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Probabilistic teleportation of an arbitrary three-particle state*

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A scheme for teleporting an arbitrary and unknown three-particle state from a sender to either one of two receivers is proposed. The quantum channel is composed of a two-particle non-maximally entangled state and two three-particle non-maximally entangled W states. An arbitrary three-particle state can be perfectly teleported probabilistically if the sender performs three generalized Bell-state measurements and sends to the two receivers the classical result of these measurements, and either one of the two receivers adopts an appropriate unitary transformation conditioned on the suitable measurement outcomes of the other receiver. All kinds of unitary transformations are given in detail.

Keywords: probabilistic teleportation, arbitrary three-particle state, unitary transformation, three-particle entangled W state

PACC: 0365, 4250

Quantum teleportation has been paid much attention both theoretically and experimentally in recent years due to its important applications in quantum communication and quantum calculation. Quantum teleportation was originally proposed by Bennett *et al.*[1] In their scheme, an unknown state was transmitted from a sender (Alice) to a receiver (Bob) via a quantum channel with the help of some classical information. Teleportation of the polarized photon and a single coherent mode of the radiation field has been realized experimentally by using parametric down-conversion.[2−4] Davidovich *et al*[5] have presented a protocol to teleport an unknown atomic state between two high-Q cavities initially prepared in an entangled Fock state. Cirac and Parkins[6] also investigated another quantum electronic dynamics (QED) proposal for quantum teleportation of an atomic state.

Recently, more and more attention has been paid to teleportation of an unknown entangled state. Shi *et al*[7−17] have investigated teleportation of a two-particle entangled state under various conditions. The proposals for teleportation of a three-particle entangled state have been put forward by Yang *et al.*[18−23] However, how to probabilistically teleport an arbitrary three-particle state is an important and interesting problem in quantum information. Dai *et al*[24] have presented a scheme to probabilistically teleport an arbitrary three-particle state by using a partial-entangled four-particle state and a partial-entangled pair.

In this paper, we propose another scheme to teleport an arbitrary and unknown three-particle state. In contrast to the scheme in Ref.[24], the present one has the following advantages. First, such an arbitrary and unknown state is transmitted from a sender to either one of two receivers Bob and Charlie, whereas the previous scheme[24] considered only the quantum teleportation from a sender to a receiver. Second, the quantum channel is different. In our present protocol, the quantum channel is composed of a two-particle non-maximally entangled state and two three-particle non-maximally entangled W states, whereas in the previous scheme, a partial-entangled four-particle state and a partial-entangled pair are used as the quantum channels. Third, the present scheme may be more easily implemented than the previous scheme[24] due to the fact that three-particle entanglement is much easier to prepare and maintain than four-particle entangle-

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ment. We show that the probabilistic teleportation of the arbitrary and unknown three-particle state can be realized if Alice performs three generalized Bell-state measurements on her side while either one of the two receivers introduces an appropriate unitary transformation in each receiver's laboratory with the help of the other receiver's simple measurement and classical communication. All possible unitary transformations performed by each receiver are given in more detail.

Suppose that the sender (Alice) wants to transmit an arbitrary and unknown three-particle state from a sender to either one of two receivers Bob and Charlie who are spatially separated. The arbitrary and unknown three-particle state that will be teleported can be expressed as

$$\begin{aligned}|\Psi\rangle_{123} =&(x_0|000\rangle + x_1|001\rangle + x_2|010\rangle \\ &+ x_3|100\rangle + x_4|110\rangle + x_5|101\rangle \\ &+ x_6|011\rangle + x_7|111\rangle)_{123}, \end{aligned} \tag{1}$$

where $\sum_{i=0}^{7}|x_i|^2 = 1$. Without loss of generality, the quantum channel, composed of two three-particle non-maximally entangled W states and a two-particle non-maximally entangled state, reads as follows:

$$|\Psi\rangle_{456} = (a|001\rangle + b|010\rangle + c|100\rangle)_{456}, \tag{2}$$

$$|\Psi\rangle_{789} = (d|001\rangle + e|010\rangle + f|100\rangle)_{789}, \tag{3}$$

$$|\Psi\rangle_{10,11} = (g|00\rangle + h|11\rangle)_{10,11}, \tag{4}$$

where the real coefficients a, b and c satisfy $|a|^2 + |b|^2 + |c|^2 = 1$, and $|c|$ is smaller than the other two coefficients' absolute value; real coefficients d, e and f satisfy $|d|^2+|e|^2+|f|^2 = 1$, and $|f|$ is smaller than the other two coefficients' absolute value; real coefficients g and h satisfy $|g|^2 + |h|^2 = 1$, and $|g| \geq |h|$. Particles 4, 7 and 10 belong to the sender Alice while particles (5, 8) and particles (6, 9) belongs to the receivers Bob and Charlie, respectively. But particle 11 only belongs to one of receivers, Bob or Charlie. Therefore, the state of the whole system composed of particles (1, 2, 3) and the quantum channel is given by

$$|\Psi\rangle = |\Psi\rangle_{123} \otimes |\Psi\rangle_{456} \otimes |\Psi\rangle_{789} \otimes |\Psi\rangle_{10,11}. \tag{5}$$

