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Quantum radiation of non-stationary Kerr–Newman–de Sitter black hole*

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By introducing a new tortoise coordinate transformation, we investigate the quantum thermal and non-thermal radiations of a non-stationary Kerr–Newman–de Sitter black hole. The accurate location and radiate temperature of the event horizon as well as the maximum energy of the non-thermal radiation are derived. It is shown that the radiate temperature and the maximum energy are related to not only the evaporation rate, but also the shape of the event horizon, moreover the maximum energy depends on the electromagnetic potential. Finally, we use the results to reduce the non-stationary Kerr–Newman black hole, the non-stationary Kerr black hole, the stationary Kerr–Newman–de Sitter black hole, and the static Schwarzshild black hole.

Keywords: quantum thermal radiation, quantum non-thermal radiation, tortoise coordinate transformation, Kerr–Newman–de Sitter black hole

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1. Introduction

In 1973, Hawking proved that the temperature of black hole existed on the event horizon, and black hole could emit radiation from its event horizon.[1−3] Subsequently the non-thermal radiation near the event horizon was proved.[4−6] Since then, the important subject of revealing the thermal properties of various black holes has aroused much attention in black hole physics. In the past few decades, much effort[7−9] was devoted to the quantum radiation of scalar and Dirac fields in some static or stationary black holes. A realistic black hole in astrophysics must radiate or absorb matter surrounding it, so it is non-stationary and evolves in time. Thus, the thermal properties of non-stationary space–times are more interesting than those of static or stationary black holes, and more worth being studied. At present, quantum radiation by black holes has been studied extensively in the non-static and non-stationary space–times.[10−13] But the quantum radiation for the non-stationary Kerr–Newman–de Sitter black hole has been unknown to us, in which there exists a cosmological constant. In recent years, there has been a renewed interest in the cosmological constant as it is found to be present in the inflationary scenario of the early universe. Moreover, when solving the problems about the quantum radiation of black holes, one usually uses the generalized tortoise coordinate transformation. Recently, some people have found that by using a kind of new tortoise coordinate transformation the problems about the quantum thermal and non-thermal radiations near the event horizon can be also solved. In this paper, we use the definition of the event horizon to determine its locations, and then use a kind of new tortoise coordinate transformation to discuss the quantum thermal and non-thermal radiations of a non-stationary Kerr–Newman–de Sitter black hole. Finally we use the results to reduce the non-stationary Kerr–Newman black hole, the non-stationary Kerr black hole, the stationary Kerr–Newman–de Sitter black hole, and the static Schwarzshild black hole.

2. Space–time of non-stationary Kerr–Newman–de Sitter black hole

According to Refs.[14,15], the metric for non-stationary Kerr–Newman–de Sitter black hole in a re-

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tarded time u has the form

$$\begin{aligned}\mathrm{d}s^2 =&A\frac{1}{\Sigma}[\Delta_\lambda - \Delta_\theta a^2\sin^2\theta]\mathrm{d}u^2\\ &+2\sqrt{A}[\mathrm{d}u - a\sin^2\theta\mathrm{d}\varphi]\mathrm{d}r\\ &-\frac{\Sigma}{\Delta_\theta}\mathrm{d}\theta^2 + A\frac{2a}{\Sigma}[\Delta_\theta\\ &\times(r^2+a^2) - \Delta_\lambda]\sin^2\theta\mathrm{d}u\mathrm{d}\varphi\\ &-A\frac{1}{\Sigma}[\Delta_\theta(r^2+a^2)^2\\ &-\Delta_\lambda a^2\sin^2\theta]\sin^2\theta\mathrm{d}\varphi^2. \end{aligned}\tag{1}$$

By using the advanced time ν to replace the retarded time u, we can obtain the following metric for the non-stationary Kerr–Newman–de Sitter black hole in the advanced time ν:

$$\begin{aligned}\mathrm{d}s^2 =&A\frac{1}{\Sigma}[\Delta_\lambda - \Delta_\theta a^2\sin^2\theta]\mathrm{d}\nu^2\\ &-2\sqrt{A}[\mathrm{d}\nu - a\sin^2\theta\mathrm{d}\varphi]\mathrm{d}r\\ &-\frac{\Sigma}{\Delta_\theta}\mathrm{d}\theta^2 + A\frac{2a}{\Sigma}[\Delta_\theta(r^2+a^2)\\ &-\Delta_\lambda]\sin^2\theta\mathrm{d}\nu\mathrm{d}\varphi\\ &-A\frac{1}{\Sigma}[\Delta_\theta(r^2+a^2)^2\\ &-\Delta_\lambda a^2\sin^2\theta]\sin^2\theta\mathrm{d}\varphi^2, \end{aligned}\tag{2}$$

where

$$\begin{aligned}&\Delta_\lambda = r^2+a^2-2Mr+Q^2-\frac{1}{3}\Lambda r^2(r^2+a^2),\\ &A=\left[1+\frac{1}{3}\Lambda a^2\right]^{-2},\\ &\Delta_\theta = 1+\frac{1}{3}\Lambda a^2\cos^2\theta,\\ &\Sigma = r^2+a^2\cos^2\theta. \end{aligned}\tag{3}$$

Here Λ is the cosmological constant and $A = \left[1+(1/3)\,\Lambda a^2\right]^{-2}$ is a constant multiple to be determined. $M(\nu)$ and $Q(\nu)$ are the mass and charge of the non-stationary Kerr–Newman–de Sitter black hole respectively, they change with advanced time ν. The total angular momentum of the black hole is given by $aM(\nu)$ with a being a constant. From Eqs.(2) and (3), we can easily acquire the inverse tensors in Eq.(2) as follows:

$$\begin{aligned}&g^{00} = -\frac{1}{A}\frac{1}{\Sigma\Delta_\theta}a^2\sin^2\theta,\\ &g^{01} = g^{10} = -\frac{1}{\sqrt{A}}\frac{1}{\Sigma}(r^2+a^2),\\ &g^{03} = g^{30} = -\frac{1}{A}\frac{a}{\Sigma\Delta_\theta},\\ &g^{11} = -\frac{\Delta_\lambda}{\Sigma},\\ &g^{13} = g^{31} = -\frac{1}{\sqrt{A}}\frac{a}{\Sigma},\\ &g^{22} = -\frac{\Delta_\theta}{\Sigma},\\ &g^{33} = -\frac{1}{A\Sigma\Delta_\theta\sin^2\theta}, \end{aligned}$$

and

$$g = -A^2\Sigma^2\sin^2\theta.\tag{4}$$

The relevant electromagnetic potential is given as follows:

$$A_\mu = \left[\frac{Qr\sqrt{A}}{\Sigma},0,0,-\frac{Qr\sqrt{A}a\sin^2\theta}{\Sigma}\right].\tag{5}$$

