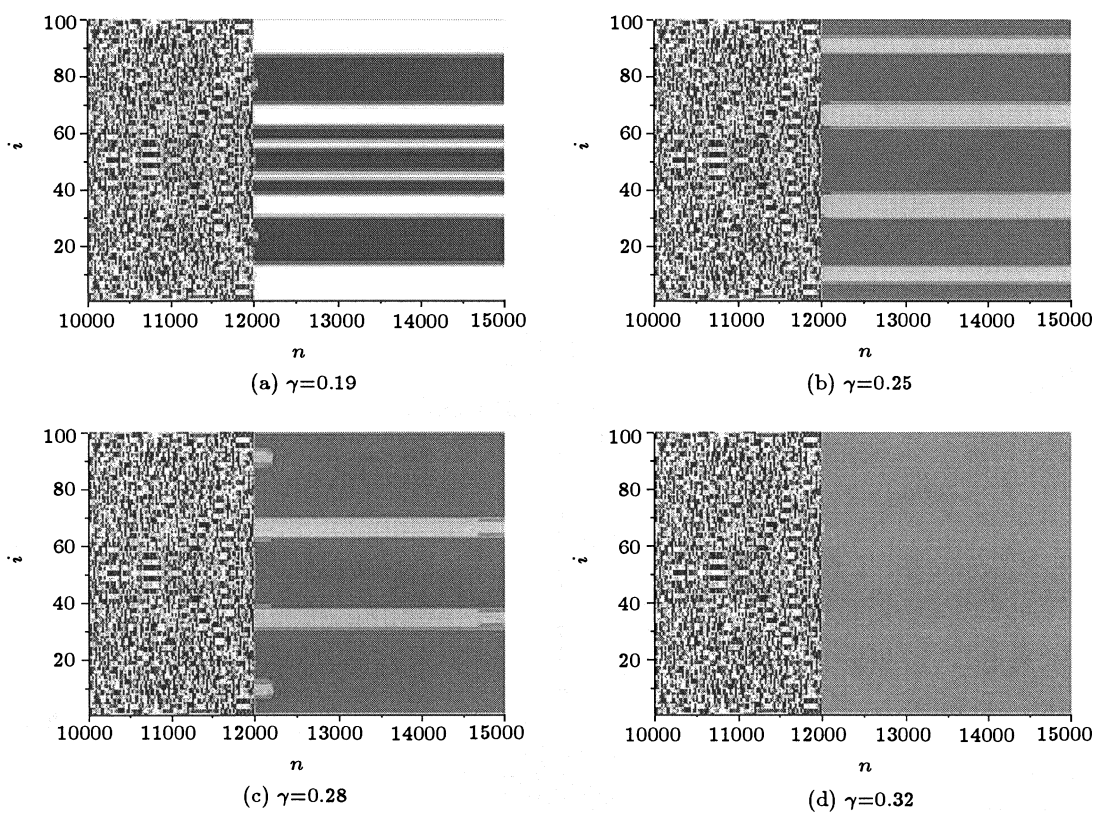


Fig.4. Control results of the coupled Bragg acousto-optic bistable map lattice.

system exhibits spatiotemporally chaotic behaviours. The nonlinear feedback control in the CBML system is represented by

$$x_{n+1}(i) = (1-\gamma)x_{n+1}(i) + \gamma x_n(i), \qquad (5)$$

where γ is the feedback strength. The scheme diagram of nonlinear feedback control is shown in Fig.1. The system parameters are the same as above, and the initial condition is $x_0(i) = 0.2$, and the boundary condition is $x_n(0) = x_n(L+1) = 0$. The turbulence of Eq.(5) is suppressed to stable periodic orbits by varying the feedback strength γ. Figures 4(a)–(d) show the space–time portraits with γ=0.19, 0.22, 0.25, 0.32, where the controls are initiated at $n = 12000$ and the temporal states with $n < 10000$ are left out, and the portraits every 8 steps are plotted along the time axis.