In order to realize teleportation, Alice must carry out the Bell-state measurements on particles (1,4), particles (2,7), and particles (3,10) separately. After the three Bell-state measurements, all possible 64 results are

$$\begin{aligned}{}_{3,10}\langle\Phi^{\pm}|_{27}\langle\Phi^{\pm}|_{14}\langle\Phi^{\pm}|\,\Psi\rangle =&\frac{1}{2\sqrt{2}}(x_0adg|01010\rangle + x_0aeg|01100\rangle + x_0bdg|10010\rangle \\ &+ x_0beg|10100\rangle \pm_3 x_1adh|01011\rangle \pm_3 x_1aeh|01101\rangle \pm_3 x_1bdh|10011\rangle \\ &\pm_3 x_1beh|10101\rangle \pm_1 x_3cdg|00010\rangle \pm_1 x_3ceg|00100\rangle \pm_1 \pm_3 x_5cdh|00011\rangle \\ &\pm_1 \pm_3 x_5ceh|00101\rangle \pm_2 x_2afg|01000\rangle \pm_2 x_2bfg|10000\rangle \pm_2 \pm_3 x_6afh|01001\rangle \\ &\pm_2 \pm_3 x_6bfh|10001\rangle \pm_1 \pm_2 x_4cfg|00000\rangle \pm_1 \pm_2 \pm_3 x_7cfh|00001\rangle)_{5689,11}; \end{aligned} \tag{6}$$

$$\begin{aligned}{}_{3,10}\langle\Psi^{\pm}|_{27}\langle\Phi^{\pm}|_{14}\langle\Phi^{\pm}|\,\Psi\rangle =&\frac{1}{2\sqrt{2}}(x_0adh|01011\rangle + x_0aeh|01101\rangle + x_0bdh|10011\rangle \\ &+ x_0beh|10101\rangle \pm_3 x_1adg|01010\rangle \pm_3 x_1aeg|01100\rangle \pm_3 x_1bdg|10010\rangle \\ &\pm_3 x_1beg|10100\rangle \pm_1 x_3cdh|00011\rangle \pm_1 x_3ceh|00101\rangle \pm_1 \pm_3 x_5cdg|00010\rangle \\ &\pm_1 \pm_3 x_5ceg|00100\rangle \pm_2 x_2afh|01001\rangle \pm_2 x_2bfh|10001\rangle \pm_2 \pm_3 x_6afg|01000\rangle \\ &\pm_2 \pm_3 x_6bfg|10000\rangle \pm_1 \pm_2 x_4cfh|00001\rangle \pm_1 \pm_2 \pm_3 x_7cfg|00000\rangle)_{5689,11}; \end{aligned} \tag{7}$$

$$\begin{aligned}{}_{3,10}\langle\Phi^{\pm}|_{27}\langle\Psi^{\pm}|_{14}\langle\Phi^{\pm}|\,\Psi\rangle =&\frac{1}{2\sqrt{2}}(x_0afg|01000\rangle + x_0bfg|10000\rangle \pm_3 x_1afh|01001\rangle \\ &\pm_3 x_1bfh|10001\rangle \pm_2 x_2adg|01010\rangle \pm_2 x_2aeg|01100\rangle \pm_2 x_2bdg|10010\rangle \\ &\pm_2 x_2beg|10100\rangle \pm_2 \pm_3 x_6adh|01011\rangle \pm_2 \pm_3 x_6aeh|01101\rangle \pm_2 \pm_3 x_6bdh|10011\rangle \\ &\pm_2 \pm_3 x_6bdh|10101\rangle \pm_1 x_3cfg|00000\rangle \pm_1 \pm_2 x_4cdg|00010\rangle \pm_1 \pm_2 x_4ceg|00100\rangle \\ &\pm_1 \pm_3 x_5cfh|00001\rangle \pm_1 \pm_2 \pm_3 x_7cdh|00011\rangle \pm_1 \pm_2 \pm_3 x_7ceh|00101\rangle)_{5689,11}; \end{aligned} \tag{8}$$

$$
\begin{aligned}
{}_{3,10}\langle\Psi^{\pm}|_{27}\langle\Psi^{\pm}|_{14}\langle\Phi^{\pm}|\,\Psi\rangle =& \frac{1}{2\sqrt{2}}(x_0afh|01001\rangle + x_0bfh|10001\rangle \pm_3 x_1afg|01000\rangle \\
& \pm_3 x_1bfg|10000\rangle \pm_2 x_2adh|01011\rangle \pm_2 x_2aeh|01101\rangle \pm_2 x_2bdh|10011\rangle \\
& \pm_2 x_2beh|10101\rangle \pm_2 \pm_3 x_6adg|01010\rangle \pm_2 \pm_3 x_6aeg|01100\rangle \pm_2 \pm_3 x_6bdg|10010\rangle \\
& \pm_2 \pm_3 x_6bdg|10100\rangle \pm_1 x_3cfh|00001\rangle \pm_1 \pm_2 x_4cdh|00011\rangle \pm_1 \pm_2 x_4ceh|00101\rangle \\
& \pm_1 \pm_3 x_5cfg|00000\rangle \pm_1 \pm_2 \pm_3 x_7cdg|00010\rangle \pm_1 \pm_2 \pm_3 x_7ceg|00100\rangle)_{5689,11}; \quad (9)
\end{aligned}
$$