The null tetrads are constructed[16] as follows:

$$\begin{aligned}&n_\mu = \frac{A}{\Sigma}(r^2+a^2)\left\{1,0,0,-a\sin^2\theta\right\},\\ &l_\mu = \frac{1}{r^2+a^2}\left\{\frac{\Delta_\lambda}{2},-\frac{\Sigma}{\sqrt{A}},0,-\frac{\Delta_\lambda}{2}a\sin^2\theta\right\},\\ &m_\mu = \Delta_\theta\left\{\sqrt{A}\mathrm{i}a\sin\theta,0,-\frac{\Sigma}{\Delta_\theta},-\sqrt{A}\mathrm{i}(r^2+a^2)\sin\theta\right\},\\ &\bar{m}_\mu = \frac{1}{2\Sigma}\left\{-\sqrt{A}\mathrm{i}a\sin\theta,0,-\frac{\Sigma}{\Delta_\theta},\sqrt{A}\mathrm{i}(r^2+a^2)\sin\theta\right\},\\ &n^\mu = \frac{\sqrt{A}\left(r^2+a^2\right)}{\Sigma}\left\{0,-1,0,0\right\},\\ &l^\mu = \frac{1}{A(r^2+a^2)}\left\{(r^2+a^2),\frac{\sqrt{A}\Delta_\lambda}{2},0,a\right\},\\ &m^\mu = \left\{\frac{\mathrm{i}a\sin\theta}{\sqrt{A}},0,\Delta_\theta,\frac{\mathrm{i}}{\sqrt{A}\sin\theta}\right\},\\ &\bar{m}^\mu = \frac{1}{2\Sigma\Delta_\theta}\left\{-\frac{\mathrm{i}a\sin\theta}{\sqrt{A}},0,\Delta_\theta,-\frac{\mathrm{i}}{\sqrt{A}\sin\theta}\right\}. \end{aligned}\tag{6}$$

The null tetrads (6) satisfy the null, orthogonal and metric conditions as follows:

$$\begin{aligned}&l^\mu l_\mu = n^\mu n_\mu = m^\mu m_\mu = \bar{m}^\mu\bar{m}_\mu = 0,\\ &l^\mu m_\mu = l^\mu\bar{m}_\mu = n^\mu m_\mu = n^\mu\bar{m}_\mu = 0,\\ &l^\mu n_\mu = -m^\mu\bar{m}_\mu = 1, \end{aligned}$$

and

$$g^{\mu\nu} = l^\mu n^\nu + n^\mu l^\nu - m^\mu\bar{m}^\nu - \bar{m}^\mu m^\nu.\tag{7}$$

The black hole district can be defined as the light-like district where no information can arrive, and the boundary area around the black hole district is called the event horizon. According to York's "definition of work"[17] (The event horizon should be light-like and the photon should stay on the event horizon for a long time). So, the event horizon should be determined by

$$\dot{r}_\mathrm{H}\approx 0,\quad\text{and}\quad\ddot{r}_\mathrm{H}\approx 0.\tag{8}$$

Because

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}\nu} =& l^a\nabla_a \\ =& \frac{1}{A}\frac{\partial}{\partial\nu} + \frac{\Delta_\lambda}{2\sqrt{A}(r^2+a^2)}\frac{\partial}{\partial r} \\ &+ \frac{a}{A(r^2+a^2)}\frac{\partial}{\partial\varphi},\end{aligned} \tag{9}$$

we have

$$\dot{r} = \frac{\mathrm{d}r}{\mathrm{d}\nu} = \frac{\Delta_\lambda}{2\sqrt{A}(r^2+a^2)}, \tag{10}$$

$$\begin{aligned}\ddot{r} =& \frac{\mathrm{d}^2r}{\mathrm{d}\nu^2} = \frac{\mathrm{d}\dot{r}}{\mathrm{d}\nu} \\ =& \frac{1}{A\sqrt{A}\,(r^2+a^2)}\left(Q\dot{Q} - \dot{M}r\right) + \dot{r}\Pi,\end{aligned} \tag{11}$$

where

$$\begin{aligned}\Pi =& \frac{1}{\sqrt{A}\,(r^2+a^2)}\left(r - M - \frac{2}{3}\Lambda r^3\right. \\ &\left. - \frac{1}{3}\Lambda a^2 r - \frac{\Delta_\lambda r}{2\sqrt{A}\,(r^2+a^2)}\right).\end{aligned} \tag{12}$$

As $\ddot{r}_{\mathrm{H}} \approx 0$ near the event horizon, we can obtain the value of $\dot{r}_{\mathrm{H}}$ on the event horizon

$$\dot{r}_{\mathrm{H}} = \frac{\left(\dot{M}r_{\mathrm{H}} - Q\dot{Q}\right)}{A\sqrt{A}\Pi\,(r_{\mathrm{H}}^2+a^2)}, \tag{13}$$

where $\dot{r}_{\mathrm{H}}$ represents the evaporation rate of non-stationary black hole near the event horizon. So, when $\dot{r}_{\mathrm{H}} > 0$, the outer event horizon is expanded gradually, and the black holes are called the absorbing black holes. On the contrary, when $\dot{r}_{\mathrm{H}} < 0$, the outer event horizon is contracted gradually, and the black holes are called the evaporating black holes. And substituting Eqs.(13) and (12) into Eq.(10), we can have

$$r_{\mathrm{H}}^4 - \left(\frac{3}{\Lambda} - a^2\right)r_{\mathrm{H}}^2 + \frac{6M_l}{\Lambda}r_{\mathrm{H}} - \frac{3}{\Lambda}\left(a^2+Q_l^2\right) = 0, \tag{14}$$

where

$$\begin{aligned}M_l =& M + \frac{1}{A\Pi}L_M, \quad Q_l^2 = Q^2 + \frac{2Q}{A\Pi}L_Q, \\ L_M =& \frac{\partial M}{\partial\nu}, \quad \text{and} \quad L_Q = \frac{\partial Q}{\partial\nu}.\end{aligned} \tag{15}$$

Under the following condition:

$$\begin{aligned}&\left(\frac{3}{\Lambda} - a^2\right) > 0, \quad M_l > 0, \\ &\left(\frac{3}{\Lambda} - a^2\right)^2 - \frac{36}{\Lambda}\left(a^2+Q_l^2\right) > 0,\end{aligned}$$

and

$$\begin{aligned}&\left[\left(\frac{3}{\Lambda} - a^2\right)^2 - \frac{36}{\Lambda}\left(a^2+Q_l^2\right)\right]^3 \\ >& \left[\left(\frac{3}{\Lambda} - a^2\right)^3 - \frac{108}{\Lambda}\left(a^2+Q_l^2\right)\left(\frac{3}{\Lambda} - a^2\right)\right. \\ &\left. + \frac{486M_l^2}{\Lambda^2}\right]^2.\end{aligned}$$

