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Exact travelling wave solutions for (1+1)-dimensional dispersive long wave equation

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A complete discrimination system for the fourth order polynomial is given. As an application, we have reduced a (1+1)-dimensional dispersive long wave equation with general coefficients to an elementary integral form and obtained its all possible exact travelling wave solutions including rational function type solutions, solitary wave solutions, triangle function type periodic solutions and Jacobian elliptic functions double periodic solutions. This method can be also applied to many other similar problems.

Keywords: complete discrimination system for polynomial, (1+1)-dimensional dispersive long wave equation, travelling wave solution

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1. Introduction

There exist many methods of obtaining the exact solutions for nonlinear evolution equations, such as homogeneous balance method,[1−3] varied hyperbolic function expansion method,[4] mixing exponential method,[5] varied elliptic function expansion method,[6,7] sine–cosine method,[8] variable separation approach[9] and nonlinear transformation method[10] and so on. Fan[11,12] introduced an algebraic method into which many known methods were integrated. But there exist allegedly few methods of being able to give all possible exact travelling wave solutions for the equation considered. Recently, Liu[13] introduced a simpler method than Fan's, that is, trial equation method. If a nonlinear equation can be reduced to an elementary integral form[14−16] directly or by using the trial equation method,[13] then in some cases all possible exact travelling wave solutions can be obtained. In order to apply these methods, we often need to classify the roots of a polynomial according to its coefficients. Yang *et al*[17] have solved the problem and obtained a complete discrimination system for polynomial with words coefficients. In the present paper, we give the complete discrimination system for the fourth order polynomial according to the theory in Ref.[17]. As an application, we consider a (1+1)-dimensional dispersive long wave equation (DLWE)[18−20] with general coefficients, and reduce it to the form of elementary integral, and obtain its all possible exact travelling wave solutions which include rational function type solutions, solitary wave solutions, triangle function type periodic solutions and Jacobian elliptic functions double periodic solutions.

The solutions of the DLWE are very helpful for coastal and harbor design. Therefore many papers are devoted to this equation,[18−20] and many types of its exact solutions have been obtained. But there is not any known method that can give all possible exact travelling wave solutions. We can search for such a method by using a complete discrimination system for the fourth order polynomial. Moreover, this method can be applied to many other equations such as double Sine–Gordon equation, combined KdV–MKdV equation and (2+1)-dimensional breaking soliton equation.

In Section 2, we give the complete discrimination system for the fourth order polynomial. In Section 3, we obtain all possible exact travelling wave solutions for the DLWE. In Section 4, we draw a conclusion.

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2. A complete discrimination system for the fourth order polynomial

Definition 1[17] Given a polynomial $f(x) = a_n x^n + \cdots + a_1 x + a_0$. The Sylverster matrix of $f(x)$ and $f'(x)$ is called the discrimination matrix of $f(x)$, and denoted by Discr(f). D_k denotes the determinant of the submatrix of Discr(f), formed by the first $2k$ rows and the first $2k$ columns, for $k = 1, \ldots, n$. We call the n-tuple $\{D_1, \ldots, D_n\}$ the discriminant sequence of polynomial $f(x)$.

Let M=Discr(f), M_k denotes the submatrix formed by the first $2k$ rows of M, for k=1,..., n; and $M(k,i)$ refers to the submatrix formed by the first $2k-1$ columns and the $(2k+i)$th columns of M_k, for k=1,..., n; i=0,...,$n-k$.

Definition 2[17] We construct polynomials $\Delta_k(f) = \sum_{i=0}^{k} \det(M(n-k,i))x^{k-i}$, for $k = 0, \ldots, n-1$. We call the n-tuple $\{\Delta_0(f),\ldots, \Delta_{n-1}(f)\}$ the multiple factor sequence of $f(x)$.

Definition 3[17] U denotes the union of all multiple factor sequences at different levels, that is, U=$\{f(x),\ \Delta_k(f),\ \Delta_j(\Delta_k(f)),\ \Delta_i(\Delta_j(\Delta_k(f))),\ldots\}$. Every polynomial in U has a discriminant sequence, and all of them form a complete discrimination system of $f(x)$.