$$
\begin{aligned}
{}_{3,10}\langle\Phi^{\pm}|_{27}\langle\Phi^{\pm}|_{14}\langle\Psi^{\pm}|\,\Psi\rangle =& \frac{1}{2\sqrt{2}}(x_0cdg|00010\rangle + x_0ceg|00100\rangle \pm_3 x_1cdh|00011\rangle \\
& \pm_3 x_1ceh|00101\rangle \pm_2 x_2cfg|00000\rangle \pm_2 \pm_3 x_6cfh|00001\rangle \pm_1 x_3adg|01010\rangle \\
& \pm_1 x_3aeg|01100\rangle \pm_1 x_3bdg|10010\rangle \pm_1 x_3beg|10100\rangle \pm_1 \pm_2 x_4afg|01000\rangle \\
& \pm_1 \pm_2 x_4bfg|10000\rangle \pm_1 \pm_3 x_5adh|01011\rangle \pm_1 \pm_3 x_5aeh|01101\rangle \pm_1 \pm_3 x_5bdh|10011\rangle \\
& \pm_1 \pm_3 x_5beh|10101\rangle \pm_1 \pm_2 \pm_3 x_7afh|01001\rangle \pm_1 \pm_2 \pm_3 x_7bfh|10001\rangle)_{5689,11}; \quad (10)
\end{aligned}
$$

$$
\begin{aligned}
{}_{3,10}\langle\Psi^{\pm}|_{27}\langle\Phi^{\pm}|_{14}\langle\Psi^{\pm}|\,\Psi\rangle =& \frac{1}{2\sqrt{2}}(x_0cdh|00011\rangle + x_0ceh|00101\rangle \pm_3 x_1cdg|00010\rangle \\
& \pm_3 x_1ceg|00100\rangle \pm_2 x_2cfh|00001\rangle \pm_2 \pm_3 x_6cfg|00000\rangle \pm_1 x_3adh|01011\rangle \\
& \pm_1 x_3aeh|01101\rangle \pm_1 x_3bdh|10011\rangle \pm_1 x_3beh|10101\rangle \pm_1 \pm_2 x_4afh|01001\rangle \\
& \pm_1 \pm_2 x_4bfh|10001\rangle \pm_1 \pm_3 x_5adg|01010\rangle \pm_1 \pm_3 x_5aeg|01100\rangle \pm_1 \pm_3 x_5bdg|10010\rangle \\
& \pm_1 \pm_3 x_5beg|10100\rangle \pm_1 \pm_2 \pm_3 x_7afg|01000\rangle \pm_1 \pm_2 \pm_3 x_7bfg|10000\rangle)_{5689,11}; \quad (11)
\end{aligned}
$$

$$
\begin{aligned}
{}_{3,10}\langle\Phi^{\pm}|_{27}\langle\Psi^{\pm}|_{14}\langle\Psi^{\pm}|\,\Psi\rangle =& \frac{1}{2\sqrt{2}}(x_0cfg|00000\rangle \pm_3 x_1cfh|00001\rangle \pm_2 x_2cdg|00010\rangle \\
& \pm_2 x_2ceg|00100\rangle \pm_2 \pm_3 x_6cdh|00011\rangle \pm_2 \pm_3 x_6ceh|00101\rangle \pm_1 x_3afg|01000\rangle \\
& \pm_1 x_3bfg|10000\rangle \pm_1 \pm_2 x_4adg|01010\rangle \pm_1 \pm_2 x_4aeg|01100\rangle \pm_1 \pm_2 x_4bdg|10010\rangle \\
& \pm_1 \pm_2 x_4beg|10100\rangle \pm_1 \pm_3 x_5afh|01001\rangle \pm_1 \pm_3 x_5bfh|10001\rangle \pm_1 \pm_2 \pm_3 x_7adh|01011\rangle \\
& \pm_1 \pm_2 \pm_3 x_7aeh|01101\rangle \pm_1 \pm_2 \pm_3 x_7bdh|10011\rangle \pm_1 \pm_2 \pm_3 x_7beh|10101\rangle)_{5689,11}; \quad (12)
\end{aligned}
$$

$$
\begin{aligned}
{}_{3,10}\langle\Psi^{\pm}|_{27}\langle\Psi^{\pm}|_{14}\langle\Psi^{\pm}|\,\Psi\rangle =& \frac{1}{2\sqrt{2}}(x_0cfh|00001\rangle \pm_3 x_1cfg|00000\rangle \pm_2 x_2cdh|00011\rangle \\
& \pm_2 x_2ceh|00101\rangle \pm_2 \pm_3 x_6cdg|00010\rangle \pm_2 \pm_3 x_6ceg|00100\rangle \pm_1 x_3afh|01001\rangle \\
& \pm_1 x_3bfh|10001\rangle \pm_1 \pm_2 x_4adh|01011\rangle \pm_1 \pm_2 x_4aeh|01101\rangle \pm_1 \pm_2 x_4bdh|10011\rangle \\
& \pm_1 \pm_2 x_4beh|10101\rangle \pm_1 \pm_3 x_5afg|01000\rangle \pm_1 \pm_3 x_5bfg|10000\rangle \pm_1 \pm_2 \pm_3 x_7adg|01010\rangle \\
& \pm_1 \pm_2 \pm_3 x_7aeg|01100\rangle \pm_1 \pm_2 \pm_3 x_7bdg|10010\rangle \pm_1 \pm_2 \pm_3 x_7beg|10100\rangle)_{5689,11}, \quad (13)
\end{aligned}
$$

where the Bell-states, in which the particles $(i,\ j)$ are measured, are defined as

$$|\Phi^{\pm}\rangle_{ij} = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)_{ij}, \quad (14)$$

$$|\Psi^{\pm}\rangle_{ij} = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)_{ij}. \quad (15)$$