Eq.(14) has the following roots: when r_{H} is for the outer horizon, the root is

$$r_{\mathrm{H}} = r_+ = f_1^+ - f_2^+ + f_3^+; \tag{16}$$

when r_{H} is for the inner horizon, the root is

$$r_{\mathrm{H}} = r_- = -f_1^- - f_2^- + f_3^-; \tag{17}$$

when r_{H} is for the cosmological horizon, the root is

$$r_{\mathrm{H}} = r_{\mathrm{c}} = f_1^{\mathrm{c}} + f_2^{\mathrm{c}} + f_3^{\mathrm{c}}, \tag{18}$$

where

$$\begin{aligned}f_1^{\mathrm{H}} =& \left\{\frac{1}{6}\left(\frac{3}{\Lambda} - a^2\right) + \frac{1}{6}\left[\left(\frac{3}{\Lambda} - a^2\right)^2 - \frac{36}{\Lambda}\left(a^2+Q_l^2\right)\right]^{\frac{1}{2}}\cos\left(\frac{1}{3}\psi\right)\right\}^{\frac{1}{2}}, \\ f_2^{\mathrm{H}} =& \left\{\frac{1}{6}\left(\frac{3}{\Lambda} - a^2\right) - \frac{1}{6}\left[\left(\frac{3}{\Lambda} - a^2\right)^2 - \frac{36}{\Lambda}\left(a^2+Q_l^2\right)\right]^{\frac{1}{2}}\cos\left(\frac{1}{3}\psi + \frac{1}{3}\pi\right)\right\}^{\frac{1}{2}}, \\ f_3^{\mathrm{H}} =& \left\{\frac{1}{6}\left(\frac{3}{\Lambda} - a^2\right) - \frac{1}{6}\left[\left(\frac{3}{\Lambda} - a^2\right)^2 - \frac{36}{\Lambda}\left(a^2+Q_l^2\right)\right]^{\frac{1}{2}}\cos\left(\frac{1}{3}\psi - \frac{1}{3}\pi\right)\right\}^{\frac{1}{2}}, \\ \cos\psi =& \frac{-\left(\frac{3}{\Lambda} - a^2\right)^3 - 108\left(\frac{1}{\Lambda}\right)\left(\frac{3}{\Lambda} - a^2\right)\left(a^2+Q_l^2\right) + 486\frac{M_l^2}{\Lambda^2}}{\left[\left(\frac{3}{\Lambda} - a^2\right)^2 - \frac{36}{\Lambda}\left(a^2+Q_l^2\right)\right]^{3/2}}.\end{aligned} \tag{19}$$

From Eqs.(16), (17), and (18), we can obtain that the shape of each event horizon is of a spherical surface in the zero-order approximation. And by using this method to calculate the event horizon, $\dot{r}_{\rm H}$ cannot exist in the formula, and it directly gives the relationships between $r_{\rm H}$ and Q, M. So this method can give a more specific location of each event horizon than the traditional method where the null surface equation is used.

3. Research on the quantum thermal radiation by using a new tortoise coordinate transformation

In the curved space–time, the Klein–Gordon equation of the particle with the charge e and mass μ is

$$\frac{1}{\sqrt{-g}}\left(\frac{\partial}{\partial x^{\mu}}-{\rm i}eA_{\mu}\right)\left[\sqrt{-g}g^{\mu\nu}\left(\frac{\partial}{\partial x^{\nu}}-{\rm i}eA_{\nu}\right)\right]\Phi-\mu^2\Phi=0. \tag{20}$$

Substituting Eqs.(4) and (5) into Eq.(20) yields the Klein–Gordon equation under the Lorentz condition

$$\begin{aligned}&-\frac{a^2\sin^2\theta}{A\Sigma\Delta_{\theta}}\times\frac{\partial^2\Phi}{\partial\nu^2}-\frac{2(r^2+a^2)}{\sqrt{A}\Sigma}\times\frac{\partial^2\Phi}{\partial\nu\partial r}-\frac{2a}{A\Sigma\Delta_{\theta}}\times\frac{\partial^2\Phi}{\partial\nu\partial\varphi}-\frac{2a}{\sqrt{A}\Sigma}\times\frac{\partial^2\Phi}{\partial r\partial\varphi}-\frac{\Delta_{\theta}}{\Sigma}\times\frac{\partial^2\Phi}{\partial\theta^2}-\frac{\Delta_{\lambda}}{\Sigma}\times\frac{\partial^2\Phi}{\partial r^2}\\&-\frac{1}{A\Sigma\Delta_{\theta}\sin^2\theta}\times\frac{\partial^2\Phi}{\partial\varphi^2}-\frac{2r}{\sqrt{A}\Sigma}\times\frac{\partial\Phi}{\partial\nu}-\left[\frac{2{\rm i}eQr}{\Sigma}+\frac{2(r-M-(1/3)\Lambda r^3-(1/3)\Lambda a^2r)}{\Sigma}\right]\frac{\partial\Phi}{\partial r}\\&+\frac{(1/3)\Lambda a^2\sin2\theta\sin\theta-\cos\theta\Delta_{\theta}}{\Sigma\sin\theta}\frac{\partial\Phi}{\partial\theta}-\mu^2\Phi=0.\end{aligned} \tag{21}$$

Introducing a new tortoise coordinate transformation[18,19]

$$r_*=\frac{1}{2\kappa}\ln\left[r-r_{\rm H}(\nu,\theta)\right],\quad \nu_*=\nu-\nu_0,\quad\text{and}\quad\theta_*=\theta-\theta_0 \tag{22}$$

