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Applications of F-expansion method to the coupled KdV system*

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An extended F-expansion method for finding periodic wave solutions of nonlinear evolution equations in mathematical physics is presented, which can be thought of as a concentration of extended Jacobi elliptic function expansion method proposed more recently. By using the homogeneous balance principle and the extended F-expansion, more periodic wave solutions expressed by Jacobi elliptic functions for the coupled KdV equations are derived. In the limit cases, the solitary wave solutions and the other type of travelling wave solutions for the system are also obtained.

Keywords: coupled KdV equations, extended F-expansion method, Jacobi elliptic functions, periodic wave solutions, solitary wave solutions

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1. Introduction

Recently the periodic wave solutions of many nonlinear wave equations have been obtained by using Jacobi elliptic function expansion method.[1−8] More recently, the F-expansion method[9−12] has been proposed, which can be thought of as any kind of Jacobi elliptic function expansion method and hence an over-all generalization of it. In this paper, in order to simultaneously obtain more periodic wave solutions expressed by various Jacobi elliptic functions to nonlinear wave equations, we introduce an extended F-expansion method and apply it to find the exact solutions for the coupled KdV equations. It will be shown that more types of periodic solutions to the coupled KdV equations, than those obtained in Ref.[13], can be derived by using the extended F-expansion method.

This paper is organized as follows. In the next section, we briefly describe the extended F-expansion method. In Section 3, we use the extended F-expansion method to obtain the concentration formulae of the solutions. In Section 4, from the concentration formulae in the Section 3, many periodic wave solutions can be obtained. In Section 5, in the limit cases, the solitary wave solutions and the other type of travelling wave solutions are also obtained. Finally, in Section 6, some conclusions are made.

2. Description of an extended F-expansion method

Consider a nonlinear wave equation for $u(x,t)$,

$$\Phi(u, u_x, u_t, u_{xx}, u_{tt}, u_{xt}, u_{xxx}, \cdots) = 0, \qquad (1)$$

in which both nonlinear terms and higher-order derivatives of $u(x,t)$ are involved. In general, the left-hand side of Eq.(1) is a polynomial in u and its various derivatives. The extended F-expansion method for solving Eq.(1) proceeds in the following four steps:

Step 1 Seek travelling wave solution of Eq.(1) by taking

$$u(x,t) = u(\xi), \quad \xi = kx + \omega t + \xi_0, \qquad (2)$$

where k, ω are constants to be determined, ξ_0 is an arbitrary real constant. Substituting Eq.(2) into Eq.(1) yields an ordinary differential equation (ODE) for $u(\xi)$,

$$\Phi(u, ku', ku', k^2u'', \omega^2u'', k\omega u'', k^3u''', \cdots) = 0. \qquad (3)$$

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Step 2 Suppose that $u(\xi)$ can be expressed by

$$u(\xi) = \sum_{n=-N}^{N} a_n F^n(\xi) + \sum_{n=1}^{N} b_n F^{n-1}(\xi)G(\xi), \quad a_N \neq 0, \tag{4}$$

where a_n, b_n are constants to be determined, $F(\xi)$ and $G(\xi)$ satisfy first-order ODEs

$$\begin{aligned} &F'^2(\xi) = P_1 F^4(\xi) + Q_1 F^2(\xi) + R_1, \\ &(F'' = 2P_1 F^3 + Q_1 F), \end{aligned} \tag{5}$$

and

$$\begin{aligned} &G'^2(\xi) = P_2 G^4(\xi) + Q_2 G^2(\xi) + R_2, \\ &(G'' = 2P_2 G^3 + Q_2 G), \end{aligned} \tag{6}$$

respectively. Both ODEs (5) and (6) are all included in Appendix B provided P_i, Q_i, $R_i (i = 1, 2)$ are selected such that the solutions of ODE (5) and (6) are different Jacobi elliptic functions satisfying the relation (see Appendix A)

$$G^2 = \mu F^2 + \nu, \tag{7}$$

where μ, ν are constants, integer N can be determined by considering the homogeneous balance[14−20] between governing nonlinear terms and the highest order derivatives of u in Eq.(3). Substituting Eq.(4) into ODE (3), and using ODEs (5), (6) together with Eq.(7), the left-hand side of ODE (3) can be converted into a polynomial in $F^{-1}(\xi)$, $F(\xi)$ and $G(\xi)$. Setting the coefficients of the polynomial to zero yields a set of algebraic equations for a_i, b_j $(i = -N, -N+1, \cdots, 0, 1, \cdots, N,\ j = 1, 2, \cdots, N)$, ω, and k.