In the present paper, we consider the following fourth order polynomial:

$$F(w) = w^4 + pw^2 + qw + r. \tag{1}$$

According to the theory in Ref.[17], its complete discrimination system is computed as follows:

$$D_1 = 4, \quad D_2 = -p, \quad D_3 = -2p^3 + 8pr - 9q^2,$$

$$D_4 = -p^3q^2 + 4p^4r + 36pq^2r - 32p^2r^2 - \frac{27}{4}q^4 + 64r^3,$$

$$E_2 = 9p^2 - 32pr, \tag{2}$$

where E_2 is the discriminant of $\Delta_2(F)$.

By use of Eqs.(2), we can classify the roots of Eq.(1). In Section 3, we will give the details of the classification.

The essential ideal of our method is to reduce the equation considered to the following elementary integral:

$$\int \frac{\mathrm{d}w}{\sqrt{F(w)}} = \pm(\xi - \xi_0),$$

where $F(w)$ is a polynomial. According to the complete discrimination system for $F(w)$, we classify the roots of $F(w)$ and solve the respective integral.

3. All possible exact travelling wave solutions for (1+1)-dimensional DLWE

Consider a (1+1)-dimensional dispersive long wave equation with general coefficients,

$$u_t + \varepsilon_1 u u_x + \varepsilon_2 v_x = 0, \tag{3}$$

$$v_t + \eta_1 (uv)_x + \eta_2 u_{xxx} = 0. \tag{4}$$

As a particular case, we obtain a (1+1)-dimensional dispersive long wave equation[18−20] when parameters take their values as follows:

$$\varepsilon_1 = \varepsilon_2 = \eta_1 = 1,\ \eta_2 = \frac{1}{3}. \tag{5}$$

In order to find travelling wave solutions, let

$$u = u(\xi), \quad v = v(\xi), \quad \xi = k(x - ct). \tag{6}$$

Substituting the values (5) into Eqs.(3) and (4) yields

$$-cu' + \varepsilon_1 uu' + \varepsilon_2 v' = 0, \tag{7}$$

$$-cv' + \eta_1 (uv)' + \eta_2 k^2 u''' = 0. \tag{8}$$

Integrating Eqs.(7) and (8) once and further carrying out the transformation yield

$$(u')^2 = a_4 u^4 + a_3 u^3 + a_2 u^2 + a_1 u + a_0, \tag{9}$$

$$v = -\frac{\varepsilon_1}{2\varepsilon_2} u^2 + \frac{c}{\varepsilon_2} u + c_1, \tag{10}$$

where

$$a_4 = \frac{\varepsilon_1 \eta_1}{4\varepsilon_2 \eta_2 k^2},$$

$$a_3 = -\frac{2c\eta_1 + c\varepsilon_1}{3\varepsilon_2 \eta_2 k^2},$$

$$a_2 = \frac{c^2 - \eta_1 \varepsilon_2 c_1}{\eta_2 \varepsilon_2 k^2}, \tag{11}$$

and a_0, a_1, c_1 are arbitrary constants. In order to solve Eq.(9), when $a_4 > 0$, we take the transformation as follows:

$$w = (a_4)^{\frac{1}{4}} \left(u + \frac{a_3}{4a_4} \right), \quad \xi_1 = (a_4)^{\frac{1}{4}} \xi. \tag{12}$$

Combining the expression (12) with Eq.(8) yields

$$w_{\xi_1}^2 = F(w) = w^4 + pw^2 + qw + r, \tag{13}$$

where

$$p = \frac{a_2}{\sqrt{a_4}}, \quad q = \left(\frac{a_3^3}{8a_4^2} - \frac{a_2 a_3}{2a_4} + a_1 \right) (a_4)^{-\frac{1}{4}},$$

and

$$r = \frac{-3a_3^4}{256a_4^3} + \frac{a_2a_3^2}{16a_4^2} - \frac{a_1a_3}{4a_4} + a_0. \quad (14)$$

If $a_4 < 0$, then we take the following transformation:

$$w = (-a_4)^{\frac{1}{4}}\left(u + \frac{a_3}{4a_4}\right), \quad \text{and} \quad \xi_1 = (-a_4)^{\frac{1}{4}}\xi. \quad (15)$$

Combining the expression Eq.(15) with Eq.(9) yields

$$w_{\xi_1}^2 = -F(w) = -(w^4 + pw^2 + qw + r), \quad (16)$$

where

$$p = \frac{-a_2}{\sqrt{-a_4}}, \quad q = \left(-\frac{a_3^3}{8a_4^2} + \frac{a_2a_3}{2a_4} - a_1\right)(-a_4)^{-\frac{1}{4}},$$

and

$$r = \frac{3a_3^4}{256a_4^3} - \frac{a_2a_3^2}{16a_4^2} - \frac{a_1a_3}{4a_4} - a_0. \quad (17)$$

