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Probabilistic and controlled teleportation of tripartite state in a general form and its quantum networks

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We discuss a scheme for probabilistic and controlled teleportation of an unknown arbitrary three-particle state by constructing a peculiar non-maximally entangled state as a controlled quantum channel , which is teleported between two sides with the help of the auxiliary particle and the cooperation of the third side (Charlie) as a supervisor. In comparison with some existing schemes, on the receiver's side it is easy to have the sender's state through operating two uniform unitary transformations in turn. In addition, we also give an efficient quantum network for implementing the new scheme by means of some primitive operations.

Keywords: arbitrary tripartite state, controlled teleportation, quantum circuit
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1. Introduction

In the field of quantum information, quantum entanglement has been playing a more and more crucial role because of its non-locality properties.[1] Quantum teleportation, originally proposed by Bennett *et al*[2] in 1993, is one of the most prominent progresses in quantum information theory. It allows the transmission of an unknown qubit state from a sender "Alice" to a spatially distant receiver "Bob" via a quantum channel with the aid of some classical communication. Based on the quantum teleportation scheme, quantum teleportation has been demonstrated with the polarization photon[3] and a single coherent mode of field[4] in experiment. At present, several schemes for teleportation of entangled states have been presented. Teleportation of a two-particle state under various conditions was studied in Refs.[5–11]. Yang and Guo,[12] Lu,[13] Liu and Guo,[14] and Dai *et al*[15] proposed schemes for teleporting a three-particle entangled Greenberger–Horne–Zeilinger (GHZ)-type state and W-type state, respectively. Especially, probabilistic teleportation of an unknown arbitrary three-particle state was proposed in Refs.[16, 17] as well.

In ordinary teleportation schemes, the entanglement of two parties between the sender and the receiver plays a very important role. In Ref.[18], three-party states are considered in which one of the parties, the assistant, makes such a measurement that the remaining two parties are left with as much entanglement as possible on average. As an obvious extension of the entanglement of assistant, recently Zhou *et al*[19] proposed the controlled teleportation of single-particle state and Yan and Wang[20] presented the probabilistic and controlled teleportation of an arbitrary two-particle state. In their protocols, the entangled state shared by a sender "Alice", a receiver "Bob" and a supervisor "Charlie" serves as the quantum channel, and without the cooperation of Charlie the teleportation between Alice and Bob cannot be realized, in other words, the teleportation is supervised by Charlie.

In this paper, we study the scheme for probabilistic and controlled teleportation of an unknown tripartite state in a general form, which can be realized by making three generalized Bell-state measurements on the sender's side and operating an appropriate unitary transformation on the receiver's side with the cooperation of the third side. This scheme lies in the fact

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that it can be utilized to construct a peculiar non-maximally entangled state as a controlled quantum channel. In comparison with the scheme discussed in Refs.[16, 17], on the receiver's side it is easy to have the sender's state by operating appropriate unitary transformations, which may be useful to the future quantum communication and experiment.

It is generally accepted that all unitary operations on any great number of bits can be decomposed into the combinations of a set of one-bit quantum gates and two-bit Controlled (CNOT) gates.[21,22] In terms of only single-qubit gates, two-qubit CNOT gates, Von Neumann measurement and classical controlled operations, Gao *et al*[23–25] and Liu *et al*[26] have constructed the efficient quantum circuits for probabilistic teleportation, which is important for realizing the teleportation scheme in experiment.

This paper is organized as follows. We first present our scheme for probabilistic and controlled teleportation of an unknown tripartite state in a general form via a peculiar controlled quantum channel. Then, we give an efficient quantum network for implementing the new scheme by means of the primitive operations consisting of single-qubit gates, two-qubit controlled-Not gates, Von Neumann measurement and classically controlled operations.

2. Probabilistic and controlled teleportation of tripartite state in a general form

Suppose, the sender "Alice", the receiver "Bob" and the supervisor "Charlie" who are spatially separated are involved in the scheme of the controlled teleportation. The purpose of this scheme is to teleport an unknown arbitrary three-particle state $|\Phi\rangle_{123}$ on the sender Alice's side to Bob under the control of Charlie.

$$\begin{aligned}|\Phi\rangle_{123} =&\alpha_0|000\rangle + \alpha_1|001\rangle + \alpha_2|010\rangle \\ &+ \alpha_3|011\rangle + \alpha_4|100\rangle + \alpha_5|101\rangle \\ &+ \alpha_6|110\rangle + \alpha_7|111\rangle, \end{aligned} \tag{1}$$

where $\sum_{i=0}^{7}|\alpha_i|^2 = 1$. The above three people have shared a seven-particle non-maximally entangled state $|\varphi\rangle_{45678910}$ as their quantum channel, which reads

$$\begin{aligned}|\varphi\rangle_{45678910} =&\beta_0|0000000\rangle + \beta_1|1000011\rangle \\ &+ \beta_2|0100100\rangle + \beta_3|1100111\rangle \\ &+ \beta_4|0011000\rangle + \beta_5|1011011\rangle \\ &+ \beta_6|0111100\rangle + \beta_7|1111111\rangle, \end{aligned} \tag{2}$$

where particles $4,5,6$, particles $7,8,9$ and particle 10 belong to Alice, Bob and Charlie, respectively. The coefficients β_i $(i = 0,1,\ldots,7)$ are assumed to be nonzero real numbers such that β_0 is smaller than the others and $\sum_{i=0}^{7}\beta_i^2 = 1$, which would guarantee the success of the unitary transformation stated later. Therefore, the initial state of the whole system composed of particles $1,2,3$ and a quantum channel formed by entangled particles 4, 5, 6, 7, 8, 9 and 10 is given by

$$|\Psi\rangle = |\Phi\rangle_{123} \otimes |\varphi\rangle_{45678910}. \tag{3}$$

The diagram of this scheme is depicted in Fig.1.