Here $\pm_1$, $\pm_2$ and $\pm_3$ correspond to the superscripts for the Bell-state composed of particles (1, 4), (2, 7) and (3, 10), respectively. For instance, if Alice's measurement results are $|\Phi^{+}\rangle_{14}$, $|\Psi^{-}\rangle_{27}$ and $|\Phi^{-}\rangle_{3,10}$, i.e. the corresponding superscripts are +, − and − the state of particles 5, 6, 8, 9, 11, as shown by Eq.(8), will collapse into

$$
\begin{aligned}
|\Psi\rangle_{5689,11} =& \frac{1}{2\sqrt{2}}(x_0afg|01000\rangle + x_0bfg|10000\rangle \\
& - x_1afh|01001\rangle \\
& - x_1bfh|10001\rangle - x_2adg|01010\rangle \\
& - x_2aeg|01100\rangle - x_2bdg|10010\rangle \\
& - x_2beg|10100\rangle + x_6adh|01011\rangle \\
& + x_6aeh|01101\rangle + x_6bdh|10011\rangle \\
& + x_6bdh|10101\rangle + x_3cfg|00000\rangle \\
& - x_4cdg|00010\rangle - x_4ceg|00100\rangle \\
& - x_5cfh|00001\rangle + x_7cdh|00011\rangle \\
& + x_7ceh|00101\rangle)_{5689,11}. \qquad (16)
\end{aligned}
$$

That is, after three measurements, the entanglement among particles 1, 2 and 3 disappears, the new entanglement among particles 5, 6, 8, 9 and 11 is set up, and the entanglement swapping occurs. Next, Alice informs the two receivers of her measurement results via a classical communication. If Bob wants to receive the teleported state $|\Psi\rangle_{123}$ in Eq.(1), particle 11 belongs to the receiver Bob. First, he can ask Charlie to make the projection measurement on particles 6 and 9 in the basis $\{|0\rangle, |1\rangle\}$, respectively; then we have

$$
\begin{aligned}
|\Psi\rangle_{5689,11} =& \frac{1}{2\sqrt{2}}[(x_0afg|0100\rangle + x_0bfg|1000\rangle \\
& - x_1afh|0101\rangle \\
& - x_1bfh|1001\rangle - x_2aeg|0110\rangle \\
& - x_2beg|1010\rangle + x_6aeh|0111\rangle \\
& + x_6bdh|1011\rangle + x_3cfg|0000\rangle \\
& - x_4ceg|0010\rangle - x_5cfh|0001\rangle \\
& + x_7ceh|0011\rangle)_{568,11}|0\rangle_9 + (-x_2adg|0100\rangle \\
& - x_2bdg|1000\rangle + x_6adh|0101\rangle \\
& + x_6bdh|1001\rangle - x_4cdg|0000\rangle \\
& + x_7cdh|0001\rangle)_{568,11}|1\rangle_9]. \qquad (17)
\end{aligned}
$$

If Charlie obtains the result of $|0\rangle_9$, then

$$
\begin{aligned}
|\Psi\rangle_{568,11} =& \frac{1}{2\sqrt{2}}(x_0afg|0100\rangle + x_0bfg|1000\rangle - x_1afh|0101\rangle - x_1bfh|1001\rangle - x_2aeg|0110\rangle \\
& - x_2beg|1010\rangle + x_6aeh|0111\rangle + x_6bdh|1011\rangle + x_3cfg|0000\rangle \\
& - x_4ceg|0010\rangle - x_5cfh|0001\rangle + x_7ceh|0011\rangle)_{568,11} \\
=& \frac{1}{2\sqrt{2}}[(x_0bfg|100\rangle - x_1bfh|101\rangle - x_2beg|110\rangle + x_6bdh|111\rangle + x_3cfg|000\rangle \\
& - x_4ceg|010\rangle - x_5cfh|001\rangle + x_7ceh|011\rangle)_{58,11}|0\rangle_6 + (x_0afg|000\rangle \\
& - x_1afh|001\rangle - x_2aeg|010\rangle + x_6aeh|011\rangle)_{58,11}|1\rangle_6]. \qquad (18)
\end{aligned}
$$

Again, if Charlie measures the result of $|0\rangle_6$, the state of particles 5, 8 and 11 will become

$$
\begin{aligned}
|\Psi\rangle_{58,11} =& \frac{1}{2\sqrt{2}}(x_0bfg|100\rangle - x_1bfh|101\rangle \\
& - x_2beg|110\rangle + x_3cfg|000\rangle \\
& - x_4ceg|010\rangle - x_5cfh|001\rangle \\
& + x_6bdh|111\rangle + x_7ceh|011\rangle)_{58,11}. \qquad (19)
\end{aligned}
$$

Then, Charlie will inform the receiver Bob of his measurement results via a classical channel. Secondly, Bob needs to establish a correspondence so that the coefficients $x_i (i = 0, 1, \ldots, 7)$ can correspond to $|000\rangle_{58,11}$, $|001\rangle_{58,11}$, $|010\rangle_{58,11}$, $|100\rangle_{58,11}$, $|110\rangle_{58,11}$, $|101\rangle_{58,11}$, $|011\rangle_{58,11}$, and $|111\rangle_{58,11}$, respectively. This can be realized by performing the following unitary transformation:

$$
\begin{aligned}
U_1 =& (|0\rangle\langle 1| + |1\rangle\langle 0|)_5 \\
& \otimes (|0\rangle\langle 0| - |1\rangle\langle 1|)_8 \\
& \otimes (|0\rangle\langle 0| - |1\rangle\langle 1|)_{11} \qquad (20)
\end{aligned}
$$

on particle (5, 8, 11) shown in Eq.(19); that is,

$$
\begin{aligned}
|\Psi\rangle_{58,11} =& \frac{1}{2\sqrt{2}}(x_0bfg|000\rangle + x_1bfh|001\rangle \\
& + x_2beg|010\rangle + x_3cfg|100\rangle \\
& + x_4ceg|110\rangle + x_5cfh|101\rangle \\
& + x_6bdh|011\rangle + x_7ceh|111\rangle)_{58,11}. \qquad (21)
\end{aligned}
$$

