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Probabilistic remote preparation of a two-atom entangled state*

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A scheme for remotely preparing a two-atom entangled state via entanglement swapping in cavity quantum electronic dynamics (QED) with the help of separate measurements is proposed. And the effect of cavity decay is eliminated in our scheme.

Keywords: remote state preparation, entanglement swapping, cavity QED, two-atom entangled state
PACC: 0367, 0365

Quantum entanglement is one of the most significant features of quantum mechanics and has been an essential resource of quantum information processing, such as quantum teleportation,[1−8] quantum cryptography,[9] quantum computation[10] and so on. In quantum teleportation, an unknown qubit state can be transmitted from a sender to a spatially separate receiver via an entangled quantum channel using local operations and classical communication. Recently, an interesting application of multiparticle entanglement has been found,[11−13] which is remote state preparation (RSP). In RSP, Alice is supposed to know the state that is to be prepared remotely. While in quantum teleportation, neither Alice nor Bob knows the state that is to be teleported. Because of the property of pre-knowledge of the quantum state that is to be prepared in RSP, the communication cost will be reduced in the course of entanglement and classical communication. A comparison of remote state preparation with quantum teleportation shows the former[11−16] to be the strong trade-off in cost between the required entanglement and the classical communication.

More recently, more theoretical and experimental protocols on the generalization of RSP have been presented.[17−23] Liu and Wang have proposed schemes for remote preparation of a chosen two-particle entangled state[24] and probabilistic remote state preparation of a general qubit.[25] Yu *et al*[26] have presented a scheme to prepare remotely two instances of quantum state. But contributions on discussing remote preparation of atomic state are less.[27] So it is necessary to explore the schemes of remotely preparing atomic states. Here we present a scheme for remote preparation of a two-atom entangled state by Alice and Bob sharing two two-atom entangled states via entanglement swapping in cavity quantum electronic dynamics (QED), and the separate measurements are needed in realizing the entanglement swapping.[28]

Suppose that the sender Alice wishes to help the receiver Bob remotely prepare a two-atom entangled state

$$|\phi\rangle = \alpha|ee\rangle + \beta|gg\rangle, \tag{1}$$

where the parameters α and β are real numbers, they satisfy $|\alpha|^2+|\beta|^2 = 1$, and the state in Eq.(1) is known completely to Alice but unknown to Bob. We also suppose that Alice and Bob share the following two two-atom entangled states:

$$|\phi\rangle_{12} = a|ee\rangle_{12} + b|gg\rangle_{12}, \tag{2}$$

$$|\phi\rangle_{34} = c|ee\rangle_{34} + d|gg\rangle_{34}, \tag{3}$$

where coefficients a, b, c and d are real numbers, they satisfy $|a|^2 + |b|^2 = 1$, and $|b|$ is smaller than $|a|$, $|c|^2 + |d|^2 = 1$, and $|d|$ is smaller than $|c|$. $|e\rangle_j$ and

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$|g\rangle_j$ ($j = 1, 2, 3, 4$) are the excited and ground states of the jth atom, respectively. The atoms 1 and 3 belong to Alice, and the atoms 2 and 4 belong to Bob. Furthermore the atoms 1, 2, 3, 4 are identical.

Alice will send the two atoms 1, 3 into an optical cavity which is introduced beforehand. The two atoms will be driven simultaneously by a classical field in the cavity. The interaction Hamiltonian of atom-cavity can be expressed as

$$\begin{aligned} H =&\omega_0 \sum_{j=1}^{2} S_{z,j} + \omega_1 a^\dagger a \\ &+ \sum_{j=1}^{2} [g(a^\dagger S_j^- + a S_j^\dagger) \\ &+ \Omega(S_j^\dagger e^{-\mathrm{i}\omega_2 t} + S_j^- \mathrm{e}^{\mathrm{i}\omega_2 t})], \end{aligned} \tag{4}$$