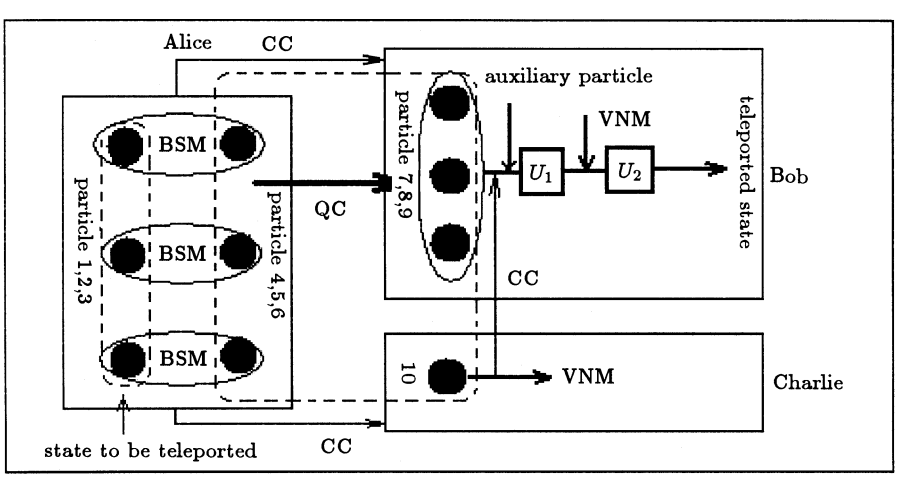


Fig.1. Schematic diagram of probabilistic and controlled teleportation, BSM denotes the Bell-states measurement, VNM denotes the Von Neumann measurement, CC denotes the classical communication, QC denotes the quantum channel.

In order to realize teleportation, Alice must perform the Bell-states measurement on particles $(1,4)$, particles $(2,5)$ and particles $(3,6)$, respectively, which will project particles $7,8,9$ and 10 into one of the following states (64 kinds in all).