Step 3 With the aid of Mathematica or Maple, solving the algebraic equations obtained in Step 2, then a_i, b_j $(i = -N, -N+1, \cdots, 0, 1, \cdots, N,\ j = 1, 2, \cdots, N)$ and ω (possibly k) can be expressed by P_i, Q_i, R_i $(i = 1, 2)$, μ and ν. Substituting these results into F-expansion (4), we can obtain a general form of travelling wave solution to Eq.(3), which is called the concentration formulae of the solutions.

Step 4 With the aid of Appendices A and B, from the concentration formulae of the solutions, many periodic wave solutions expressed by various Jacobi elliptic functions can simultaneously be obtained. In the limit cases when the modulus $m \to 1$ and $m \to 0$, the solitary wave solutions and trigonometric function solutions are also obtained.

3. Concentration formulae of the solutions

Using F-expansion method, we discuss the coupled KdV equations,

$$u_t + pvv_x + quu_x + ru_{xxx} = 0, \tag{8}$$

$$v_t + s(uv)_x + hvv_x = 0. \tag{9}$$

Assume that the travelling wave solutions to Eqs.(8) and (9) are in the form

$$u(x,t) = u(\xi), \quad v(x,t) = v(\xi), \quad \xi = kx + \omega t + \xi_0, \tag{10}$$

where k, ω are constants to be determined, ξ_0 is an arbitrary real constant. Substituting Eq.(10) into Eqs.(8) and (9) yields the ODEs for $u(\xi)$ and $v(\xi)$

$$\omega u' + kpvv' + kquu' + k^3 ru''' = 0, \tag{11}$$

$$\omega v' + ksu'v + ksuv' + khvv' \ = 0. \tag{12}$$

Performing once integration and setting the integration constant to zero, Eq.(12) becomes

$$\omega v + ksuv + \frac{1}{2}khv^2 \ = 0. \tag{13}$$

From Eq.(13), we obtain a transformation

$$v = -\frac{2s}{h}u - \frac{2\omega}{kh}. \tag{14}$$

Substituting Eq.(14) into Eq.(11) yields an ODE

$$(4ps + h^2)\omega u' + (4ps^2 + qh^2)kuu' + k^3 rh^2 u''' = 0. \tag{15}$$

Considering homogeneous balance between uu' and u''' in Eq.(15), we suppose that the solution of ODE (15) is in the form, i.e. even-expansion for short, as follows:

$$u(\xi) = a_{-2}F^{-2}(\xi) + a_0 + a_2 F^2(\xi) + b_2 F(\xi)G(\xi), \tag{16}$$

where $a_2 \neq 0$, $F(\xi)$ and $G(\xi)$ satisfy ODEs (5), (6) and the relation (7), $a_i (i = -2, 0, 2)$, b_2 are constants to be determined later.

Substituting Eq.(16) into Eq.(15), and using ODEs (5), (6) and the relation (7), the left-hand side of ODE (15) can be converted into a polynomial in $F^{-1}(\xi)$, $F(\xi)$ and $G(\xi)$. Setting the coefficients of the polynomial to zero yields a set of algebraic equations for $a_i (i = -2, 0, 2)$, b_2, ω and k:

$$
\begin{aligned}
F^4: &\quad (h^2q+4ps^2)a_2b_2+3h^2k^2r(P_1+\mu P_2)b_2=0,\\
F^3G: &\quad (h^2q+4ps^2)a_2^2+(h^2q+4ps^2)\mu b_2^2+12h^2k^2rP_1a_2=0,\\
F^2: &\quad \left[2\mu(h^2+4ps)+2h^2k^3r(3\nu P_1+9\mu\nu P_2+2\mu Q_1+2\mu Q_2)\right]b_2+2k\mu(h^2q+4ps^2)a_0b_2\\
&\quad +3k\nu(h^2q+4ps^2)a_2b_2=0,\\
FG: &\quad 2(h^2\omega+4ps\omega+4h^2k^3rQ_1)a_2+2k(h^2q+4ps^2)a_0a_2+k\nu(h^2q+4ps^2)b_2^2=0,\\
F^0: &\quad \left[\nu\omega(h^2+4ps)+h^2k^3r\nu(6\nu P_2+Q_1+3Q_2)\right]b_2+k\nu(h^2q+4ps^2)a_0b_2=0,\\
F^{-2}: &\quad -k\nu(h^2q+4ps^2)a_{-2}b_2=0,\\
F^{-3}G: &\quad (h^2\omega+4ps\omega+4h^2k^3rQ_1)a_{-2}+k(h^2q+4ps^2)a_{-2}a_0=0,\\
F^{-5}G: &\quad k(h^2q+4ps^2)a_{-2}^2+12h^2k^3rR_1a_{-2}=0.
\end{aligned}
$$