At the same time, Bob introduces another auxiliary particle A with its initial state $|0\rangle_A$ and makes another unitary transformation U_2 on particles 5, 8 11 and A. In order for Bob to reincarnate the original state under the basis $\{|0000\rangle_{58,11,A}, |0010\rangle_{58,11,A},$

$|0100\rangle_{58,11,A}$, $|1000\rangle_{58,11,A}$, $|1100\rangle_{58,11,A}$, $|1010\rangle_{58,11,A}$, $|0110\rangle_{58,11,A}$, $|1110\rangle_{58,11,A}$, $|0001\rangle_{58,11,A}$, $|0011\rangle_{58,11,A}$, $|0101\rangle_{58,11,A}$, $|1001\rangle_{58,11,A}$, $|1101\rangle_{58,11,A}$, $|1011\rangle_{58,11,A}$, $|0111\rangle_{58,11,A}$, $|1111\rangle_{58,11,A}\}$, the unitary transformation may take the form of the following 16×16 matrices:

$$U_2 = \begin{pmatrix} A_1 & A_2 \\ A_2 & -A_1 \end{pmatrix}, \tag{22}$$

where A_1 and A_2 are 8×8 matrices and may be written as

$$A_1 = \mathrm{diag}(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8), \tag{23}$$

$$B_2 = \mathrm{diag}(\sqrt{1-a_1^2}, \sqrt{1-a_2^2}, \sqrt{1-a_3^2}, \sqrt{1-a_4^2}, \sqrt{1-a_5^2}, \sqrt{1-a_6^2}, \sqrt{1-a_7^2}, \sqrt{1-a_8^2}), \tag{24}$$

where a_i $(i = 1, 2, \ldots, 8$ and $|a_i| \leq 1)$ depends on the state of particles 5, 8 and 11. For example, Bob introduces an auxiliary A with its initial state $|0\rangle_A$, the state described in Eq.(21) will become

$$\begin{aligned} |\Psi\rangle_{58,11} \otimes |0\rangle_A =& \frac{1}{2\sqrt{2}}(x_0 bfg|000\rangle + x_1 bfh|001\rangle \\ &+ x_2 beg|010\rangle + x_3 cfg|100\rangle \\ &+ x_4 ceg|110\rangle + x_5 cfh|101\rangle \\ &+ x_6 bdh|011\rangle \\ &+ x_7 ceh|111\rangle)_{58,11}|0\rangle_A, \end{aligned} \tag{25}$$

if we choose

$$\begin{aligned} &(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8) \\ =&\left(\frac{ch}{bg}, \frac{c}{b}, \frac{cfh}{beg}, \frac{h}{g}, \frac{fh}{eg}, 1, \frac{cf}{bd}, \frac{f}{e}\right). \end{aligned} \tag{26}$$

The unitary transformation U_2 will transform the state in Eq.(25) into

$$\begin{aligned} &\frac{cfh}{2\sqrt{2}}(x_0|000\rangle + x_1|001\rangle + x_2|010\rangle + x_3|100\rangle \\ &+ x_4|110\rangle + x_5|101\rangle + x_6|011\rangle + x_7|111\rangle)_{58,11}|0\rangle_A \\ &+ \frac{1}{2\sqrt{2}}[f\sqrt{(bg)^2 - (ch)^2}x_0|000\rangle_{58,11} \\ &+ fh\sqrt{b^2 - c^2}x_1|001\rangle_{58,11} \\ &+ \sqrt{(beg)^2 - (cfh)^2}x_2|010\rangle_{58,11} \\ &+ cf\sqrt{g^2 - h^2}x_3|100\rangle_{58,11} \\ &+ c\sqrt{(eg)^2 - (fh)^2}x_4|110\rangle_{58,11} \\ &+ h\sqrt{(bd)^2 - (cf)^2}x_6|011\rangle_{58,11} \\ &+ ch\sqrt{e^2 - f^2}x_7|111\rangle_{58,11}]|1\rangle_A. \end{aligned} \tag{27}$$

Of course, U_1 and U_2 may be incorporated into a single transformation $U = U_1 \otimes U_2$. Such a unitary transformation can be decomposed into universal quantum logic operations[25] that have been experimentally realized in some physical systems.[26] All possible unitary transformation U_1 and the coefficients for unitary transformation U_2 performed by Bob on the states of particles 5,8 and 11 are given in Tables 1 and 2, respectively, where $\pm_1$, $\pm_2$ and $\pm_3$ correspond to the superscripts for the Bell-state composed of particles (1, 4), (2, 7) and (3, 10), respectively. Finally, Bob measures the state of auxiliary particle A. If the result $|0\rangle_A$ is given, quantum teleportation is successfully realized with the probability of $\frac{|cfh|^2}{8}$. Otherwise, teleportation fails. It can be easily verified that 64 kinds of states have the same results for Bob, therefore the total probability of successful teleportation is $8|cfh|^2$.