where $a^\dagger$ and a are the creation operator and annihilation operator of the cavity mode, g is the atom-cavity coupling strength, $S_j^- = |g\rangle_{jj}\langle e|$, $S_j^\dagger = |e\rangle_{jj}\langle g|$ and $S_{z,j} = \frac{1}{2}(|e\rangle_{jj}\langle e| - |g\rangle_{jj}\langle g|)$ are atomic operators. ω_0, ω_1, ω_2 and Ω are atomic transition frequency ($e \leftrightarrow g$), cavity frequency, driving field frequency and the Rabi frequency of the classical field, respectively. In the case of large detuning $\delta \gg g/2$ (g limit) and strong driving field $2\Omega \gg \delta$, the effective Hamiltonian of this interaction system is[29]

$$\begin{aligned} H_{\mathrm{eff}} =&\lambda \Bigg[\frac{1}{2} \sum_{j=1}^{2} (|e\rangle_{jj}\langle e| + |g\rangle_{jj}\langle g|) \\ &+ \sum_{j,k=1, j\neq k}^{2} (S_j^\dagger S_k^\dagger + S_j^\dagger S_k^- + \mathrm{H.c.}) \Bigg], \end{aligned} \tag{5}$$

where $\lambda = g^2/2\delta$, $\delta = \omega_0 - \omega_1$ and $\omega_0 = \omega_2$. The effects of cavity decay and thermal field are eliminated since there is no energy exchange between atoms and cavity mode. The evolution operator of the system in an interaction picture is[29]

$$U(t) = \mathrm{e}^{-\mathrm{i}H_0 t} e^{-\mathrm{i}H_{\mathrm{eff}} t}, \tag{6}$$

where $H_0 = \sum_{j=1}^{2} \Omega(S_j^\dagger + S_j^-)$.

The evolution of total system can be described as

$$\begin{aligned} &(a|ee\rangle_{12} + b|gg\rangle_{12})(c|ee\rangle_{34} + d|gg\rangle_{34}) \\ &\rightarrow ac\mathrm{e}^{-\mathrm{i}\lambda t}|ee\rangle_{24}[\cos\lambda t(\cos\Omega t|e\rangle_1 - \mathrm{i}\sin\Omega t|g\rangle_1) \times (\cos\Omega t|e\rangle_3 - \mathrm{i}\sin\Omega t|g\rangle_3) \\ &- \mathrm{i}\sin\lambda t(\cos\Omega t|g\rangle_1 - \mathrm{i}\sin\Omega t|e\rangle_1) \times (\cos\Omega t|g\rangle_3 - \mathrm{i}\sin\Omega t|e\rangle_3)] \\ &+ ad\mathrm{e}^{-\mathrm{i}\lambda t}|eg\rangle_{24}[\cos\lambda t(\cos\Omega t|e\rangle_1 - \mathrm{i}\sin\Omega t|g\rangle_1) \times (\cos\Omega t|g\rangle_3 - \mathrm{i}\sin\Omega t|e\rangle_3) \\ &- \mathrm{i}\sin\lambda t(\cos\Omega t|g\rangle_1 - \mathrm{i}\sin\Omega t|e\rangle_1) \times (\cos\Omega t|e\rangle_3 - \mathrm{i}\sin\Omega t|g\rangle_3)] \\ &+ bc\mathrm{e}^{-\mathrm{i}\lambda t}|ge\rangle_{24}[\cos\lambda t(\cos\Omega t|g\rangle_1 - \mathrm{i}\sin\Omega t|e\rangle_1) \times (\cos\Omega t|e\rangle_3 - \mathrm{i}\sin\Omega t|g\rangle_3) \\ &- \mathrm{i}\sin\lambda t(\cos\Omega t|e\rangle_1 - \mathrm{i}\sin\Omega t|g\rangle_1) \times (\cos\Omega t|g\rangle_3 - \mathrm{i}\sin\Omega t|e\rangle_3)] \\ &+ bd\mathrm{e}^{-\mathrm{i}\lambda t}|gg\rangle_{24}[\cos\lambda t(\cos\Omega t|g\rangle_1 - \mathrm{i}\sin\Omega t|e\rangle_1) \times (\cos\Omega t|g\rangle_3 - \mathrm{i}\sin\Omega t|e\rangle_3) \\ &- \mathrm{i}\sin\lambda t(\cos\Omega t|e\rangle_1 - \mathrm{i}\sin\Omega t|g\rangle_1) \times (\cos\Omega t|e\rangle_3 - \mathrm{i}\sin\Omega t|g\rangle_3)]. \end{aligned} \tag{7}$$