and applying the expression (22) to Eq.(21), we can have

$$\begin{aligned}&\left[\frac{a^2\sin^2\theta}{A\Delta_{\theta}}\times\frac{\dot{r}_{\rm H}^2}{2\kappa(r-r_{\rm H})}-\frac{2(r^2+a^2)}{\sqrt{A}}\times\frac{\dot{r}_{\rm H}}{2\kappa(r-r_{\rm H})}+\frac{\Delta_{\theta}r'^2_{\rm H}}{2\kappa(r-r_{\rm H})}+\frac{\Delta_{\lambda}}{2\kappa(r-r_{\rm H})}\right]\frac{\partial^2\Phi}{\partial r_*^2}\\&+2\left(\frac{r^2+a^2}{\sqrt{A}}-\frac{a^2\dot{r}_{\rm H}\sin^2\theta}{A\Delta_{\theta}}\right)\frac{\partial^2\Phi}{\partial\nu_*\partial r_*}-\left[\frac{\ddot{r}_{\rm H}(r-r_{\rm H})+\dot{r}_{\rm H}^2}{r-r_{\rm H}}\times\frac{a^2\sin^2\theta}{A\Delta_{\theta}}-\frac{2(r^2+a^2)}{\sqrt{A}}\times\frac{\dot{r}_{\rm H}}{r-r_{\rm H}}\right.\\&+\frac{\Delta_{\theta}[r'^2_{\rm H}+r''_{\rm H}(r-r_{\rm H})]}{r-r_{\rm H}}+\frac{\Delta_{\lambda}}{r-r_{\rm H}}+\frac{2r\dot{r}_{\rm H}}{\sqrt{A}}-2{\rm i}eQr-2(r-M-\frac{2}{3}\Lambda r^3-\frac{1}{3}\Lambda a^2r)\\&\left.-\frac{(1/3)\Lambda a^2\sin2\theta\sin\theta-\cos\theta\Delta_{\theta}}{\sin\theta}r'_{\rm H}\right]\frac{\partial\Phi}{\partial r_*}-\left(\frac{2a\dot{r}_{\rm H}}{A\Delta_{\theta}}-\frac{2a}{\sqrt{A}}\right)\frac{\partial^2\Phi}{\partial r_*\partial\varphi}-2r'_{\rm H}\Delta_{\theta}\frac{\partial^2\Phi}{\partial r_*\partial\theta_*}\\&+2\kappa(r-r_{\rm H})\left[\frac{a^2\sin^2\theta}{A\Delta_{\theta}}\times\frac{\partial^2\Phi}{\partial\nu_*^2}+\Delta_{\theta}\frac{\partial^2\Phi}{\partial\theta_*^2}+\frac{1}{A\Delta_{\theta}\sin^2\theta}\times\frac{\partial^2\Phi}{\partial\varphi^2}+\frac{2r}{\sqrt{A}}\times\frac{\partial\Phi}{\partial\nu_*}+\frac{2a}{A\Delta_{\theta}}\times\frac{\partial^2\Phi}{\partial\nu_*\partial\varphi}\right.\\&\left.+\Sigma\mu^2\Phi-\frac{(1/3)\Lambda a^2\sin2\theta\sin\theta-\cos\theta\Delta_{\theta}}{\sin\theta}\times\frac{\partial\Phi}{\partial\theta_*}\right]=0.\end{aligned} \tag{23}$$

In the vicinity of the event horizon, namely $r\to r_{\rm H}$, $\nu\to\nu_0$, and $\theta\to\theta_0$, we take the limits of each item in Eq.(23), we can arrive at

$$\alpha\frac{\partial^2\Phi}{\partial r_*^2}+2\frac{\partial^2\Phi}{\partial r_*\partial\nu_*}+2\Omega_{\rm H}\frac{\partial^2\Phi}{\partial r_*\partial\varphi}+2C_2\frac{\partial^2\Phi}{\partial r_*\partial\theta_*}+2(C_1+{\rm i}e\Phi_{\rm H})\frac{\partial\Phi}{\partial r_*}=0, \tag{24}$$

where

$$\alpha = \frac{-2\sqrt{A}\Delta_\theta r_{\rm H}\dot{r}_{\rm H} + A\Delta_\theta\left[r_{\rm H} - M - (2/3)\,\Lambda r_{\rm H}^3 - (1/3)\,\Lambda a^2 r_{\rm H}\right]}{\kappa\left[\sqrt{A}\Delta_\theta(r_{\rm H}^2 + a^2) - a^2\sin^2\theta_0\dot{r}_{\rm H}\right]}, \tag{25}$$

$$\Omega_{\rm H} = \frac{a\left(\dot{r}_{\rm H} - \sqrt{A}\Delta_\theta\right)}{a^2\dot{r}_{\rm H}\sin^2\theta_0 - \sqrt{A}\Delta_\theta(r_{\rm H}^2 + a^2)}, \tag{26}$$

$$\Phi_{\rm H} = \frac{A\Delta_\theta Q r_{\rm H}}{\sqrt{A}\Delta_\theta(r_{\rm H}^2 + a^2) - a^2\sin^2\theta_0\dot{r}_{\rm H}}, \tag{27}$$

$$C_1 = -\frac{\Phi_{\rm H}}{2\theta r_{\rm H}}\left[\frac{a^2\sin^2\theta\ddot{r}_{\rm H}}{A\Delta_\theta} - \frac{2r_{\rm H}r'_{\rm H}}{\sqrt{A}} + \Delta_\theta r_{\rm H} - \frac{(1/3)\Lambda a^2\sin 2\theta_0\sin\theta_0 - \cos\theta_0\Delta_\theta r'_{\rm H}}{\sin\theta_0}\right., \tag{28}$$

$$C_2 = \frac{Ar'_{\rm H}\Delta_\theta^2}{\dot{r}_{\rm H}a^2\sin^2\theta_0 - \sqrt{A}\Delta_\theta(r_{\rm H}^2 + a^2)}. \tag{29}$$

In order to reduce Eq.(24) to a standard form of wave equation near the event horizon, we adjust the parameter κ such that it satisfies

$$\frac{-2\sqrt{A}\Delta_\theta r_{\rm H}\dot{r}_{\rm H} + A\Delta_\theta\left[r_{\rm H} - M - (2/3)\,\Lambda r_{\rm H}^3 - (1/3)\,\Lambda a^2 r_{\rm H}\right]}{\kappa\left[\sqrt{A}\Delta_\theta(r_{\rm H}^2 + a^2) - a^2\sin^2\theta_0\dot{r}_{\rm H}\right]} = 1, \tag{30}$$

which means that the "surface gravity" of the horizon is

$$\kappa = \frac{-2\sqrt{A}\Delta_\theta r_{\rm H}\dot{r}_{\rm H} + A\Delta_\theta\left[r_{\rm H} - M - (2/3)\,\Lambda r_{\rm H}^3 - (1/3)\,\Lambda a^2 r_{\rm H}\right]}{\sqrt{A}\Delta_\theta(r_{\rm H}^2 + a^2) - \dfrac{a^2\sin^2\theta_0}{2(r_{\rm H}^2 + a^2)}\left(\dfrac{a^2 r_{\rm H}^2\sin^2\theta_0}{\sqrt{A}\Delta_\theta} + \sqrt{A}\Delta_{\rm H} + \sqrt{A}\Delta_\theta r'^2_{\rm H}\right)}, \tag{31}$$