$$
\begin{aligned}
{}_{36}\langle\phi^{\pm}|_{25}\langle\phi^{\pm}|_{14}\langle\phi^{\pm}|\,\Psi\rangle =&\frac{1}{4}[\alpha_0\beta_0|000\rangle \pm^a \alpha_4\beta_1|001\rangle \pm^b \alpha_2\beta_2|010\rangle \pm^b \pm^a\alpha_6\beta_3|011\rangle\\
&\pm^c \alpha_1\beta_4|100\rangle \pm^c \alpha_5\beta_5|101\rangle \pm^c \pm^b\alpha_3\beta_6|110\rangle \pm^c \pm^b \pm^a \alpha_7\beta_7|111\rangle]_{789}|+\rangle_{10}\\
&+\frac{1}{4}[\alpha_0\beta_0|000\rangle \mp^a \alpha_4\beta_1|001\rangle \pm^b \alpha_2\beta_2|010\rangle \mp^b \pm^a\alpha_6\beta_3|011\rangle\\
&\pm^c \alpha_1\beta_4|100\rangle \mp^c \alpha_5\beta_5|101\rangle \pm^c \pm^b\alpha_3\beta_6|110\rangle \mp^c \pm^b \pm^a \alpha_7\beta_7|111\rangle]_{789}|-\rangle_{10}, \qquad (4)\\
{}_{36}\langle\psi^{\pm}|_{25}\langle\phi^{\pm}|_{14}\langle\phi^{\pm}|\,\Psi\rangle =&\frac{1}{4}[\pm^c\alpha_1\beta_0|000\rangle \pm^c \pm^a\alpha_5\beta_1|001\rangle \pm^c \pm^b\alpha_3\beta_2|010\rangle \pm^c \pm^b \pm^a \alpha_7\beta_3|011\rangle\\
&+\alpha_0\beta_4|100\rangle \pm^a \alpha_4\beta_5|101\rangle \pm^b \alpha_2\beta_6|110\rangle \pm^c \pm^b\alpha_6\beta_7|111\rangle]_{789}|+\rangle_{10}\\
&+\frac{1}{4}[\pm^c\alpha_1\beta_0|000\rangle \mp^c \pm^a\alpha_5\beta_1|001\rangle \pm^c \pm^b\alpha_3\beta_2|010\rangle \mp^c \pm^b \pm^a \alpha_7\beta_3|011\rangle\\
&+\alpha_0\beta_4|100\rangle \mp^a \alpha_4\beta_5|101\rangle \pm^b \alpha_2\beta_6|110\rangle \mp^c \pm^b\alpha_6\beta_7|111\rangle]_{789}|-\rangle_{10}, \qquad (5)\\
{}_{36}\langle\phi^{\pm}|_{25}\langle\psi^{\pm}|_{14}\langle\phi^{\pm}|\,\Psi\rangle =&\frac{1}{4}[\pm^b\alpha_2\beta_0|000\rangle \pm^b \pm^a\alpha_6\beta_1|001\rangle + \alpha_0\beta_2|010\rangle \pm^a \alpha_4\beta_3|011\rangle\\
&\pm^c \pm^b\alpha_3\beta_4|100\rangle \pm^c \pm^b \pm^a \alpha_7\beta_5|101\rangle \pm^c \alpha_1\beta_6|110\rangle \pm^c \pm^a\alpha_5\beta_7|111\rangle]_{789}|+\rangle_{10}\\
&+\frac{1}{4}[\pm^b\alpha_2\beta_0|000\rangle \mp^b \pm^a\alpha_6\beta_1|001\rangle + \alpha_0\beta_2|010\rangle \mp^a \alpha_4\beta_3|011\rangle\\
&\pm^c \pm^b\alpha_3\beta_4|100\rangle \mp^c \pm^b \pm^a \alpha_7\beta_5|101\rangle \pm^c \alpha_1\beta_6|110\rangle \mp^c \pm^a\alpha_5\beta_7|111\rangle]_{789}|-\rangle_{10}, \qquad (6)\\
{}_{36}\langle\psi^{\pm}|_{25}\langle\psi^{\pm}|_{14}\langle\phi^{\pm}|\,\Psi\rangle =&\frac{1}{4}[\pm^c \pm^b \alpha_3\beta_0|000\rangle \pm^c \pm^b \pm^a \alpha_7\beta_1|001\rangle \pm^c \alpha_1\beta_2|010\rangle \pm^c \pm^a\alpha_5\beta_3|011\rangle\\
&\pm^b \alpha_2\beta_4|100\rangle \pm^b \pm^a\alpha_6\beta_5|101\rangle + \alpha_0\beta_6|110\rangle \pm^a \alpha_4\beta_7|111\rangle]_{789}|+\rangle_{10}\\
&+\frac{1}{4}[\pm^c \pm^b \alpha_3\beta_0|000\rangle \mp^c \pm^b \pm^a \alpha_7\beta_1|001\rangle \pm^c \alpha_1\beta_2|010\rangle \mp^c \pm^a\alpha_5\beta_3|011\rangle\\
&\pm^b \alpha_2\beta_4|100\rangle \mp^b \pm^a\alpha_6\beta_5|101\rangle + \alpha_0\beta_6|110\rangle \mp^a \alpha_4\beta_7|111\rangle]_{789}|-\rangle_{10}, \qquad (7)\\
{}_{36}\langle\phi^{\pm}|_{25}\langle\phi^{\pm}|_{14}\langle\psi^{\pm}|\,\Psi\rangle =&\frac{1}{4}[\pm^a\alpha_4\beta_0|000\rangle + \alpha_0\beta_1|001\rangle \pm^b \pm^a\alpha_6\beta_2|010\rangle \pm^b \alpha_2\beta_3|011\rangle\\
&\pm^c \pm^a\alpha_5\beta_4|100\rangle \pm^c \alpha_1\beta_5|101\rangle \pm^c \pm^b \pm^a \alpha_7\beta_6|110\rangle \pm^c \pm^b\alpha_3\beta_7|111\rangle]_{789}|+\rangle_{10}\\
&+\frac{1}{4}[\pm^a\alpha_4\beta_0|000\rangle - \alpha_0\beta_1|001\rangle \pm^b \pm^a\alpha_6\beta_2|010\rangle \mp^b \alpha_2\beta_3|011\rangle\\
&\pm^c \pm^a\alpha_5\beta_4|100\rangle \mp^c \alpha_1\beta_5|101\rangle \pm^c \pm^b \pm^a \alpha_7\beta_6|110\rangle \mp^c \pm^b\alpha_3\beta_7|111\rangle]_{789}|-\rangle_{10}, \qquad (8)\\
{}_{36}\langle\psi^{\pm}|_{25}\langle\phi^{\pm}|_{14}\langle\psi^{\pm}|\,\Psi\rangle =&\frac{1}{4}[\pm^c \pm^a \alpha_5\beta_0|000\rangle \pm^c \alpha_1\beta_1|001\rangle \pm^c \pm^b \pm^a \alpha_7\beta_2|010\rangle \pm^c \pm^b\alpha_3\beta_3|011\rangle\\
&\pm^a \alpha_4\beta_4|100\rangle + \alpha_0\beta_5|101\rangle \pm^b \pm^a\alpha_6\beta_6|110\rangle \pm^b \alpha_2\beta_7|111\rangle]_{789}|+\rangle_{10}\\
&+\frac{1}{4}[\pm^c \pm^a \alpha_5\beta_0|000\rangle \mp^c \alpha_1\beta_1|001\rangle \pm^c \pm^b \pm^a \alpha_7\beta_2|010\rangle \mp^c \pm^b\alpha_3\beta_3|011\rangle\\
&\pm^a \alpha_4\beta_4|100\rangle - \alpha_0\beta_5|101\rangle \pm^b \pm^a\alpha_6\beta_6|110\rangle \mp^b \alpha_2\beta_7|111\rangle]_{789}|-\rangle_{10}, \qquad (9)\\
{}_{36}\langle\phi^{\pm}|_{25}\langle\psi^{\pm}|_{14}\langle\psi^{\pm}|\,\Psi\rangle =&\frac{1}{4}[\pm^b \pm^a \alpha_6\beta_0|000\rangle \pm^c \pm^b\alpha_2\beta_1|001\rangle \pm^a \alpha_4\beta_2|010\rangle + \alpha_0\beta_3|011\rangle\\
&\pm^c \pm^b \pm^a \alpha_7\beta_4|100\rangle \pm^c \pm^b\alpha_3\beta_5|101\rangle \pm^c \pm^a\alpha_5\beta_6|110\rangle \pm^c \alpha_1\beta_7|111\rangle]_{789}|+\rangle_{10}\\
&+\frac{1}{4}[\pm^b \pm^a \alpha_6\beta_0|000\rangle \mp^c \pm^b\alpha_2\beta_1|001\rangle \pm^a \alpha_4\beta_2|010\rangle - \alpha_0\beta_3|011\rangle\\
&\pm^c \pm^b \pm^a \alpha_7\beta_4|100\rangle \mp^c \pm^b\alpha_3\beta_5|101\rangle \pm^c \pm^a\alpha_5\beta_6|110\rangle \mp^c \alpha_1\beta_7|111\rangle]_{789}|-\rangle_{10}, \qquad (10)\\
{}_{36}\langle\psi^{\pm}|_{25}\langle\psi^{\pm}|_{14}\langle\psi^{\pm}|\,\Psi\rangle =&\frac{1}{4}[\pm^c \pm^b \pm^a\alpha_7\beta_0|000\rangle \pm^c \pm^b\alpha_3\beta_1|001\rangle \pm^c \pm^a\alpha_5\beta_2|010\rangle \pm^c \alpha_1\beta_3|011\rangle\\
&\pm^b \pm^a\alpha_6\beta_4|100\rangle \pm^b \alpha_2\beta_5|101\rangle \pm^a \alpha_4\beta_6|110\rangle + \alpha_0\beta_7|111\rangle]_{789}|+\rangle_{10}\\
&+\frac{1}{4}[\pm^c \pm^b \pm^a\alpha_7\beta_0|000\rangle \mp^c \pm^b\alpha_3\beta_1|001\rangle \pm^c \pm^a\alpha_5\beta_2|010\rangle \mp^c \alpha_1\beta_3|011\rangle\\
&\pm^b \pm^a\alpha_6\beta_{41}|100\rangle \mp^b \alpha_2\beta_5|101\rangle \pm^a \alpha_4\beta_6|110\rangle - \alpha_0\beta_7|111\rangle]_{789}|-\rangle_{10}, \qquad (11)
\end{aligned}
$$