Then Alice will measure the states of atoms 1, 3. If her measurement is $|ee\rangle_{13}$, the states of atoms 2, 4 will become

$$\begin{aligned} |\psi\rangle_{24} =& \mathrm{e}^{-\mathrm{i}\lambda t} ac(\mathrm{e}^{-\mathrm{i}\lambda t}\cos^2\Omega t + \mathrm{i}\sin\lambda t)|ee\rangle_{24} \\ &- \mathrm{e}^{-\mathrm{i}\lambda t} bd(\mathrm{e}^{-\mathrm{i}\lambda t}\sin^2\Omega t + \mathrm{i}\sin\lambda t)|gg\rangle_{24} \\ &- \frac{\mathrm{i}}{2}\mathrm{e}^{-2\mathrm{i}\lambda t} ad\sin 2\Omega t|eg\rangle_{24} \\ &- \frac{\mathrm{i}}{2}\mathrm{e}^{-2i\lambda t} bc\sin 2\Omega t|ge\rangle_{24}. \end{aligned} \tag{8}$$

Since Alice has completely known the parameters α and β, she can select the atomic velocity to meet the condition $\cos\lambda t = \alpha$ (or $\sin\lambda t = \beta$) and obtain a conclusive interaction time. In addition, she can make the Rabi frequency Ω satisfy the condition $\Omega t = \pi$ through modulating the classical field. After the interaction and measurements, Alice will inform Bob of her measurement through a classical channel. The states of atoms 2, 4 on Bob's side will become

$$|\psi\rangle_{24} = \mathrm{e}^{-\mathrm{i}\lambda t}(\alpha ac|ee\rangle_{24} - \mathrm{i}\beta bd|gg\rangle_{24}). \tag{9}$$

After a rotation operation, the state in Eq.(9) will become

$$(\alpha ac|ee\rangle_{24} + \beta bd|gg\rangle_{24}). \tag{10}$$

In order to achieve the goal of remotely preparing a two-atom entangled state, Bob will introduce an identical auxiliary atom a with the initial state of $|g\rangle_a$ and a single-mode vacuum cavity field, the joint state of atoms 2, 4 and a will become

$$(\alpha ac|ee\rangle_{24} + \beta bd|gg\rangle_{24}) \otimes |g\rangle_a. \tag{11}$$

Similarly Bob will simultaneously send the two atoms 2, a into the single-mode vacuum cavity. In the case of $\delta' \gg g'$ and $\delta' = \omega_0' - \omega'$, where g' is the atom-cavity coupling strength, ω_0' and ω' are the atomic transition frequency and the cavity frequency, respectively. There is no energy exchange between the atoms and the cavity mode.

The effective Hamiltonian of the system in the interaction picture is[30]

$$H_{\rm eff}' = \lambda' \left(\sum_{j'=1}^{2} |e\rangle_{j'j'}\langle e| + \sum_{j',k'=1,j'\neq k'}^{2} S_{j'}^{\dagger} S_{k'}^{-} \right), \tag{12}$$

where $\lambda' = (g')^2/\delta'$, $S_{j'}^{-} = |g\rangle_{j'j'}\langle e|$ and $S_{j'}^{\dagger} = |e\rangle_{j'j'}\langle g|$. $|e\rangle_{j'}$ and $|g\rangle_{j'}$ are the excited and ground states of the j'th atom, respectively. After the interaction time t', the state in the system will evolve to

$$\begin{aligned} &{\rm e}^{-{\rm i}\lambda' t'}(\alpha ac\cos\lambda' t'|eeg\rangle_{24a} \\ &- {\rm i}\alpha ac\sin\lambda' t'|gee\rangle_{24a}) + \beta bd|ggg\rangle_{24a}. \end{aligned} \tag{13}$$