4. Synchronization of spatiotemporal chaos in coupled bistable map lattice systems

Chaos synchronization has shown many potential applications in engineering. Techniques for synchronizing chaos have been very successful in low-dimensional systems. Synchronizing spatiotemporal systems are facing a challenge, however, because the chaotic states in such systems are typically high dimensional and involve multiple stable or unstable modes. We consider two identical CBML systems but with different initial conditions, one of them is a drive system and the other is a response system. The drive and response systems are described respectively by the following equations:

$$x_{n+1}(i) = (1-\varepsilon)f(x_n(i)) + \frac{\varepsilon}{2}(f(x_n(i+1)) + f(x_n(i-1))), \qquad (6)$$

$$y_{n+1}(i) = (1-\varepsilon)f(y_n(i)) + \frac{\varepsilon}{2}(f(y_n(i+1)) + f(y_n(i-1))), \qquad (7)$$

where $x_n(i)$ denotes the drive system and $y_n(i)$ the response system. The local dynamical functions $f(x_n(i))$ and $f(y_n(i))$ are the same as those in the acousto-optic bistable map system of Eq.(2). We take the nonlinear feedback control method as follows:

$$y_{n+1}(i) = (1-\gamma)y_{n+1}(i) + \gamma x_{n+1}(i), \qquad (8)$$

where γ is the feedback strength. The schematic diagram of nonlinear feedback synchronization method is shown in Fig.5. The initial conditions of the drive and response systems are 0.2 and 0.21 respectively. The other parameters are the same as the above. The feedback strength γ is 0.35. The space–time portraits of the $x_n(i)$ system and the $y_n(i)$ system are shown in Figs.6(a) and 6(b) respectively. The difference in $x_n(i) - y_n(i)$ between the drive and response systems is plotted in Fig.6(c). The synchronization feedback is initiated at n= 7000 and the temporal states with $n < 5000$ are left out.

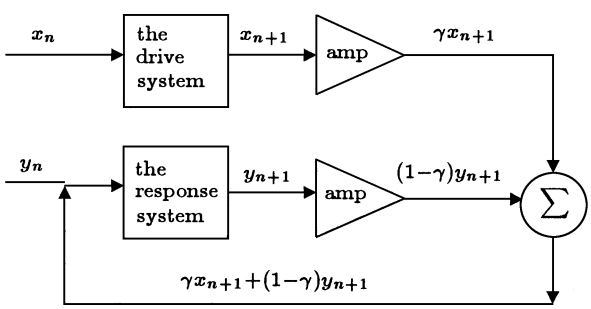


Fig.5. The scheme diagram of synchronization using the nonlinear feedback method.

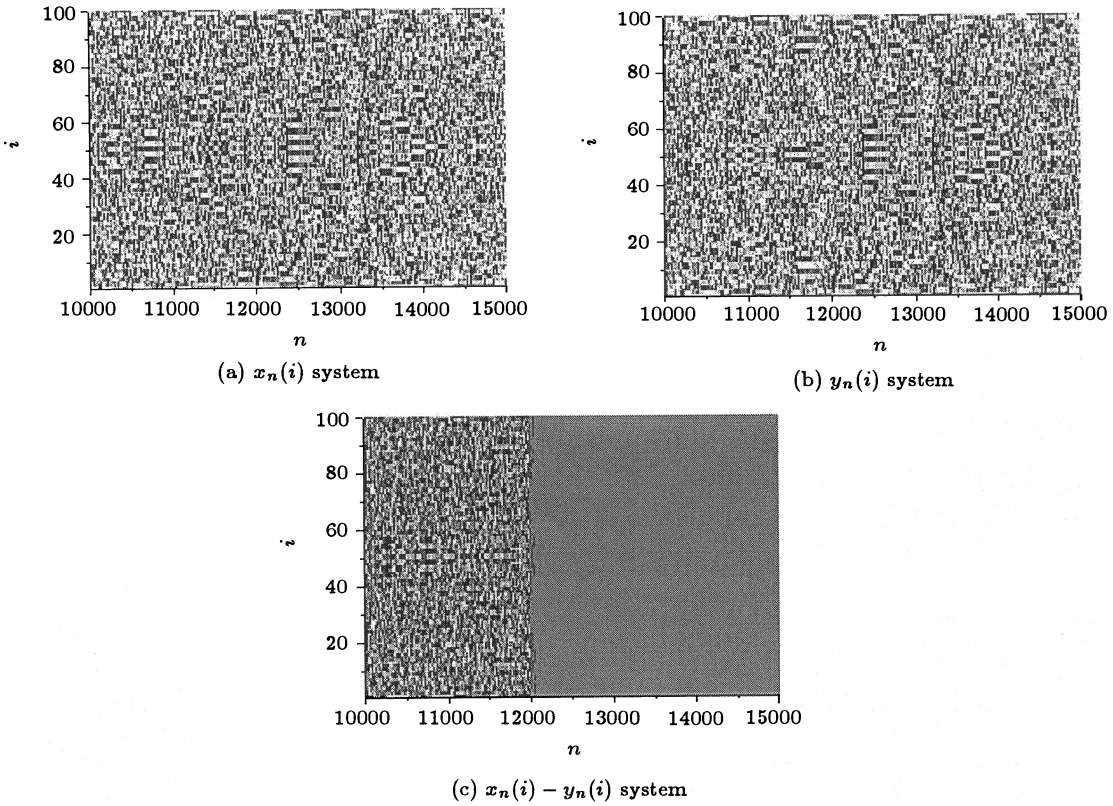


Fig.6. Space–time diagram for synchronizing the coupled Bragg acousto-optic bistable map system Eq.(7) when $\gamma = 0.35$.