where we use the null surface condition of non-stationary Kerr–Newman–de Sitter black hole. By substituting Eq.(30) into Eq.(24), and carrying out the separation variable to Eq.(24) in the following form:

$$\Phi = R(r_*)\Theta(\theta_*)\mathrm{e}^{\mathrm{i}\varsigma\varphi - i\omega\nu_*}, \tag{32}$$

in which ω is the Klein–Gordon particle's energy and ς is the projection of the Klein–Gordon particle's angular momentum on the φ axis, and by substituting Eqs.(30) and (32) into Eq.(24) , we can have

$$\begin{aligned}&\Theta' = \lambda\Theta,\\ &R'' + 2\left(A + \mathrm{i}\varsigma\Omega_{\rm H} + \mathrm{i}e\Phi_{\rm H} - \mathrm{i}\omega\right)R' = 0,\end{aligned} \tag{33}$$

where λ is a real constant used in the process of variable separation, and $A = \lambda C_2 + C_1$. So, Eq.(33) has the following solution:

$$\begin{aligned}&\Theta = c_1\mathrm{e}^{\lambda\theta_*},\\ &R = c_2\mathrm{e}^{2\mathrm{i}(\omega - \varsigma\Omega_{\rm H} - e\Phi_{\rm H})r_* - 2Ar_*} + c_3,\end{aligned} \tag{34}$$

where c_1, c_2, and c_3 are constants, θ_* is the polar angular. Solving Eq.(24) yields the radial wave function of the charged particles in-going and out-going non-stationary Kerr–Newman–de Sitter black hole:

$$\begin{aligned}&R_{\rm in} = R_{\rm in}^+ = \mathrm{e}^{-\mathrm{i}\omega\nu_*},\\ &R_{\rm out} = R_{\rm out}^+ = \mathrm{e}^{-\mathrm{i}\omega\nu_*}\mathrm{e}^{2\mathrm{i}(\omega - \varsigma\Omega_{\rm H} - e\Phi_{\rm H})r_*}\mathrm{e}^{-2Ar_*}.\end{aligned} \tag{35}$$

In Eq.(35), $R_{\rm in}^+$ is analytical on the outer event horizon, while $R_{\rm out}^+$ has a logarithm singularity. By using the Darmour–Ruffini stretch method of analysis, and extending it to the inside of event horizon, we have

$$\begin{aligned}\bar{R}_{\rm out}^+ =& \mathrm{e}^{-\mathrm{i}\omega\nu_*}\mathrm{e}^{2\mathrm{i}(\omega - \varsigma\Omega_{\rm H} - e\Phi_{\rm H})r_*}\mathrm{e}^{-2Ar_*}\\ &\times \mathrm{e}^{\frac{\mathrm{i}\pi A}{\kappa}}\mathrm{e}^{\frac{\pi(\omega - \varsigma\Omega_{\rm H} - e\Phi_{\rm H})}{\kappa}}.\end{aligned} \tag{36}$$

With the Damour–Ruffini–Sannan method, we obtain the following spectrum of out-going wave on the outer event horizon:

$$N_\omega^+ = \frac{1}{\mathrm{e}^{(\omega - \zeta\Omega_{\rm H} - e\Phi_{\rm H})/K_{\rm B}T^+} \pm 1}, \tag{37}$$

where

$$T^+ = \frac{\kappa}{2\pi} = \frac{-2\sqrt{A}\Delta_\theta r_{\rm H}\dot{r}_{\rm H} + A\Delta_\theta\left[r_{\rm H} - M - (2/3)\,\Lambda r_{\rm H}^3 - (1/3)\,\Lambda a^2 r_{\rm H}\right]}{2\pi\left[\sqrt{A}\Delta_\theta(r_{\rm H}^2 + a^2) - \dfrac{a^2\sin^2\theta_0}{2(r_{\rm H}^2 + a^2)}\left(\dfrac{a^2\dot{r}_{\rm H}^2\sin^2\theta_0}{\sqrt{A}\Delta_\theta} + \sqrt{A}\Delta_{\rm H} + \sqrt{A}\Delta_\theta r'^2_{\rm H}\right)\right]}. \tag{38}$$

In the same way, we have the spectrum of out-going wave on the cosmological event horizon as follows:

$$N_w^{\rm c}=\frac{1}{{\rm e}^{(\omega-\varsigma\Omega_{\rm H}-e\,\Phi_{\rm H})/K_{\rm B}T^{\rm c}}\pm1}, \tag{39}$$

where

$$T^{\rm c}=\frac{\kappa}{2\pi}=\frac{-2\sqrt{A}\Delta_\theta r_{\rm c}\dot{r}_{\rm c}+A\Delta_\theta\left[r_{\rm c}-M-(2/3)\,\Lambda r_{\rm c}^3-(1/3)\,\Lambda a^2 r_{\rm c}\right]}{2\pi\left[\sqrt{A}\Delta_\theta(r_{\rm c}^2+a^2)-\dfrac{a^2\sin^2\theta_0}{2(r_{\rm c}^2+a^2)}\left(\dfrac{a^2\dot{r}_{\rm H}^2\sin^2\theta_0}{\sqrt{A}\Delta_\theta}+\Delta_{\rm c}+\Delta_\theta r'^2_{\rm c}\right)\right]}. \tag{40}$$

In Eqs.(37) and (39), $\Omega_{\rm H}$ and $\Phi_{\rm H}$ are the dragging angular velocity and electro-magnetic potential of non-stationary Kerr–Newman–de Sitter black hole respectively. Obviously, Eqs.(37) and (39) give out the radiation spectrum of quantum thermal radiation of charged particles in the non-stationary Kerr–Newman–de Sitter black hole. Through Eqs.(26), (38), and (40), we can see that by using the new tortoise coordinate transformation the problems about the quantum thermal radiation can be also studied, and that the radiated temperature as well as the dragging angular velocity is related to not only the advanced time ν but also the angle θ. This is an interesting result.