Table 1. Unitary transformation U_1 on the states of particles 5,8 and 11.

States of particles 5, 8 and 11	Unitary transformation U_1
Eq.(6)$\|\Psi\rangle_{58,11}$	$(\|0\rangle\langle 1\| \pm_1 \|1\rangle\langle 0\|)_5 \otimes (\|0\rangle\langle 1\| \pm_2 \|1\rangle\langle 0\|)_8 \otimes (\|0\rangle\langle 0\| \pm_3 \|1\rangle\langle 1\|)_{11}$
Eq.(7)$\|\Psi\rangle_{58,11}$	$(\|0\rangle\langle 1\| \pm_1 \|1\rangle\langle 0\|)_5 \otimes (\|0\rangle\langle 1\| \pm_2 \|1\rangle\langle 0\|)_8 \otimes (\|0\rangle\langle 1\| \pm_3 \|1\rangle\langle 0\|)_{11}$
Eq.(8)$\|\Psi\rangle_{58,11}$	$(\|0\rangle\langle 1\| \pm_1 \|1\rangle\langle 0\|)_5 \otimes (\|0\rangle\langle 0\| \pm_2 \|1\rangle\langle 1\|)_8 \otimes (\|0\rangle\langle 0\| \pm_3 \|1\rangle\langle 1\|)_{11}$
Eq.(9)$\|\Psi\rangle_{58,11}$	$(\|0\rangle\langle 1\| \pm_1 \|1\rangle\langle 0\|)_5 \otimes (\|0\rangle\langle 0\| \pm_2 \|1\rangle\langle 1\|)_8 \otimes (\|0\rangle\langle 1\| \pm_3 \|1\rangle\langle 0\|)_{11}$
Eq.(10)$\|\Psi\rangle_{58,11}$	$(\|0\rangle\langle 0\| \pm_1 \|1\rangle\langle 1\|)_5 \otimes (\|0\rangle\langle 1\| \pm_2 \|1\rangle\langle 0\|)_8 \otimes (\|0\rangle\langle 0\| \pm_3 \|1\rangle\langle 1\|)_{11}$
Eq.(11)$\|\Psi\rangle_{58,11}$	$(\|0\rangle\langle 0\| \pm_1 \|1\rangle\langle 1\|)_5 \otimes (\|0\rangle\langle 1\| \pm_2 \|1\rangle\langle 0\|)_8 \otimes (\|0\rangle\langle 1\| \pm_3 \|1\rangle\langle 0\|)_{11}$
Eq.(12)$\|\Psi\rangle_{58,11}$	$(\|0\rangle\langle 0\| \pm_1 \|1\rangle\langle 1\|)_5 \otimes (\|0\rangle\langle 0\| \pm_2 \|1\rangle\langle 1\|)_8 \otimes (\|0\rangle\langle 0\| \pm_3 \|1\rangle\langle 1\|)_{11}$
Eq.(13)$\|\Psi\rangle_{58,11}$	$(\|0\rangle\langle 0\| \pm_1 \|1\rangle\langle 1\|)_5 \otimes (\|0\rangle\langle 0\| \pm_2 \|1\rangle\langle 1\|)_8 \otimes (\|0\rangle\langle 1\| \pm_3 \|1\rangle\langle 0\|)_{11}$

Table 2. Values $a_i (i = 1, \cdots, 8)$ of unitary transformation U_2 corresponding to the states of particles 5, 8 and 11.

States of particles 5, 8 and 11	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
Eq.(6)$\|\Psi\rangle_{58,11}$	$\frac{cfh}{beg}$	$\frac{cf}{be}$	$\frac{ch}{bg}$	$\frac{fh}{eg}$	$\frac{h}{g}$	$\frac{f}{e}$	$\frac{c}{b}$	1
Eq.(7)$\|\Psi\rangle_{58,11}$	$\frac{cf}{be}$	$\frac{cfh}{beg}$	$\frac{c}{b}$	$\frac{f}{e}$	1	$\frac{fh}{eg}$	$\frac{ch}{bg}$	$\frac{h}{g}$
Eq.(8)$\|\Psi\rangle_{58,11}$	$\frac{ch}{bg}$	$\frac{c}{b}$	$\frac{cfh}{beg}$	$\frac{h}{g}$	$\frac{fh}{eg}$	1	$\frac{cf}{bd}$	$\frac{f}{e}$
Eq.(9)$\|\Psi\rangle_{58,11}$	$\frac{c}{b}$	$\frac{ch}{bg}$	$\frac{cf}{be}$	1	$\frac{f}{e}$	$\frac{h}{g}$	$\frac{cfh}{bdg}$	$\frac{fh}{eg}$
Eq.(10)$\|\Psi\rangle_{58,11}$	$\frac{fh}{eg}$	$\frac{f}{e}$	$\frac{h}{g}$	$\frac{cfh}{beg}$	$\frac{ch}{bg}$	$\frac{cf}{be}$	1	$\frac{c}{b}$
Eq.(11)$\|\Psi\rangle_{58,11}$	$\frac{f}{e}$	$\frac{fh}{eg}$	1	$\frac{cf}{be}$	$\frac{c}{b}$	$\frac{cfh}{beg}$	$\frac{h}{g}$	$\frac{ch}{bg}$
Eq.(12)$\|\Psi\rangle_{58,11}$	$\frac{h}{g}$	1	$\frac{fh}{eg}$	$\frac{ch}{bg}$	$\frac{cfh}{beg}$	$\frac{c}{b}$	$\frac{f}{e}$	$\frac{cf}{be}$
Eq.(13)$\|\Psi\rangle_{58,11}$	1	$\frac{h}{g}$	$\frac{f}{e}$	$\frac{c}{b}$	$\frac{cf}{be}$	$\frac{ch}{bg}$	$\frac{fh}{eg}$	$\frac{cfh}{beg}$