where $|\phi^{\pm}\rangle_{ij} = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$ and $|\psi^{\pm}\rangle_{ij} = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$ are four Bell-states of particles i and j and $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$, and where $\pm^a$, $\pm^b$ and $\pm^c$ correspond to the superscripts for the Bell-states composed of particles $(1,4)$, particles $(2,5)$ and particles $(3,6)$, respectively and $\mp^a = -\pm^a$, $\mp^b = -\pm^b$, and $\mp^c = -\pm^c$. For example, if Alice's measurements are$|\psi^-\rangle_{14}$, $|\psi^+\rangle_{25}$, and $|\phi^+\rangle_{36}$, namely the corresponding superscripts are $-, +$ and $+$ in Eq.(10), the states of particles $7, 8, 9$ and 10 will collapse into

$$
\begin{aligned}
&{}_{36}\langle\phi^+|_{25}\langle\psi^+|_{14}\langle\psi^-|\,\Psi\rangle \\
=&\frac{1}{4}[-\alpha_6\beta_0|000\rangle + \alpha_2\beta_1|001\rangle \\
&- \alpha_4\beta_2|010\rangle + \alpha_0\beta_3|011\rangle \\
&- \alpha_7\beta_4|100\rangle + \alpha_3\beta_5|101\rangle \\
&- \alpha_5\beta_6|110\rangle + \alpha_1\beta_7|111\rangle]_{789}|+\rangle_{10} \\
&+ \frac{1}{4}[-\alpha_6\beta_0|000\rangle - \alpha_2\beta_1|001\rangle \\
&- \alpha_4\beta_2|010\rangle - \alpha_0\beta_3|011\rangle \\
&- \alpha_7\beta_4|100\rangle - \alpha_3\beta_5|101\rangle \\
&- \alpha_5\beta_6|110\rangle - \alpha_1\beta_7|111\rangle]_{789}|-\rangle_{10}. \qquad (12)
\end{aligned}
$$