In order to obtain the solutions to Eqs.(13) and (16), according to the complete discrimination system for the fourth order polynomial given in Section 2, nine cases are discussed separately as follows:

Case 1 $D_2 < 0$, $D_3 = 0$, and $D_4 = 0$. $F(w)$ has a pair of conjugate complex roots of multiplicities two, i.e.

$$F(w) = ((w-l)^2 + s^2)^2, \quad (18)$$

where $s > 0$. For $a_4 > 0$, by use of Eq.(13), we have

$$\xi_1 - \xi_0 = \int \frac{\mathrm{d}w}{(w-l)^2 + s^2} = \frac{1}{s}\arctan\frac{w-l}{s}, \quad (19)$$

where ξ_0 is an integral constant hereafter. The solution of Eq.(13) is

$$w = s\tan(s(\xi_1 - \xi_0)) + l, \quad (20)$$

which is a triangular function periodic solution.

Case 2 $D_2 = 0$, $D_3 = 0$, and $D_4 = 0$. $F(w)$ has a real root zero of multiplicity four, i.e.

$$F(w) = w^4. \quad (21)$$

When $a_4 > 0$, by using Eq.(13), we have

$$\xi_1 - \xi_0 = \int \frac{\mathrm{d}w}{w^2} = -w^{-1}. \quad (22)$$

We derive the solution of Eq.(13) as follows:

$$w = -(\xi_1 - \xi_0)^{-1}, \quad (23)$$

which is a rational solution.

Case 3 $D_2 > 0$, $D_3 = 0$, $D_4 = 0$, and $E_2 > 0$. $F(w)$ has two distinct real roots of multiplicities two such that

$$F(w) = (w-\alpha)^2(w-\beta)^2, \quad (24)$$

where $\alpha > \beta$. For $a_4 > 0$, from Eq.(13) we have

$$\pm(\xi_1 - \xi_0) = \int \frac{dw}{(w-\alpha)(w-\beta)} = \frac{1}{\alpha-\beta}\ln\left|\frac{w-\alpha}{w-\beta}\right|. \quad (25)$$

For $w > \alpha$ or $w < \beta$, by using Eq.(25) we have

$$w = \frac{\beta-\alpha}{\mathrm{e}^{(\alpha-\beta)(\xi_1-\xi_0)} - 1} + \beta = \frac{\beta-\alpha}{2}\left[\coth\frac{(\alpha-\beta)(\xi_1-\xi_0)}{2} - 1\right] + \beta. \quad (26)$$

When $\beta < w < \alpha$, by using Eq.(25), we have

$$w = \frac{\beta-\alpha}{-\mathrm{e}^{(\alpha-\beta)(\xi_1-\xi_0)} - 1} + \beta = \frac{\beta-\alpha}{2}\left[\tanh\frac{(\alpha-\beta)(\xi_1-\xi_0)}{2} - 1\right] + \beta, \quad (27)$$

where the expressions (26) and (27) are solitary wave solutions.

Case 4 $D_2 > 0$, $D_3 = 0$, $D_4 = 0$, and $E_2 = 0$. $F(w)$ has a real root of multiplicity three and a real root of multiplicity one, i.e.

$$F(w) = (w-\alpha)^3(w-\beta). \quad (28)$$

When $a_4 > 0$, integrating Eq.(13) yields

$$\pm(\xi_1 - \xi_0) = \int \frac{\mathrm{d}w}{(w-\alpha)\sqrt{(w-\alpha)(w-\beta)}} = \frac{2}{\beta-\alpha}\sqrt{\frac{w-\beta}{w-\alpha}}. \quad (29)$$

When $w > \alpha$, $w > \beta$ or $w < \alpha$, $w < \beta$, the solution of Eq.(13) is

$$w = \frac{4(\alpha-\beta)}{(\beta-\alpha)^2(\xi_1-\xi_0)^2 - 4} + \alpha. \quad (30)$$

For $a_4 < 0$, by using Eq.(16), we have

$$\pm(\xi_1 - \xi_0) = \int \frac{\mathrm{d}w}{(w-\alpha)\sqrt{(w-\alpha)(\beta-w)}} = \frac{2}{\alpha-\beta}\sqrt{\frac{\beta-w}{w-\alpha}}. \quad (31)$$

When $w > \alpha$, $w < \beta$ or $w < \alpha, w > \beta$, the solution of Eq.(16) is

$$w = \frac{4(\alpha-\beta)}{-(\beta-\alpha)^2(\xi_1-\xi_0)^2 - 4} + \alpha, \quad (32)$$

where the expressions (30) and (32) are rational solutions.