Similarly, if Charlie wants to receive the teleported state $|\Psi\rangle_{123}$ in Eq.(1), particle 11 will belong to the receiver Charlie. First, he can ask Bob to do the projection measurements on his particles 8 and 5 in the basis $\{|0\rangle, |1\rangle\}$, then we have

$$\begin{aligned}|\Psi\rangle_{5689,11} =& \frac{1}{2\sqrt{2}}[(x_0 afg|0100\rangle + x_0 bfg|1000\rangle \\ & - x_1 afh|0101\rangle \\ & - x_1 bfh|1001\rangle - x_2 adg|0110\rangle \\ & - x_2 bdg|1010\rangle + x_6 adh|0111\rangle \\ & + x_6 bdh|1011\rangle + x_3 cfg|0000\rangle \\ & - x_4 cdg|0010\rangle - x_5 cfh|0001\rangle \\ & + x_7 cdh|0011\rangle)_{569,11}|0\rangle_8 + (-x_2 aeg|0100\rangle \\ & - x_2 beg|1000\rangle + x_6 aeh|0101\rangle \\ & + x_6 bdh|1001\rangle - x_4 ceg|0000\rangle \\ & + x_7 ceh|0001\rangle)_{569,11}|1\rangle_8]. \end{aligned} \tag{28}$$

If Bob obtains the result of $|0\rangle_8$, then

$$\begin{aligned}|\Psi\rangle_{569,11} =& \frac{1}{2\sqrt{2}}[(x_0 afg|0100\rangle + x_0 bfg|1000\rangle \\ & - x_1 afh|0101\rangle \\ & - x_1 bfh|1001\rangle - x_2 adg|0110\rangle \\ & - x_2 bdg|1010\rangle + x_6 adh|0111\rangle \\ & + x_6 bdh|1011\rangle + x_3 cfg|0000\rangle \\ & - x_4 cdg|0010\rangle - x_5 cfh|0001\rangle \\ & + x_7 cdh|0011\rangle)_{569,11}. \end{aligned} \tag{29}$$

Again, if Bob measures the result of $|0\rangle_5$, the state of particles 6, 9 and 11 will become

$$\begin{aligned}|\Psi\rangle_{69,11} =& \frac{1}{2\sqrt{2}}[(x_0 afg|100\rangle - x_1 afh|101\rangle \\ & - x_2 adg|110\rangle + x_3 cfg|000\rangle \\ & - x_4 cdg|010\rangle - x_5 cfh|001\rangle \\ & + x_6 adh|111\rangle + x_7 cdh|011\rangle)_{69,11}. \end{aligned} \tag{30}$$

Then Charlie will successfully receive the teleported state $|\Psi\rangle_{123}$ in Eq.(1) with the probability of $\frac{|cfh|^2}{8}$, as soon as he can do the same thing as Bob does, but with different unitary transformation. Also, it can be easily proven that 64 kinds of states have the same results for Charlie, and the total probability of successful teleportation is $8|cfh|^2$. Tables 3 and 4 give all possible unitary transformation U_1 and the coefficients for the unitary transformation U_2 which Charlie makes on the states of particles 6, 9 and 11.

Table 3. Unitary transformation U_1 on the states of particles 6, 9 and 11.

States of particles 6, 9 and 11	Unitary transformation U_1
Eq.(6)$\|\Psi\rangle_{69,11}$	$(\|0\rangle\langle 1\| \pm_1 \|1\rangle\langle 0\|)_6 \otimes (\|0\rangle\langle 1\| \pm_2 \|1\rangle\langle 0\|)_9 \otimes (\|0\rangle\langle 0\| \pm_3 \|1\rangle\langle 1\|)_{11}$
Eq.(7)$\|\Psi\rangle_{69,11}$	$(\|0\rangle\langle 1\| \pm_1 \|1\rangle\langle 0\|)_6 \otimes (\|0\rangle\langle 1\| \pm_2 \|1\rangle\langle 0\|)_9 \otimes (\|0\rangle\langle 1\| \pm_3 \|1\rangle\langle 0\|)_{11}$
Eq.(8)$\|\Psi\rangle_{69,11}$	$(\|0\rangle\langle 1\| \pm_1 \|1\rangle\langle 0\|)_6 \otimes (\|0\rangle\langle 0\| \pm_2 \|1\rangle\langle 1\|)_9 \otimes (\|0\rangle\langle 0\| \pm_3 \|1\rangle\langle 1\|)_{11}$
Eq.(9)$\|\Psi\rangle_{69,11}$	$(\|0\rangle\langle 1\| \pm_1 \|1\rangle\langle 0\|)_6 \otimes (\|0\rangle\langle 0\| \pm_2 \|1\rangle\langle 1\|)_9 \otimes (\|0\rangle\langle 1\| \pm_3 \|1\rangle\langle 0\|)_{11}$
Eq.(10)$\|\Psi\rangle_{69,11}$	$(\|0\rangle\langle 0\| \pm_1 \|1\rangle\langle 1\|)_6 \otimes (\|0\rangle\langle 1\| \pm_2 \|1\rangle\langle 0\|)_9 \otimes (\|0\rangle\langle 0\| \pm_3 \|1\rangle\langle 1\|)_{11}$
Eq.(11)$\|\Psi\rangle_{69,11}$	$(\|0\rangle\langle 0\| \pm_1 \|1\rangle\langle 1\|)_6 \otimes (\|0\rangle\langle 1\| \pm_2 \|1\rangle\langle 0\|)_9 \otimes (\|0\rangle\langle 1\| \pm_3 \|1\rangle\langle 0\|)_{11}$
Eq.(12)$\|\Psi\rangle_{69,11}$	$(\|0\rangle\langle 0\| \pm_1 \|1\rangle\langle 1\|)_6 \otimes (\|0\rangle\langle 0\| \pm_2 \|1\rangle\langle 1\|)_9 \otimes (\|0\rangle\langle 0\| \pm_3 \|1\rangle\langle 1\|)_{11}$
Eq.(13)$\|\Psi\rangle_{69,11}$	$(\|0\rangle\langle 0\| \pm_1 \|1\rangle\langle 1\|)_6 \otimes (\|0\rangle\langle 0\| \pm_2 \|1\rangle\langle 1\|)_9 \otimes (\|0\rangle\langle 1\| \pm_3 \|1\rangle\langle 0\|)_{11}$