Bob can select the atomic velocity to meet the condition $\cos\lambda' t' = \dfrac{bd}{ac}$ (or $\sin\lambda' t' = \sqrt{1 - \left(\dfrac{bd}{ac}\right)^2}$), the state in Eq.(13) will become

$$\begin{aligned} &bd(\alpha|ee\rangle_{24} + \beta|gg\rangle_{24}) \otimes |g\rangle_a \\ &- {\rm i}\alpha\sqrt{(ac)^2 - (bd)^2}|gee\rangle_{24a}. \end{aligned} \tag{14}$$

Then Bob will measure the state of the auxiliary atom a. If the measurement is $|g\rangle_a$, Bob will obtain the state

$$bd(\alpha|ee\rangle_{24} + \beta|gg\rangle_{24}). \tag{15}$$

The remote preparation of the state $\alpha|ee\rangle_{24} + \beta|gg\rangle_{24}$ is successful with the probability of $|bd|^2$.

Through analyses, we find that the measurements $|eg\rangle_{13}$, $|ge\rangle_{13}$ and $|gg\rangle_{13}$ also can successfully realize the remote preparation of the state $\alpha|ee\rangle_{24} + \beta|gg\rangle_{24}$ if appropriate operations are performed on atoms 2, 4. But for the two results $|eg\rangle_{13}$ and $|ge\rangle_{13}$, Bob must introduce two single-mode vacuum cavity fields and two auxiliary atoms a and b, and simultaneously send atoms 2, a and 4, b into the two single-mode vacuum cavity fields, respectively. By combining the appropriate interaction time and separate measurements, the remote preparation of the state in Eq.(1) will be realized. By considering the four cases, the total successful probability is obtained to be $4|bd|^2$.

We must note that the state $\alpha|ee\rangle + \beta|gg\rangle$ can also be realized by the state $\alpha|e\rangle + \beta|g\rangle$ which should be prepared beforehand and the state $a|e\rangle + b|g\rangle$ using the operations in this paper (the process of introducing auxiliary atom in this paper is necessary).

In our scheme, if $a{=}b{=}c{=}d{=}\dfrac{1}{\sqrt{2}}$, Alice and Bob will share two two-atom maximally entangled states, the state in Eq.(10) will become

$$\frac{1}{2}(\alpha|ee\rangle_{24} + \beta|gg\rangle_{24}). \tag{16}$$

Under this condition, it is unnecessary for Bob to introduce any vacuum cavity field and auxiliary atom to realize the remote preparation of the two-atom entangled state in Eq.(1). And the total successful probability can reach 1.

We continue to discuss the feasibility of our scheme. According to the form of the effective Hamiltonian in Eq.(5), namely, in the case of large detuning and strong driving field, the effects of cavity decay and thermal field are all eliminated. But from the effective Hamiltonian in Eq.(12), the effect of cavity decay can be eliminated, while the effect of thermal field still exists. So our scheme is insensitive to cavity decay, but it is sensitive to thermal field. In the process of preparation of the quantum state and the separate measurements, our scheme is related only to the interaction of atoms and cavity and there is no quantum information transfer between atoms and cavity, so the scheme is reliable by present cavity QED techniques. In addition, the two atoms must be sent into the cavity simultaneously, this will produce errors between experiment and theory, but the effect of the errors on the fidelity of the resulting state is slight.

We have proposed a scheme for remotely preparing a two-atom entangled state by Alice and Bob sharing two two-atom entangled states via entanglement swapping in cavity QED. The successful probability of the remote preparation is $4|bd|^2$. If Alice and Bob share two two-atom maximally entangled states, the successful probability of the remote preparation reaches 1. In this case, there is no need to introduce any vacuum cavity field and auxiliary atom to realize the remote state preparation, so the effects of cavity decay and thermal field are all eliminated. We

must emphasis that three cavity QED and two auxiliary atoms have to be introduced to realize the remote preparation of the two-atom entangled state if Alice's measurement is $|eg\rangle_{13}$ or $|ge\rangle_{13}$.

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