After that, Alice informs Bob and Charlie of the measurements via a classical communication. For assisting Alice and Bob, Charlie should make a measurement on particle 10 using the base $\{|+\rangle_{10}, |-\rangle_{10}\}$ and transmit her results to Bob over a classical communication channel, then the particles $7, 8, 9$ in Bob's place collapse into one of 128 kinds of states, again. For instance, if Charlie obtains $|+\rangle$, then in Eq.(12) particles $7, 8, 9$ collapse into the following state:

$$
\begin{aligned}
&\frac{1}{4}[-\alpha_6\beta_0|000\rangle + \alpha_2\beta_1|001\rangle - \alpha_4\beta_2|010\rangle + \alpha_0\beta_3|011\rangle \\
&- \alpha_7\beta_4|100\rangle + \alpha_3\beta_5|101\rangle \\
&- \alpha_5\beta_6|110\rangle + \alpha_1\beta_7|111\rangle]_{789}, \qquad (13)
\end{aligned}
$$

otherwise, the state is

$$
\begin{aligned}
&\frac{1}{4}[-\alpha_6\beta_0|000\rangle - \alpha_2\beta_1|001\rangle - \alpha_4\beta_2|010\rangle - \alpha_0\beta_3|011\rangle \\
&- \alpha_7\beta_4|100\rangle - \alpha_3\beta_5|101\rangle \\
&- \alpha_5\beta_6|110\rangle - \alpha_1\beta_7|111\rangle]_{789}. \qquad (14)
\end{aligned}
$$

In order to make the teleportation successful, Bob needs to recover the original state $|\Phi\rangle_{123}$ on his side from the states of particles $7, 8, 9$. Firstly, Bob introduces an auxiliary particle A with an initial state $|0\rangle_A$ and performs a collective unitary transformation U_1 on particles $7, 8, 9$ and particle A. Under the base of particles $7, 8, 9$ and $A\{|0000\rangle, |0001\rangle, \ldots |1111\rangle\}$, the unitary transformation U_1 (a 16×16 matrix) may take the form

$$
U_1 = \begin{pmatrix}
I & 0 & 0 & \cdots & 0 \\
0 & u_1 & 0 & \cdots & 0 \\
0 & 0 & u_2 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & u_7
\end{pmatrix}, \qquad (15)
$$

where $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$,

$$
u_i = \begin{pmatrix}
\beta_0/\beta_i & -\sqrt{1-\beta_0^2/\beta_i^2} \\
\sqrt{1-\beta_0^2/\beta_i^2} & \beta_0/\beta_i
\end{pmatrix}
$$

$(i = 1, 2, \ldots, 7)$. For example, the unitary transformation U_1 will transform the state in Eq.(13) into

$$
\begin{aligned}
&\frac{\beta_0}{4}[-\alpha_6|000\rangle + \alpha_2|001\rangle - \alpha_4|010\rangle + \alpha_0|011\rangle - \alpha_7|100\rangle + \alpha_3|101\rangle - \alpha_5|110\rangle \\
&+ \alpha_1|111\rangle]_{789}|0\rangle_A + \frac{1}{4}[\alpha_2\beta_1\sqrt{1-\beta_0^2/\beta_1^2}|001\rangle - \alpha_4\beta_2\sqrt{1-\beta_0^2/\beta_2^2}|010\rangle \\
&+ \alpha_0\beta_3\sqrt{1-\beta_0^2/\beta_3^2}|011\rangle - \alpha_7\beta_4\sqrt{1-\beta_0^2/\beta_4^2}|100\rangle + \alpha_3\beta_5\sqrt{1-\beta_0^2/\beta_5^2}|101\rangle \\
&- \alpha_5\beta_6\sqrt{1-\beta_0^2/\beta_6^2}|110\rangle + \alpha_1\beta_7\sqrt{1-\beta_0^2/\beta_7^2}|111\rangle]_{789}|1\rangle_A. \qquad (16)
\end{aligned}
$$

Similarly, Eq.(14) turns into

$$\frac{\beta_0}{4}[-\alpha_6|000\rangle-\alpha_2|001\rangle-\alpha_4|010\rangle-\alpha_0|011\rangle-\alpha_7|100\rangle-\alpha_3|101\rangle-\alpha_5|110\rangle$$
$$-\alpha_1|111\rangle]_{789}|0\rangle_A-\frac{1}{4}[\alpha_2\beta_1\sqrt{1-\beta_0^2/\beta_1^2}|001\rangle+\alpha_4\beta_2\sqrt{1-\beta_0^2/\beta_2^2}|010\rangle$$
$$+\alpha_0\beta_3\sqrt{1-\beta_0^2/\beta_3^2}|011\rangle+\alpha_7\beta_4\sqrt{1-\beta_0^2/\beta_4^2}|100\rangle+\alpha_3\beta_5\sqrt{1-\beta_0^2/\beta_5^2}|101\rangle$$
$$+\alpha_5\beta_6\sqrt{1-\beta_0^2/\beta_6^2}|110\rangle+\alpha_1\beta_7\sqrt{1-\beta_0^2/\beta_7^2}|111\rangle]_{789}|1\rangle_A. \quad (17)$$