Solving the algebraic equations obtained above and using the identities (See Appendix C):

$$P_1=\mu P_2,\quad Q_1=Q_2+3\nu P_2,$$

we have three solutions which are:

Case 1

$$
\begin{aligned}
&a_{-2}=0,\\
&a_0=-\frac{h^2\omega+4ps\omega+4h^2k^3rQ_1}{k(h^2q+4ps^2)},\\
&a_2=-\frac{12h^2k^2rP_1}{h^2q+4ps^2},\\
&b_2=0,
\end{aligned}
\tag{17}
$$

$k\neq 0,\omega$ is an arbitrary real constant.

Case 2

$$
\begin{aligned}
&a_{-2}=-\frac{12h^2k^2rR_1}{h^2q+4ps^2},\\
&a_0=-\frac{h^2\omega+4ps\omega+4h^2k^3rQ_1}{k(h^2q+4ps^2)},\\
&a_2=-\frac{12h^2k^2rP_1}{h^2q+4ps^2},\\
&b_2=0,
\end{aligned}
\tag{18}
$$

$k\neq 0,\omega$ is an arbitrary real constant.

Case 3

$$
\begin{aligned}
&a_{-2}=0,\\
&a_0=-\frac{h^2\omega+4ps\omega+9h^2k^3r\nu P_2+4h^2k^3rQ_2}{k(h^2q+4ps^2)},\\
&a_2=-\frac{6h^2k^2r\mu P_2}{h^2q+4ps^2},\\
&b_2=\pm\frac{6h^2k^2r\sqrt{\mu}P_2}{h^2q+4ps^2},
\end{aligned}
\tag{19}
$$

$k\neq 0,\omega$ is an arbitrary real constant.

Substituting (17)–(19) into Eq.(16) yields three kinds of concentration formulae of the solutions to Eq.(8):

$$
\begin{aligned}
u(x,t)=&-\frac{12h^2k^2rP_1}{h^2q+4ps^2}F^2(\xi)\\
&-\frac{h^2\omega+4ps\omega+4h^2k^3rQ_1}{k(h^2q+4ps^2)};
\end{aligned}
\tag{20}
$$

$$
\begin{aligned}
u(x,t)=&-\frac{12h^2k^2r}{h^2q+4ps^2}[R_1F^{-2}(\xi)+P_1F^2(\xi)]\\
&-\frac{h^2\omega+4ps\omega+4h^2k^3rQ_1}{k(h^2q+4ps^2)};
\end{aligned}
\tag{21}
$$

$$
\begin{aligned}
u(x,t)=&-\frac{6h^2k^2r}{h^2q+4ps^2}[\mu P_2F^2(\xi)\\
&\pm\sqrt{\mu}P_2F(\xi)G(\xi)]\\
&-\frac{h^2\omega+4ps\omega+9h^2k^3r\nu P_2+4h^2k^3rQ_2}{k(h^2q+4ps^2)},
\end{aligned}
\tag{22}
$$

where $\xi=kx+\omega t+\xi_0$, k is a nonzero real constant, and ω and ξ_0 are arbitrary real constants.

4. Periodic wave solutions

With the aid of Appendices A and B, from the concentration formulae of the solutions we could obtain many periodic wave solutions expressed by various Jacobi elliptic functions to Eq.(8), by using the transformation (14), and the solutions to Eq.(9) can be obtained as well. We discuss the following three cases:

Case 1

From Appendix B, taking $F(\xi)=\mathrm{sn}\xi$, $P_1=m^2$, $Q_1=-1-m^2$, substituting these into Eq.(20) and using Eq.(14) yields

$$\begin{cases} u_1 = -\dfrac{12h^2k^2rm^2}{h^2q+4ps^2}\mathrm{sn}^2\xi - \dfrac{h^2\omega+4ps\omega-4h^2k^3r(1+m^2)}{k(h^2q+4ps^2)} \\ v_1 = \dfrac{24hk^2rsm^2}{h^2q+4ps^2}\mathrm{sn}^2\xi + \dfrac{2h[s\omega-q\omega-4k^3rs(1+m^2)]}{k(h^2q+4ps^2)} \end{cases},$$

where $\xi = kx + \omega t + \xi_0$.