Case 5 $D_2D_3 < 0$, and $D_4 = 0$. $F(w)$ has a real roots of multiplicity two and a pair of conjugate complex roots, i.e.

$$F(w) = (w-\alpha)^2((w-l)^2+s^2). \tag{33}$$

By using Eq.(13) we have

$$\begin{aligned}\pm(\xi_1-\xi_0) =& \int \frac{\mathrm{d}w}{(w-\alpha)\sqrt{(w-l)^2+s^2}} \\ =& \frac{1}{\sqrt{(\alpha-l)^2+s^2}} \\ & \times \ln\left|\frac{\gamma w+\delta-\sqrt{(w-l)^2+s^2}}{w-\alpha}\right|, \end{aligned}\tag{34}$$

where

$$\gamma = \frac{\alpha-2l}{\sqrt{(\alpha-l)^2+s^2}},$$

and

$$\delta = \sqrt{(\alpha-l)^2+s^2} - \frac{\alpha(\alpha-2l)}{\sqrt{(\alpha-l)^2+s^2}}. \tag{35}$$

Correspondingly we have the solution of Eq.(13)

$$w = \frac{(\mathrm{e}^{\pm\sqrt{(\alpha-l)^2+s^3}(\xi-\xi_0)}-\gamma)+\sqrt{(\alpha-l)^2+s^2}(2-\gamma)}{(\mathrm{e}^{\pm\sqrt{(\alpha-l)^2+s^2}(\xi-\xi_0)}-\gamma)^2-1}, \tag{36}$$

which is a solitary wave solution.

Case 6 $D_2 > 0$, $D_3 > 0$, and $D_4 > 0$. $F(w)$ has four distinct real roots such that

$$F(w) = (w-\alpha_1)(w-\alpha_2)(w-\alpha_3)(w-\alpha_4), \tag{37}$$

where $\alpha_1 > \alpha_2 > \alpha_3 > \alpha_4$. When $a_4 > 0$, if $w > \alpha_1$ or $w < \alpha_4$, then we take the following transformation:

$$w = \frac{\alpha_2(\alpha_1-\alpha_4)\sin^2\phi - \alpha_1(\alpha_2-\alpha_4)}{(\alpha_1-\alpha_4)\sin^2\phi - (\alpha_2-\alpha_4)}. \tag{38}$$

If $\alpha_3 < w < \alpha_2$, then we take the transformation of

$$w = \frac{\alpha_4(\alpha_2-\alpha_3)\sin^2\phi - \alpha_3(\alpha_2-\alpha_4)}{(\alpha_2-\alpha_3)\sin^2\phi - (\alpha_2-\alpha_4)}. \tag{39}$$

Combining the expression (38) or (39) with Eq.(13) yields

$$\begin{aligned}\xi_1-\xi_0 =& \int \frac{\mathrm{d}w}{\sqrt{(w-\alpha_1)(w-\alpha_2)(w-\alpha_3)(w-\alpha_4)}} \\ =& \frac{2}{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}\int \frac{d\phi}{\sqrt{1-m^2\sin^2\phi}}, \end{aligned}\tag{40}$$

where $m^2 = \dfrac{(\alpha_1-\alpha_4)(\alpha_2-\alpha_3)}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}$. By use of Eq.(40) and the definition of Jacobian elliptic sine function,[21] we have

$$\mathrm{sn}\left(\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}(\xi_1-\xi_0), m\right) = \sin\phi. \tag{41}$$

Combining Eq.(41) with the expressions (38) and (39) yields the following solutions of Eq.(13) with corresponding conditions:

$$w = \frac{\alpha_2(\alpha_1-\alpha_4)\mathrm{sn}^2\left(\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}(\xi_1-\xi_0), m\right) - \alpha_1(\alpha_2-\alpha_4)}{(\alpha_1-\alpha_4)\mathrm{sn}^2\left(\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}(\xi_1-\xi_0), m\right) - (\alpha_2-\alpha_4)}, \tag{42}$$

and

$$w = \frac{\alpha_4(\alpha_2-\alpha_3)\mathrm{sn}^2\left(\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}(\xi_1-\xi_0), m\right) - \alpha_3(\alpha_2-\alpha_4)}{(\alpha_2-\alpha_3)\mathrm{sn}^2\left(\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}(\xi_1-\xi_0), m\right) - (\alpha_2-\alpha_4)}. \tag{43}$$