Secondly, Bob will make a measurement on auxiliary particle A. If the outcome is $|1\rangle_A$, the teleportation fails; otherwise, the states of particles $7,8,9$ will further collapse into the desired state $|\Phi\rangle_{789}$. For example, in Eq.(16) Bob has a probability of $|\beta_0|^2/16$ to obtain the following state:

$$[-\alpha_6|000\rangle+\alpha_2|001\rangle-\alpha_4|010\rangle+\alpha_0|011\rangle$$
$$-\alpha_7|100\rangle+\alpha_3|101\rangle-\alpha_5|110\rangle+\alpha_1|111\rangle]_{789}, \quad (18)$$

and in Eq.(17) Bob also has the probability $|\beta_0|^2/16$ to obtain the following state:

$$[-\alpha_6|000\rangle-\alpha_2|001\rangle-\alpha_4|010\rangle-\alpha_0|011\rangle$$
$$-\alpha_7|100\rangle-\alpha_3|101\rangle-\alpha_5|110\rangle-\alpha_1|111\rangle]_{789}. \quad (19)$$

Table 1. Unitary transformation U_2 corresponding to the states of particles 7, 8 and 9.

States of particles 7, 8 and 9	The corresponding operation
${}_{36}\langle\phi^{\pm}|_{25}\langle\phi^{\pm}|_{14}\langle\phi^{\pm}|\,\Psi\rangle$	S_{79}
${}_{36}\langle\psi^{\pm}|_{25}\langle\phi^{\pm}|_{14}\langle\phi^{\pm}|\,\Psi\rangle$	$(\sigma_X)_9S_{79}$
${}_{36}\langle\phi^{\pm}|_{25}\langle\psi^{\pm}|_{14}\langle\phi^{\pm}|\,\Psi\rangle$	$C_{78}S_{79}$
${}_{36}\langle\psi^{\pm}|_{25}\langle\psi^{\pm}|_{14}\langle\phi^{\pm}|\,\Psi\rangle$	$(\sigma_X)_9(\sigma_X)_8S_{79}$
${}_{36}\langle\phi^{\pm}|_{25}\langle\phi^{\pm}|_{14}\langle\psi^{\pm}|\,\Psi\rangle$	$(\sigma_X)_7S_{79}$
${}_{36}\langle\psi^{\pm}|_{25}\langle\phi^{\pm}|_{14}\langle\psi^{\pm}|\,\Psi\rangle$	$(\sigma_X)_8C_{87}C_{89}(\sigma_X)_8C_{87}C_{89}S_{79}$
${}_{36}\langle\phi^{\pm}|_{25}\langle\psi^{\pm}|_{14}\langle\psi^{\pm}|\,\Psi\rangle$	$(\sigma_X)_9C_{97}C_{98}(\sigma_X)_9C_{97}C_{98}S_{79}$
${}_{36}\langle\psi^{\pm}|_{25}\langle\psi^{\pm}|_{14}\langle\psi^{\pm}|\,\Psi\rangle$	$(\sigma_X)_9(\sigma_X)_8(\sigma_X)_7S_{79}$

Finally, Bob may operate another unitary transformation U_2 on particles $7,8,9$ and then choose appropriate unitary operations $(I,\sigma_X,\sigma_Y,\sigma_Z)$ in order to recover the original state he wants. All possible unitary transformations on the states of particles $7,8,9$ are listed in Table 1. For instance, the unitary transformation $U_2=(\sigma_X)_9C_{97}C_{98}(\sigma_X)_9C_{97}C_{98}S_{79}$, where σ_X is a Pauli operation, C_{ij} is CNOT gate of control qubit i and target qubit j and S_{ij} is the SWAP operation which is defined as $S_{ij}|m\rangle_i\otimes|n\rangle_j=|n\rangle_i\otimes|m\rangle_j$, will transform the state expressed by Eq.(18) into

$$[\alpha_0|000\rangle+\alpha_1|001\rangle+\alpha_2|010\rangle+\alpha_3|011\rangle$$
$$-\alpha_4|100\rangle-\alpha_5|101\rangle-\alpha_6|110\rangle-\alpha_7|111\rangle]_{789}, \quad (20)$$

and Eq.(19) turns into

$$[-\alpha_0|000\rangle-\alpha_1|001\rangle-\alpha_2|010\rangle-\alpha_3|011\rangle$$
$$-\alpha_4|100\rangle-\alpha_5|101\rangle-\alpha_6|110\rangle-\alpha_7|111\rangle]_{789}. \quad (21)$$

Then for Eq.(20), by using σ_Z on the particles 7 Bob may obtain the desired state of particles $7,8,9$ and for Eq.(21), Bob does not need any operation to have the original state. By using a similar method, Bob may make the teleportation successful in the other outcomes of particles $7,8,9$. For the sake of saving space we will not write them out.

Now we calculate the probability of successful teleportation in this scheme. Equation (16) shows that the probability of successful teleportation is $|\beta_0|^2/16$. It can be easily proven that there exist 128 kinds of states that share the same probability, therefore in all synthesized cases, the probability of successful teleportation in this scheme is $\frac{|\beta_0|^2}{16}\times128=8|\beta_0|^2$, which depends on the smallest coefficient of the entangled state used as the quantum channel. Evidently, if coefficients of the entangled state are all on equality i.e. the maximally entangled state, auxiliary particle A is not needed and the total probability reaches 1, i.e. the probabilistic and controlled teleportation becomes the ordinary controlled one.