Taking $F(\xi) = \mathrm{ns}\xi$, $P_1 = 1$, $Q_1 = -1 - m^2$, substituting these into Eq.(20) and using Eq.(14) yields

$$\begin{cases} u_2 = -\dfrac{12h^2k^2r}{h^2q+4ps^2}\mathrm{ns}^2\xi - \dfrac{h^2\omega+4ps\omega-4h^2k^3r(1+m^2)}{k(h^2q+4ps^2)} \\ v_2 = \dfrac{24hk^2rs}{h^2q+4ps^2}\mathrm{ns}^2\xi + \dfrac{2h[s\omega-q\omega-4k^3rs(1+m^2)]}{k(h^2q+4ps^2)} \end{cases},$$

where $\xi = kx + \omega t + \xi_0$.

Taking $F(\xi) = \mathrm{nc}\xi$, $P_1 = 1 - m^2$, $Q_1 = 2m^2 - 1$, substituting these into Eq.(20) and using Eq.(14) yields

$$\begin{cases} u_3 = -\dfrac{12h^2k^2r(1-m^2)}{h^2q+4ps^2}\mathrm{nc}^2(\xi) - \dfrac{h^2\omega+4ps\omega+4h^2k^3r(2m^2-1)}{k(h^2q+4ps^2)} \\ v_3 = \dfrac{24hk^2rs(1-m^2)}{h^2q+4ps^2}\mathrm{nc}^2(\xi) + \dfrac{2h[s\omega-q\omega+4k^3rs(2m^2-1)]}{k(h^2q+4ps^2)} \end{cases},$$

where $\xi = kx + \omega t + \xi_0$.

Taking $F(\xi) = \mathrm{nd}\xi$, $P_1 = m^2 - 1$, $Q_1 = 2 - m^2$, substituting these into Eq.(20) and using Eq.(14) yields

$$\begin{cases} u_4 = -\dfrac{12h^2k^2r(m^2-1)}{h^2q+4ps^2}\mathrm{nd}^2\xi - \dfrac{h^2\omega+4ps\omega+4h^2k^3r(2-m^2)}{k(h^2q+4ps^2)} \\ v_4 = \dfrac{24hk^2rs(m^2-1)}{h^2q+4ps^2}\mathrm{nd}^2\xi + \dfrac{2h[s\omega-q\omega+4k^3rs(2-m^2)]}{k(h^2q+4ps^2)} \end{cases},$$

where $\xi = kx + \omega t + \xi_0$.

Case 2

With the aid of Appendix B, substituting $F(\xi) = \mathrm{sn}\xi$, $F^{-1}(\xi) = \mathrm{ns}\xi$, $P_1 = m^2$, $Q_1 = -1 - m^2$, and $R_1 = 1$ into Eq.(21) and using Eq.(14) yields

$$\begin{cases} u_5 = -\dfrac{12h^2k^2r}{h^2q+4ps^2}(\mathrm{ns}^2\xi + m^2\mathrm{sn}^2\xi) - \dfrac{h^2\omega+4ps\omega-4h^2k^3r(1+m^2)}{k(h^2q+4ps^2)} \\ v_5 = \dfrac{24hk^2rs}{h^2q+4ps^2}(\mathrm{ns}^2\xi + m^2\mathrm{sn}^2\xi) + \dfrac{2h[s\omega-q\omega-4k^3rs(1+m^2)]}{k(h^2q+4ps^2)} \end{cases},$$

where $\xi = kx + \omega t + \xi_0$.

In u_5 and v_5, if $\mathrm{sn}\xi$ and $\mathrm{ns}\xi$ are replaced by $\mathrm{cd}\xi$ and $\mathrm{dc}\xi$ respectively, then we have

$$\begin{cases} u_6 = -\dfrac{12h^2k^2r}{h^2q+4ps^2}(\mathrm{dc}^2\xi + m^2\mathrm{cd}^2\xi) - \dfrac{h^2\omega+4ps\omega-4h^2k^3r(1+m^2)}{k(h^2q+4ps^2)} \\ v_6 = \dfrac{24hk^2rs}{h^2q+4ps^2}(\mathrm{dc}^2\xi + m^2\mathrm{cd}^2\xi) + \dfrac{2h[s\omega-q\omega-4k^3rs(1+m^2)]}{k(h^2q+4ps^2)} \end{cases},$$

where $\xi = kx + \omega t + \xi_0$.