The portraits every 8 steps are plotted along the time axis. Figure 6(c) shows that the difference in $x_n(i) - y_n(i)$ goes to zero, indicating successful synchronization between the drive and response systems. In order to check the validity of this method, the noise term $A_p\xi$ is added in the response system Eq.(7) as follows:

$$y_{n+1}(i) = (1-\gamma)y_{n+1}(i) + \gamma x_{n+1}(i) + A_{\mathrm{p}}\xi, \qquad (9)$$

where ξ is the random noise in a range (0,1) and A_{p} is the noise amplitude. We calculate the largest conditional Lyapunov exponents (λ_{MCLE}) while the random noise amplitude A_{p} is not more than 0.001 in Fig.7. The largest conditional Lyapunov exponents are negative as $\gamma > \gamma_{\mathrm{c}}$, so the synchronization can be realized. Figure 8(a) shows that the identity synchronization is not obtained as $\gamma = 0.35$. Figure 8(b) shows that the identity synchronization can be obtained as $\gamma = 0.58$. The response system is identical with the drive system by increasing the feedback strength γ. The numerical result indicates that this method is reliable.

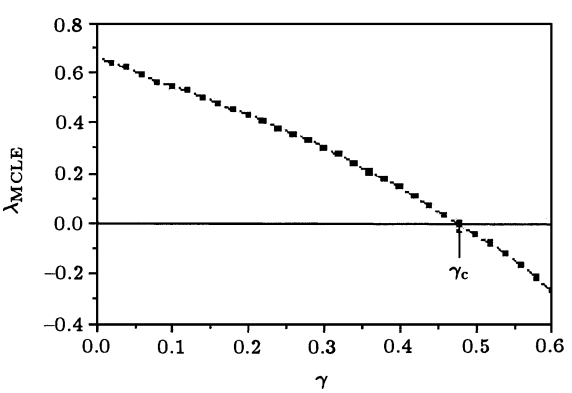


Fig.7. The largest conditional Lyapunov exponent spectrum adding a random noise ($A_{\mathrm{p}} < 0.001$).

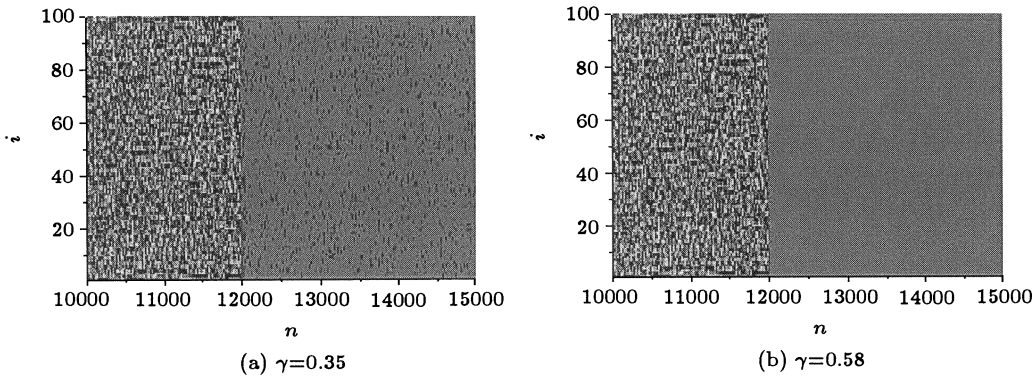


Fig.8. Space–time diagram of $x_n(i) - y_n(i)$ for the coupled Bragg acousto-optic bistable map system (7) adding a random noise ($A_{\rm p} < 0.001$).

5. Conclusion

We have presented the results of controlling and synchronizing the spatially coupled acousto-optic bistable map lattice systems using nonlinear feedback technology. This method consists of the feedback of the nonlinear mapping signals rather than the linearized signal around the fixed point. It can increase the range of the controlled and synchronized signals and decrease the sensitivity down to a noise level. The synchronization threshold can be obtained by analysing the largest conditional Lyapunov exponent spectrum, so the minimum feedback strength of achieving the synchronization can be obtained. This method is very important in that some systems have been affected by noise. This method can be also easily implemented in practical systems just by adjusting the coupled strength without any pre-knowledge of the dynamic system required. It is of particular importance in practical communications.

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