From the above analysis, if Charlie does not cooperate with Alice and Bob, the teleportation will not be realized between Alice and Bob. In other words, Charlie can be regarded as a supervisor in the teleportation process.

3. Quantum networks for the above scheme

In general, any unitary operation can be decomposed into universal operations consisting of local unitary operations on a single qubit and two-bit

Controlled-NOT (CNOT) gates.[21,22] Based on the above scheme, we can construct an efficient quantum network for implementing this new scheme by means of the primitive operations consisting of single-qubit gates, two-qubit controlled-Not gates, Von Neumann measurement and classically controlled operations in the following.

To begin with, we array efficient gates for the unitary operation U_1. Evidently, the implementation of u_i in Eq.(15) plays an essential role in this teleportation. In Ref.[27], a four-bit controlled gate is constructed from the NOT gate and controlled-controlled-NOT (Toffoli gate).[22] The four-qubit controlled u_i gate, which applies u_i to the fourth qubit if the first three qubits have the value 1, can be expressed in Fig.2, where we assume $\cos\dfrac{\theta_i}{2}=\dfrac{\beta_0}{\beta_i}$, then

$$u_i = R_y(\theta_i) = \begin{pmatrix} \cos\dfrac{\theta_i}{2} & -\sin\dfrac{\theta_i}{2} \\ \sin\dfrac{\theta_i}{2} & \cos\dfrac{\theta_i}{2} \end{pmatrix} \quad (i=1,2,\ldots,7),$$

$A_i = R_y\left(\dfrac{\theta_i}{2}\right)$ and $B_i = R_y\left(-\dfrac{\theta_i}{2}\right)$ which represent rotations of θ_i and $-\theta_i$ around the y-axis, respectively, and the dot "•" denotes the control qubit, and the cross "$\oplus$" represents the control qubit. For instance, the state $|1110\rangle$ turns into $|111\rangle\left(\dfrac{\beta_0}{\beta_i}|0\rangle + \sqrt{1-\beta_0^2/\beta_1^2}|1\rangle\right)$ after the operation of four-qubit controlled u_i gate. Therefore, from Eqs.(16) and (17), the sequence of the gate U_1 is shown in Fig.3 ,which is the same as that in Ref.[24], where σ_X is a Pauli operation.

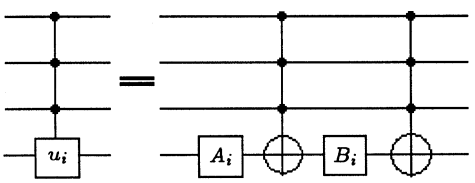


Fig.2. Two equivalent circuits using a set of single-bit gates and four-qubit controlled NOT gates.

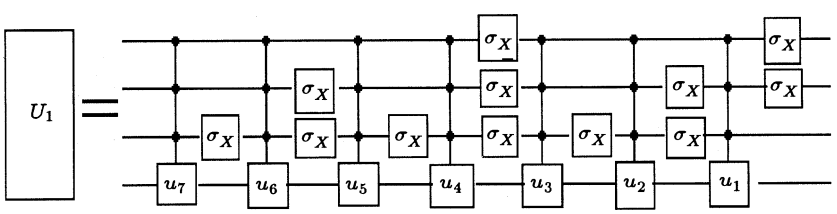


Fig.3. Network implementing the U_1 operation.

Based on the above description and the unitary operation U_1, the network of probabilistic and controlled teleportation of an unknown tripartite state $|\Phi\rangle_{123}$ in a general form is shown in Fig.4. Here, $R_i = \begin{pmatrix} a_i & -b_i \\ b_i & a_i \end{pmatrix}$ $(a_i^2+b_i^2=1, i=1,2,3)$ is a single-qubit rotation transformation, and H is a Hadamard gate defined as $H=\dfrac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ and the six top lines represent Alice's system, the seventh-tenth lines are for Bob's system, where the tenth line is for the auxiliary qubit introduced by Bob, while the bottom line is for Charlie's system. If the measurement on the auxiliary qubit in the output state is $|0\rangle_A$, the teleportation is successful with the final state of particles $7,8,9$ reconstructed as the initial state to be teleported. From Fig.4, we see that the teleportation can be divided into three steps. The first step is that we carefully choose a_i and $b_i (i=1,2,3)$ to make the state of particles $4,5,6,7,8,9$ and 10 the quantum channel like Eq.(2), indicated by the first dash line. The second step is a purification process in which Bob can concentrate the state $|\varphi\rangle_{45678910}$ into a maximally entangled state by introducing an auxiliary particle A and performing a collective unitary operation U_1. The final step is the standard teleportation using the maximal entanglement as the quantum channel.

Note that a controlled unitary operation on any number of qubits followed by a measurement of the control qubit can be replace by the measurement of the control qubit prior to the controlled operation.[28,29] Therefore, Fig.4 can be expressed as Fig.5.