For $a_4 < 0$, if $\alpha_1 > w > \alpha_2$, then we take the transformation as follows:

$$w = \frac{\alpha_3(\alpha_1-\alpha_2)\sin^2\phi - \alpha_2(\alpha_1-\alpha_3)}{(\alpha_1-\alpha_2)\sin^2\phi - (\alpha_1-\alpha_3)}; \tag{44}$$

if $\alpha_4 < w < \alpha_3$, then we take the following transformation:

$$w = \frac{\alpha_1(\alpha_3-\alpha_4)\sin^2\phi - \alpha_4(\alpha_3-\alpha_1)}{(\alpha_3-\alpha_4)\sin^2\phi - (\alpha_3-\alpha_1)}. \tag{45}$$

Similarly we have the following solutions of Eq.(16)

$$w=\frac{\alpha_3(\alpha_1-\alpha_2)\mathrm{sn}^2\left(\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}(\xi_1-\xi_0),m\right)-\alpha_2(\alpha_1-\alpha_3)}{(\alpha_1-\alpha_2)\mathrm{sn}^2\left(\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}(\xi_1-\xi_0),m\right)-(\alpha_1-\alpha_3)}, \tag{46}$$

and

$$w=\frac{\alpha_1(\alpha_3-\alpha_4)\mathrm{sn}^2\left(\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}(\xi_1-\xi_0),m\right)-\alpha_4(\alpha_3-\alpha_1)}{(\alpha_3-\alpha_4)\mathrm{sn}^2\left(\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}(\xi_1-\xi_0),m\right)-(\alpha_3-\alpha_1)}, \tag{47}$$

where $m^2=\dfrac{(\alpha_1-\alpha_2)(\alpha_3-\alpha_4)}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}$. The expressions (42), (43), (46) and (47) are elliptic functions double periodic solutions.

Case 7 $D_2D_3\geq 0$ and $D_4<0$. $F(w)$ has two distinct real roots and a pair of conjugate complex roots, i.e.

$$F(w)=(w-\alpha)(w-\beta)((w-l)^2+s^2), \tag{48}$$

where $\alpha>\beta$ and $s>0$. We take the transformation as follows:

$$w=\frac{a\cos\phi+b}{c\cos\phi+d}, \tag{49}$$

where

$$\begin{aligned}
a&=\frac{1}{2}(\alpha+\beta)c-\frac{1}{2}(\alpha-\beta)d,\\
b&=\frac{1}{2}(\alpha+\beta)d-\frac{1}{2}(\alpha-\beta)c,\\
c&=\alpha-l-\frac{s}{m_1},\\
d&=\alpha-l-sm_1,\\
E&=\frac{s^2+(\alpha-l)(\beta-l)}{s(\alpha-\beta)},\\
m_1&=E\pm\sqrt{E^2+1}.
\end{aligned} \tag{50}$$

Choosing the sign of m_1 such that $a_4m_1<0$. Combining the expression (49) with Eq.(13) yields

$$\begin{aligned}
\xi_1-\xi_0&=\int\frac{\mathrm{d}w}{\sqrt{\pm(w-\alpha)(w-\beta)((w-l)^2+s^2)}}\\
&=\frac{2mm_1}{\sqrt{\mp 2sm_1(\alpha-\beta)}}\int\frac{\mathrm{d}\phi}{\sqrt{1-m^2\sin^2\phi}},
\end{aligned} \tag{51}$$

where $m^2=\dfrac{1}{1+m_1^2}$. By use of Eq.(51) and the definition of Jacobian elliptic cosine function[21], we have

$$\mathrm{cn}\left(\frac{\sqrt{\mp 2sm_1(\alpha-\beta)}}{2mm_1}(\xi_1-\xi_0),m\right)=\cos\phi. \tag{52}$$

Combining the expression (52) with the expression (49) yields the solutions of Eqs.(13) and (16) under the corresponding condition (50) (here the positive sign for Eq.(16), and the negative for Eq.(13))

$$w=\frac{a\ \mathrm{cn}\left(\frac{\sqrt{\mp 2sm_1(\alpha-\beta)}}{2mm_1}(\xi_1-\xi_0),m\right)+b}{c\ \mathrm{cn}\left(\frac{\sqrt{\mp 2sm_1(\alpha-\beta)}}{2mm_1}(\xi_1-\xi_0),m\right)+d}, \tag{53}$$

where the expression (53) is an elliptic function double periodic solution.