Table 4. Values $a_i \quad (i = 1, \cdots, 8)$ of unitary transformation U_2 corresponding to the states of particles 6, 9 and 11.

States of particles 6, 9 and 11	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
Eq.(6)$\|\Psi\rangle_{69,11}$	$\frac{cfh}{adg}$	$\frac{cf}{ad}$	$\frac{ch}{ag}$	$\frac{fh}{dg}$	$\frac{h}{g}$	$\frac{f}{d}$	$\frac{c}{a}$	1
Eq.(7)$\|\Psi\rangle_{69,11}$	$\frac{cf}{ad}$	$\frac{cfh}{adg}$	$\frac{c}{a}$	$\frac{f}{d}$	1	$\frac{fh}{dg}$	$\frac{ch}{ag}$	$\frac{h}{g}$
Eq.(8)$\|\Psi\rangle_{69,11}$	$\frac{ch}{ag}$	$\frac{c}{a}$	$\frac{cfh}{adg}$	$\frac{h}{g}$	$\frac{fh}{dg}$	1	$\frac{cf}{ad}$	$\frac{f}{d}$
Eq.(9)$\|\Psi\rangle_{69,11}$	$\frac{c}{a}$	$\frac{ch}{ag}$	$\frac{cf}{ad}$	1	$\frac{f}{d}$	$\frac{h}{g}$	$\frac{cfh}{adg}$	$\frac{fh}{dg}$
Eq.(10)$\|\Psi\rangle_{69,11}$	$\frac{fh}{dg}$	$\frac{f}{d}$	$\frac{h}{g}$	$\frac{cfh}{adg}$	$\frac{ch}{ag}$	$\frac{cf}{ad}$	1	$\frac{c}{a}$
Eq.(11)$\|\Psi\rangle_{69,11}$	$\frac{f}{d}$	$\frac{fh}{dg}$	1	$\frac{cf}{ad}$	$\frac{c}{a}$	$\frac{cfh}{adg}$	$\frac{h}{g}$	$\frac{ch}{ag}$
Eq.(12)$\|\Psi\rangle_{69,11}$	$\frac{h}{g}$	1	$\frac{fh}{dg}$	$\frac{ch}{ag}$	$\frac{cfh}{adg}$	$\frac{c}{a}$	$\frac{f}{d}$	$\frac{cf}{ad}$
Eq.(13)$\|\Psi\rangle_{69,11}$	1	$\frac{h}{g}$	$\frac{f}{d}$	$\frac{c}{a}$	$\frac{cf}{ad}$	$\frac{ch}{ag}$	$\frac{fh}{dg}$	$\frac{cfh}{adg}$

From the above analysis, we can see that, in each case, the total probability of successful teleportation for each receiver is $8|cfh|^2$. If $|a| = |b| = |c| = \frac{1}{\sqrt{3}}$, $|d| = |e| = |f| = \frac{1}{\sqrt{3}}$, $|g| = |h| = \frac{1}{\sqrt{2}}$, i.e. for a quantum channel composed of two three-particle maximally entangled W states and a two-particle maximally entangled state, the total probability equals $\frac{4}{9}$. Obviously, the unitary transformation made by each receiver in this situation is reduced to the unitary transformations U_1 operated on either the particles (5,8,11) only, or particles (6, 9, 11), in which the auxiliary particle A and unitary transformations U_2 are not needed. Thus probabilistic teleportation of an arbitrary and unknown three-particle state from a sender to either one of two different receivers becomes the usual standard teleportation via a maximally entangled quantum channel.

In conclusion, we have proposed a protocol for teleporting an arbitrary and unknown three-particle state from a sender to either one of two receivers by using two three-particle non-maximally entangled W states and a two-particle non-maximally entangled state as the quantum channel. The results show that, for such a non-maximally entangled quantum channel, there is still a certain probability of successful teleportation if the sender performs three generalized Bell-state measurements and sends to the two receivers the classical results of these measurements, and either one of the two receivers adopts an appropriate unitary transformation conditioned on the suitable measurement outcomes of the other receiver. The probability of success is determined by the smaller coefficients of the non-maximally entangled states used as quantum channel. Recently, some experimental schemes[27−31] for preparing the W state have been proposed; therefore, we believe that this scheme will be realized by experiment.

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