4. Research on the quantum non-thermal radiation by using a new tortoise coordinate transformation

In the curved space–time, the Hamilton–Jacobi equation[20,21] of the particle with the charge e and mass μ_0 is

$$g^{\mu\nu}\left(\frac{\partial s}{\partial x^\mu}-eA_\mu\right)\left(\frac{\partial s}{\partial x^\nu}-eA_\nu\right)-\mu_0^2=0, \tag{41}$$

where $s\,(\nu,r,\theta,\varphi)$ is the Hamilton principal function, A_μ is an electromagnetic four-potential, and given by Eq.(5) for the axial-symmetry of the non-stationary Kerr–Newman–de Sitter black hole. By substituting the expressions (4) and (5) into Eq.(41), we can arrive at

$$\begin{aligned}&\frac{a^2\sin^2\theta}{A\Delta_\theta}\left(\frac{\partial s}{\partial\nu}\right)^2+\frac{2\left(r^2+a^2\right)}{\sqrt{A}}\frac{\partial s}{\partial\nu}\frac{\partial s}{\partial r}+\Delta_\lambda\left(\frac{\partial s}{\partial r}\right)^2\\&+\Delta_\theta\left(\frac{\partial s}{\partial\theta}\right)^2+\frac{1}{A\Delta_\theta\sin^2\theta}\left(\frac{\partial s}{\partial\varphi}\right)^2\\&+\frac{2a}{A\Delta_\theta}\frac{\partial s}{\partial\nu}\frac{\partial s}{\partial\varphi}+\frac{2a}{\sqrt{A}}\frac{\partial s}{\partial r}\frac{\partial s}{\partial\varphi}\\&-\frac{2e}{A\Delta_\theta}\left(a^2\sin^2\theta A_0+aA_3\right)\frac{\partial s}{\partial\nu}\\&+\frac{e^2A_3^2}{A\Delta_\theta\sin^2\theta}+\frac{a^2\sin^2\theta e^2A_0^2}{A\Delta_\theta}\\&-\frac{2e}{\sqrt{A}}\left[\left(r^2+a^2\right)A_0+aA_3\right]\frac{\partial s}{\partial\nu}\\&-\frac{2e}{A\Delta_\theta}\left(aA_0+\frac{A_3}{\sin^2\theta}\right)\frac{\partial s}{\partial\varphi}\\&+\frac{2ae^2A_0A_3}{A\Delta_\theta}+\mu_0^2\Sigma=0.\end{aligned} \tag{42}$$

We now introduce the new tortoise coordinate transformation as follows:

$$r_*=\frac{1}{2\kappa}\ln\left[r-r_{\rm H}\left(\nu,\theta\right)\right],$$

$$\nu_*=\nu-\nu_0,\quad\text{and}\quad\theta_*=\theta-\theta_0. \tag{43}$$

There exists the killing vector $\left(\dfrac{\partial s}{\partial\varphi}\right)^\alpha$ in the curved space–time represented by Eq.(2), so, we have

$$\frac{\partial s}{\partial\varphi}=\varsigma\ \ (\text{constant}). \tag{44}$$

Let

$$\frac{\partial s}{\partial\nu_*}=-\omega,\quad\text{and}\quad\frac{\partial s}{\partial\theta_*}=p, \tag{45}$$

where ω is the energy of a particle, and p is a component of the general momentum, and substitute the expression (43) into Eq.(42), then we will have

$$G\left(\frac{\partial s}{\partial r_*}\right)^2-2D\left(\frac{\partial s}{\partial r_*}\right)+2\kappa\left(r-r_{\rm H}\right)Y=0, \tag{46}$$

where

$$G=\frac{\dfrac{a^2\sin^2\theta}{A\Delta_\theta}\dot{r}_{\rm H}^2-\dfrac{2(r^2+a^2)}{\sqrt{A}}\dot{r}_{\rm H}+\Delta_\lambda+\Delta_\theta r'^2_{\rm H}}{2\kappa\,(r-r_{\rm H})}, \tag{47}$$

$$D=-\left[\frac{a^2\sin^2\theta}{A\Delta_\theta}\,\dot{r}_{\rm H}\omega-p\Delta_\theta r'_{\rm H}-\frac{\left(r^2+a^2\right)}{\sqrt{A}}\omega-\frac{a}{A\Delta_\theta}\dot{r}_{\rm H}\varsigma+\frac{a}{\sqrt{A}}\varsigma-\frac{e}{\sqrt{A}}\left(\left(r^2+a^2\right)A_0+aA_3\right)\right],$$

$$Y=\frac{a^2\sin^2\theta}{A\Delta_\theta}\omega^2+\Delta_\theta p^2+\frac{1}{A\Delta_\theta\sin^2\theta}\varsigma^2-\frac{2a}{A\Delta_\theta}\omega\varsigma+\mu_0^2\Sigma. \tag{48}$$

In Eq.(46), each item is multiplied by the factor $\dfrac{A\Delta_\theta}{\sqrt{A}\Delta_\theta\left(r_{\rm H}^2+a^2\right)-a^2\dot{r}_{\rm H}\sin^2\theta}$. Obviously, in the vicinity of the event horizon, the coefficient of the first item in Eq.(46) is of $\dfrac{0}{0}$−type. We adjust the parameter κ such that it satisfies

$$\frac{-2\sqrt{A}\Delta_\theta r_{\rm H}\dot{r}_{\rm H}+A\Delta_\theta\left[r_{\rm H}-M-(2/3)\,\Lambda r_{\rm H}^3-(1/3)\,\Lambda a^2 r_{\rm H}\right]}{\kappa\left[\sqrt{A}\Delta_\theta(r_{\rm H}^2+a^2)-a^2\sin^2\theta_0\dot{r}_{\rm H}\right]}=1. \tag{49}$$

By solving Eq.(49), we can obtain the "surface gravity" of the horizon

$$\kappa=\frac{-2\sqrt{A}\Delta_\theta r_{\rm H}\dot{r}_{\rm H}+A\Delta_\theta\left[r_{\rm H}-M-(2/3)\,\Lambda r_{\rm H}^3-(1/3)\,\Lambda a^2 r_{\rm H}\right]}{\sqrt{A}\Delta_\theta(r_{\rm H}^2+a^2)-\dfrac{a^2\sin^2\theta_0}{2(r_{\rm H}^2+a^2)}\left(\dfrac{a^2r_{\rm H}^2\sin^2\theta_0}{\sqrt{A}\Delta_\theta}+\sqrt{A}\Delta_{\rm H}+\sqrt{A}\Delta_\theta r'^2_{\rm H}\right)}. \tag{50}$$

The solutions of Eq.(46) can be written as

$$\frac{\partial s}{\partial r_*}=\frac{1}{G}\left(D\pm\sqrt{D^2-2\kappa\,(r-r_{\rm H})\,GY}\right). \tag{51}$$