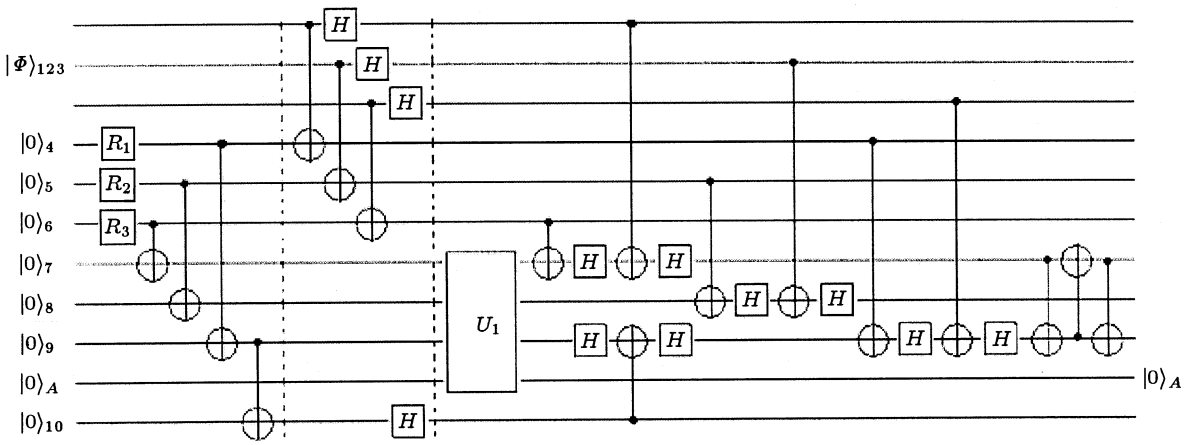


Fig.4. Quantum circuit for probabilistic and controlled teleportation of an arbitrary three-particle state.

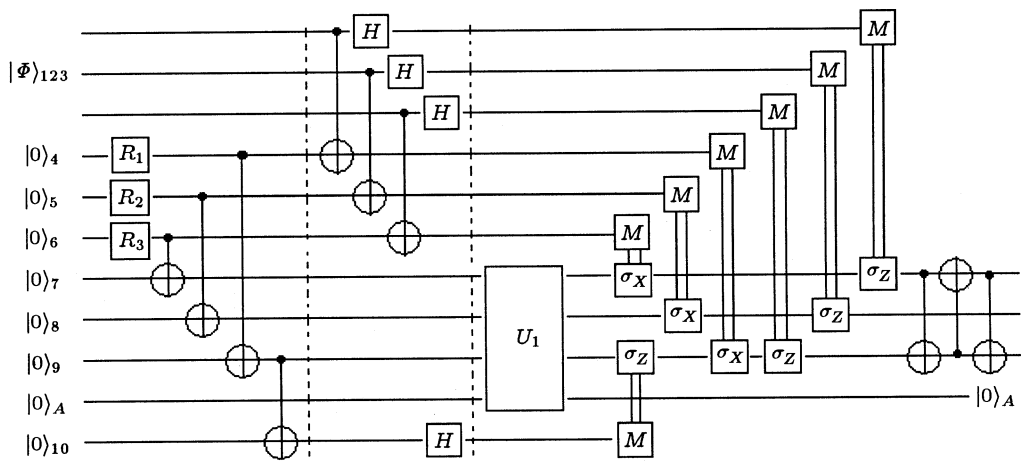


Fig.5. The same as Fig.4, but for Von Neumann measurement M and classical controlled operation being made at the end.

Here, the double line coming out of the measurement M denotes the classical information. In Fig.5, Bob, relying on the measurements made by Alice and Charlie on their own qubit, can realize the controlled operation locally. If and only if the outcome of their measurement is 1, Bob can operate the controlled-σ_X or controlled-σ_Z ($\sigma_Z = H\sigma_X H$). Clearly, if the measurement of the auxiliary particle is $|1\rangle_A$, this teleportation fails.

4. Conclusion

In summary, we have proposed a scheme for probabilistic and controlled teleportation of an unknown arbitrary tripartite state by constructing the peculiar non-maximally entangled state as the controlled quantum channel. As pointed out in Refs.[16, 17], there is still a certain probability of teleportation for such a non-maximally entangled quantum channel and the probability of success is determined by the smallest coefficient of non-maximally entangled states used as the quantum channel. Not only because does an arbitrary three-particle state include all kinds of three-particle states in a particular case, no matter whether they are entangled, for instance, if $\alpha_3 \neq 0$, $\alpha_7 \neq 0$, and $\alpha_i = 0$ ($i = 0, 1, 2, 4, 5, 6$), $|\Phi\rangle_{123} = \alpha_0|000\rangle + \alpha_7|111\rangle$ is the GHZ-type state, if $\alpha_1 \neq 0$, $\alpha_2 \neq 0$, $\alpha_4 \neq 0$, and $\alpha_i = 0$ ($i = 0, 3, 5, 6, 7$), $|\Phi\rangle_{123} = \alpha_1|001\rangle + \alpha_2|010\rangle + \alpha_4|100\rangle$ is the W-type state, but also the quantum logic networks for implementing this scheme has be provided. Our scheme may be widely useful to the future quantum communication and experiment. Finally, we point out that this scheme can also be generalized to controlled teleportation of an arbitrary and unknown multi-particle state, so we expect this scheme to be realized by experiment.

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