Case 8 $D_2D_3\leq 0$ and $D_4>0$. $F(w)$ has two pairs of conjugate complex roots, i.e.

$$F(w)=((w-l_1)^2+s_1^2)((w-l_2)^2+s_2^2), \tag{54}$$

where $s_1\geq s_2>0$. For $a_4>0$, we take the transformation as follows:

$$w=\frac{a\tan\phi+b}{c\tan\phi+d}, \tag{55}$$

where

$$\begin{aligned}
a&=l_1c+s_1d,\\
b&=l_1d-s_1c,\\
c&=-s_1-\frac{s_2}{m_1},\\
d&=l_1-l_2,\\
E&=\frac{(l_1-l_2)^2+s_1^2+s_2^2}{2s_1s_2},\\
m_1&=E+\sqrt{E^2-1}.
\end{aligned} \tag{56}$$

Combining the expression (55) with Eq.(13) yields

$$\begin{aligned}
\xi_1-\xi_0&=\int\frac{\mathrm{d}w}{\sqrt{((w-l_1)^2+s_1^2)((w-l_2)^2+s_2^2)}}\\
&=\frac{c^2+d^2}{s_2\sqrt{(c^2+d^2)(m_1^2c^2+d^2)}}\\
&\quad\times\int\frac{\mathrm{d}\phi}{\sqrt{1-m^2\sin^2\phi}},
\end{aligned} \tag{57}$$

where $m^2 = \dfrac{m_1^2 - 1}{m_1^2}$. By use of Eq.(57) and the definitions of Jacobian elliptic sine and cosine functions,[21] we have

$$\mathrm{sn}\left(\frac{s_2\sqrt{(c^2+d^2)(m_1^2c^2+d^2)}}{c^2+d^2}(\xi_1-\xi_0), m\right) = \sin\phi, \tag{58}$$

$$\mathrm{cn}\left(\frac{s_2\sqrt{(c^2+d^2)(m_1^2c^2+d^2)}}{c^2+d^2}(\xi_1-\xi_0), m\right) = \cos\phi. \tag{59}$$

Combining the expressions (58) and (59) with the expression (55) yields the solution of Eq.(13) under the corresponding condition (56),

$$w = \frac{a\ \mathrm{sn}(\eta(\xi_1-\xi_0), m) + b\ \mathrm{cn}(\eta(\xi_1-\xi_0), m)}{c\ \mathrm{sn}(\eta(\xi_1-\xi_0), m) + d\ \mathrm{cn}(\eta(\xi_1-\xi_0), m)}, \tag{60}$$

where

$$\eta = \frac{s_2\sqrt{(c^2+d^2)(m_1^2c^2+d^2)}}{c^2+d^2}. \tag{61}$$

The expression (60) is an elliptic function double periodic solution.

Case 9 $D_2 > 0$, $D_3 > 0$, and $D_4 = 0$; $F(w)$ has two single real roots and a real root with multiplicities two. Similarly, we can give respective solutions, they are omitted.

From the above description, we have obtained all possible exact solutions of Eqs.(13) and (16). By use of transformations (12) and (15), we can write all possible solutions to Eq.(9) under corresponding parameter conditions. Furthermore, by using Eqs.(9) and (10), we can give all possible exact travelling wave solutions to Eqs.(3) and (4). As a particular case, we can write all possible exact travelling wave solutions for a (1+1)-dimensional dispersive long wave equation with special parameters in Eq.(5), those solutions include rational function type solutions, solitary wave solutions, triangular function type periodic solutions and six types of Jacobian elliptic function double periodic solutions. But for simplicity, the general cases of Eqs.(3) and (4) are not discussed here.

4. Summary and conclusions

All possible exact travelling wave solutions for DLWE have been given. There are many nonlinear evolution equations such as doubled Sine–Gordon equation, nonlinear coupled scale field equation, coupled mKdV equation, Hirota equation, combined KdV–MKdV equation and (2+1)-dimensional breaking soliton equation and so on, their all or part of travelling wave solutions can be obtained by using the complete discrimination system for the fourth polynomial. This method is simple but powerful for many problems. Another feature of the present method is that the solutions given have most general parameter conditions.

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