Since s and $\dfrac{\partial s}{\partial r_*}$ cannot be imaginary numbers, we have

$$D^2-2\kappa\,(r-r_{\rm H})\,GY\geq 0. \tag{52}$$

By solving Eq.(52), we obtain

$$\omega^{\pm}=\frac{-C_2\pm\sqrt{C_2^2-C_1C_3}}{C_1}, \tag{53}$$

where

$$C_1=\left(\frac{a^2\sin^2\theta}{A\Delta_\theta}\dot{r}_{\rm H}-\frac{r^2+a^2}{\sqrt{A}}\right)^2-2\kappa\,(r-r_{\rm H})\,\frac{a^2\sin^2\theta}{A\Delta_\theta}G,$$

$$C_2=\left(\frac{a^2\sin^2\theta}{A\Delta_\theta}\dot{r}_{\rm H}-\frac{r^2+a^2}{\sqrt{A}}\right)\left[-p\Delta_\theta r'_{\rm H}-\frac{a}{A\Delta_\theta}\dot{r}_{\rm H}\varsigma+\frac{a}{\sqrt{A}}\varsigma-\frac{e}{\sqrt{A}}\left(\left(r^2+a^2\right)A_0+aA_3\right)\right]+2\kappa\,(r-r_{\rm H})\left(\frac{1}{A\Delta_\theta}\right)Ga\varsigma,$$

$$C_3=\left[p\Delta_\theta r'_{\rm H}+\frac{a}{A\Delta_\theta}\dot{r}_{\rm H}\varsigma-\frac{a}{\sqrt{A}}\varsigma+\frac{e}{\sqrt{A}}((r^2+a^2)A_0+aA_3)\right]^2-2\kappa(r-r_{\rm H})G[\Delta_\theta p^2+\frac{\varsigma^2}{A\Delta_\theta\sin^2\theta}-\mu_0^2\left(r^2+a^2\cos^2\theta\right)]. \tag{54}$$

By considering the sign of inequality in Eq.(52), we have

$$\omega>\omega^+\quad\text{and}\quad\omega<\omega^-. \tag{55}$$

In the non-stationary Kerr–Newman–de Sitter space–time, the Dirac energy level distribution is described exactly in the expressions (53)–(55), but the energy states that satisfy $\omega^-<\omega<\omega^+$ are in a forbidden region. In the vicinity of the event horizon, namely, when $r\to r_{\rm H}$, $\nu\to\nu_0$, and $\theta\to\theta_0$, we can obtain the maximum energy from Eqs.(14), (53) and (54):

$$\lim_{r\to r_{\rm H}}\omega^{\pm}=\omega^0=\frac{\sqrt{A}a\Delta_\theta\varsigma-a\dot{r}_{\rm H}\varsigma-Ap\Delta_\theta^2 r'_{\rm H}-\sqrt{A}\Delta_\theta e\left[\left(r_{\rm H}^2+a^2\right)A_0+aA_3\right]}{\sqrt{A}\Delta_\theta\left(r_{\rm H}^2+a^2\right)-a^2\sin^2\theta\dot{r}_{\rm H}}, \tag{56}$$

which means that the width of the forbidden region vanishes on the event horizon, and positive and negative energy levels are crossed near the event horizon. When $\omega^0 > \mu_0$, the Starbinsky–Unruh process must occur. This means that there appears a non-thermal quantum radiation from the region near the event horizon, the maximum energy of a particle is ω^0, while the energy of the radiated particle is $\mu_0 < \omega \leq \omega^0$. Through Eq.(56), we can easily see that this radiation is irrelevant to temperature, which differs from the thermal radiation given by Eqs.(37) and (39). Equation (56) indicates that the maximum energy ω^0 of a particle varies with radiating direction and the evaporation rate as well as the four-potential, correspondingly, the energy of the radiated particle changes with them.

5. Discussion

(1) Through Eqs.(31) and (50), we can conclude that both Hamilton–Jacobi equation and Klein–Gordon equation can give the "surface gravity" of the event horizon. In both radial equations, the coefficient of $\left(\frac{\partial s}{\partial r_*}\right)^2$ is equal to the coefficient of $\frac{\partial^2 \Phi}{\partial r_*^2}$. As $r \to r_{\rm H}$, $\nu \to \nu_0$, and $\theta \to \theta_0$, the coefficient is of $\frac{0}{0}-$ type. By calculating the limit of the two coefficients, and taking the limit value as 1, we can obtain the value of κ. By applying the formula of the thermal radial temperature $(\kappa/2\pi K_{\rm B})$, we can also acquire the thermal radiation temperature of the black hole.[22–24] From the above discussion, we can see by using a new tortoise coordinate transformation the problems about the quantum thermal and non-thermal radiations near the event horizon can be also solved, which usually are solved by using the generalized tortoise coordinate transformation.[25–28]

(2) When $\Lambda = 0$, substituting it into Eqs.(26), (31), and (56) yields

$$\kappa = \frac{2(r_{\rm H}^2 + a^2)\left[-2r_{\rm H}\dot{r}_{\rm H} + (r_{\rm H} - M)\right]}{2(r_{\rm H}^2 + a^2)^2 - a^2\sin^2\theta_0\left(a^2\dot{r}_{\rm H}^2\sin^2\theta_0 + r_{\rm H}^2 + a^2 - 2Mr_{\rm H} + Q^2 + r'^2_{\rm H}\right)}, \tag{57}$$

$$\Omega_{\rm H} = \frac{a\left(1 - \dot{r}_{\rm H}\right)}{r_{\rm H}^2 + a^2 - a^2\sin^2\theta\dot{r}_{\rm H}}, \tag{58}$$

$$\omega^0 = \frac{a\varsigma - a\dot{r}_{\rm H}\varsigma - pr'_{\rm H} - e\left[(r_{\rm H}^2 + a^2)A_0 + aA_3\right]}{(r_{\rm H}^2 + a^2) - a^2\sin^2\theta\dot{r}_{\rm H}}, \tag{59}$$

which are the results describing the non-stationary Kerr–Newman black hole, so under the circumstances, the metric of the non-stationary Kerr–Newman–de Sitter black hole comes back to that of the non-stationary Kerr–Newman black hole. And it is interesting that the "surface gravity" and the dragging angular velocity of the event horizon as well as the maximum energy of a radiated particle are related to not only the advanced time ν but also the angle θ, so the radiated temperature $(\kappa/2\pi K_{\rm B})$ on the event horizon also changes with them.

(3) When $\Lambda = 0$ and $Q = 0$, substituting them into Eqs.(26), (31), and (56) can give

$$\kappa = \frac{2(r_{\rm H}^2 + a^2)\left[-2r_{\rm H}\dot{r}_{\rm H} + (r_{\rm H} - M)\right]}{2(r_{\rm H}^2 + a^2)^2 - a^2\sin^2\theta_0\left(a^2\dot{r}_{\rm H}^2\sin^2\theta_0 + r_{\rm H}^2 + a^2 - 2Mr_{\rm H} + r'^2_{\rm H}\right)}, \tag{60}$$

$$\Omega_{\rm H} = \frac{a\left(1 - \dot{r}_{\rm H}\right)}{r_{\rm H}^2 + a^2 - a^2\sin^2\theta\dot{r}_{\rm H}}, \tag{61}$$

$$\omega^0 = \frac{a\varsigma - a\dot{r}_{\rm H}\varsigma - pr'_{\rm H}}{r_{\rm H}^2 + a^2 - a^2\sin^2\theta\dot{r}_{\rm H}}, \tag{62}$$

which are the results describing the non-stationary Kerr black hole. Equations (60), (61), and (62) give the "surface gravity" and the dragging angular velocity of the event horizon as well as the maximum energy of a radiate particle, and we can easily know that all of these parameters depend on not only the advanced

time ν but also the angle θ, so the radiated temperature ($\kappa/2\pi K_{\rm B}$) on the event horizon changes with them.

(4) When $\dot{r}_{\rm H}=0$ and $r'_{\rm H}=0$, substituting them into Eqs.(26), (31), and (56) can give

$$\kappa=\frac{2A\Delta_\theta(r_{\rm H}^2+a^2)\left[r_{\rm H}-M-(2/3)\,\Lambda r_{\rm H}^3-(1/3)\,\Lambda a^2 r_{\rm H}\right]}{2\sqrt{A}\Delta_\theta(r_{\rm H}^2+a^2)^2-\sqrt{A}a^2\sin^2\theta_0\Delta_{\rm H}}, \tag{63}$$

$$\Omega_{\rm H}=\frac{a}{r_{\rm H}^2+a^2}, \tag{64}$$

$$\omega^0=\frac{\sqrt{A}a\Delta_\theta\varsigma-\sqrt{A}\Delta_\theta e\left[\left(r_{\rm H}^2+a^2\right)A_0+aA_3\right]}{\sqrt{A}\Delta_\theta\left(r_{\rm H}^2+a^2\right)}, \tag{65}$$

which reduce the stationary Kerr–Newman–de Sitter space–time result, where $r_{\rm H}$ is given by the null surface equation of the stationary Kerr–Newman–de Sitter black hole. So the "surface gravity" and the dragging angular velocity of the event horizon as well as the maximum energy of a radial particle are constants.

(5) When $\Lambda=0$, $a=0$, $Q=0$, $\dot{r}_{\rm H}=0$, and $r'_{\rm H}=0$, under the null surface condition of the static Schwarzshild black hole, by substituting them into Eqs.(26), (31), and (56) we can have

$$\kappa=\frac{1}{4M}, \tag{66}$$

$$\Omega_{\rm H}=0, \quad \text{and} \quad \omega^0=0. \tag{67}$$

From Eq.(67), we can obtain that the dragging angular velocity of the event horizon as well as the maximum energy of a radiated particle is zero in the static Schwarzshild black hole. And the quantum non-thermal radiation cannot occur there, while the quantum thermal radiation is still carried on; the "surface gravity" of the event horizon is given by Eq.(66). The maximum energy of a radiated particle is related to the dragging angular velocity of the event horizon.[29–31]

References

[1] Hawking S W 1974 *Nature* **30** 248
Hawking S W 1975 *Commun. Math. Phys.* **43** 199
[2] Birrell N D and Davies P C W 1982 *Quantum Field Theory in the Curved Space–time* (Cambridge: Cambridge University Press)
[3] Gibbons G W and Hawking S W 1977 *Phys. Rev.* D **15** 2752
[4] Misner C W and Bull Am 1972 *Phys. Soc.* (II) **17** 472
[5] Zeldovich Y B 1972 *JETP* **35** 1085
[6] Unruh W G 1974 *Phys. Rev.* D **10** 3194
[7] Darmour T and Ruffini R 1976 *Phys. Rev.* D **14** 332
[8] Liu L and Xu D Y 1980 *Acta Phys. Sin.* **29** 1617 (in Chinese)
Xu D Y 1982 *Acta Phys. Sin.* **32** 225 (in Chinese)
Xu C M 1982 *Acta Phys. Sin.* **31** 207 (in Chinese)
Zhao Z, Gui Y X and Liu L 1981 *Acta Astrophys. Sin.* **1** 141 (in Chinese)
[9] Yang S Z and Zhao Z 1994 *J. Beijing Normal University* (Natural Science) **30** 221 (in Chinese)
[10] Yang S Z and Zhao Z 1996 *Inter. J. Theor. Phys.* **35** 2455
[11] Yang S Z and Lin L B 2002 *Chin. Phys.* **11** 619
[12] Yang S Z 1995 *Acta Phys. Sin.* **44** 373 (in Chinese)
[13] Li H L and Jiang Q Q 2004 *J. Chin. West. Normal University* (Natural Science) **25** 353 (in Chinese)
[14] Xu D Y 1998 *Class. Quant. Grav.* **15** 153
Xu D Y 1999 *Class. Quant. Grav.* **16** 343
[15] Hossain Ali M 2004 *Gen. Relat. Grav.* **36** 1171
[16] Newman E T and Penrose R 1962 *J. Math. Phys.* **3** 566
[17] Zhao Z 1999 *Thermal Property of Black Hole and Singularity of Space–time* (Beijing: Beijing Normal University Press) (in Chinese) p230
[18] Qiang L E, Gao X Q and Zhao Z 2004 *Acta Phys. Sin.* **53** 3624 (in Chinese)
[19] Zhang J Y and Zhao Z 2003 *Acta Phys. Sin.* **52** 2096 (in Chinese)
[20] Damour T 1997 *In Proceedings of the First Marcel Grossmann Meeting on General Relativity* (Amsterdam: North-Holland) p476
[21] Yang S Z 2004 *Acta Phys. Sin.* **53** 4007 (in Chinese)
[22] Ma Y and Yang S Z 1997 *Acta Phys. Sin.* **46** 2280 (in Chinese)
[23] Zhang J Y 1997 *Acta Phys. Sin.* **46** 2294 (in Chinese)
[24] Li Z H and Mi L Q 1999 *Acta Phys. Sin.* **48** 575 (in Chinese)
[25] Liu W B and Li X 1999 *Acta Phys. Sin.* **48** 992 (in Chinese)
[26] Wang B B and Liu L 2002 *Acta Phys. Sin.* **51** 1654 (in Chinese)
[27] Zhao Z and Zhu J Y 1999 *Acta Phys. Sin.* **48** 1558 (in Chinese)
[28] Wang D X 1999 *Acta Phys. Sin.* **48** 1552 (in Chinese)
[29] Zhao Z, Liu W B and Jiang Y L 2000 *Acta Phys. Sin.* **49** 379 (in Chinese)
[30] Bu H and Liu L 1998 *Acta Phys. Sin.* **46** 2280 (in Chinese)
[31] Wu S Q and Cai M 2002 *Acta Phys. Sin.* **11** 327 (in Chinese)