Substituting $F(\xi) = \mathrm{cn}\xi$, $F^{-1}(\xi) = \mathrm{nc}\xi$, $P_1 = -m^2$, $Q_1 = 2m^2 - 1$, and $R_1 = 1 - m^2$ into Eq.(21) and using Eq.(14) yields

$$\begin{cases} u_7 = -\dfrac{12h^2k^2r}{h^2q+4ps^2}[(1-m^2)\mathrm{nc}^2\xi - m^2\mathrm{cn}^2\xi] - \dfrac{h^2\omega+4ps\omega+4h^2k^3r(2m^2-1)}{k(h^2q+4ps^2)} \\ v_7 = \dfrac{24hk^2rs}{h^2q+4ps^2}[(1-m^2)\mathrm{nc}^2\xi - m^2\mathrm{cn}^2\xi] + \dfrac{2h[s\omega-q\omega+4k^3rs(2m^2-1)]}{k(h^2q+4ps^2)} \end{cases},$$

where $\xi = kx + \omega t + \xi_0$.

In the same way we can obtain other solutions:

$$\begin{cases} u_8 = -\dfrac{12h^2k^2r}{h^2q+4ps^2}[(m^2-1)\mathrm{nd}^2\xi - \mathrm{dn}^2\xi] - \dfrac{h^2\omega+4ps\omega+4h^2k^3r(2-m^2)}{k(h^2q+4ps^2)} \\ v_8 = \dfrac{24hk^2rs}{h^2q+4ps^2}[(m^2-1)\mathrm{nd}^2\xi - \mathrm{dn}^2\xi] + \dfrac{2h[s\omega-q\omega+4k^3rs(2-m^2)]}{k(h^2q+4ps^2)} \end{cases},$$

$$\begin{cases} u_9 = -\dfrac{12h^2k^2r}{h^2q+4ps^2}[\mathrm{cs}^2\xi + (1-m^2)\mathrm{sc}^2\xi] - \dfrac{h^2\omega+4ps\omega+4h^2k^3r(2-m^2)}{k(h^2q+4ps^2)} \\ v_9 = \dfrac{24hk^2rs}{h^2q+4ps^2}[\mathrm{cs}^2\xi + (1-m^2)\mathrm{sc}^2\xi] + \dfrac{2h[s\omega-q\omega+4k^3rs(2-m^2)]}{k(h^2q+4ps^2)} \end{cases},$$

$$\begin{cases} u_{10} = -\dfrac{12h^2k^2r}{h^2q+4ps^2}[\mathrm{ds}^2\xi - m^2(1-m^2)\mathrm{sd}^2\xi] - \dfrac{h^2\omega+4ps\omega+4h^2k^3r(2m^2-1)}{k(h^2q+4ps^2)} \\ v_{10} = \dfrac{24hk^2rs}{h^2q+4ps^2}[\mathrm{ds}^2\xi - m^2(1-m^2)\mathrm{sd}^2\xi] + \dfrac{2h[s\omega-q\omega+4k^3rs(2m^2-1)]}{k(h^2q+4ps^2)} \end{cases},$$

where $\xi = kx + \omega t + \xi_0$.

Case 3

From Appendix A, we choose $\mathrm{dn}^2\xi = m^2\mathrm{cn}^2\xi + 1 - m^2$, and set $F(\xi) = \mathrm{cn}\xi$, $G(\xi) = \mathrm{dn}\xi$, $\mu = m^2$, and $\nu = 1 - m^2$, and from Appendix B, we find that the respective coefficients of ODE for $\mathrm{dn}\xi$ are $P_2 = -1$, $Q_2 = 2 - m^2$. Substituting these into Eq.(22) and using Eq.(14) yields

$$\begin{cases} u_{11} = \dfrac{6h^2k^2r}{h^2q+4ps^2}[m^2\mathrm{cn}^2\xi \pm m\mathrm{cn}\xi\mathrm{dn}\xi] - \dfrac{h^2\omega+4ps\omega+h^2k^3r(5m^2-1)}{k(h^2q+4ps^2)} \\ v_{11} = -\dfrac{12hk^2rs}{h^2q+4ps^2}[\,m^2\mathrm{cn}^2\xi \pm m\mathrm{cn}\xi\mathrm{dn}\xi] + \dfrac{2h[s\omega-q\omega+k^3rs(5m^2-1)]}{k(h^2q+4ps^2)} \end{cases},$$

where $\xi = kx + \omega t + \xi_0$.

Choose $\mathrm{ns}^2\xi = \mathrm{cs}^2\xi + 1$, and set $F(\xi) = \mathrm{cs}\xi$, $G(\xi) = \mathrm{ns}\xi$, and $\mu = \nu = 1$, from Appendix B, we find that the respective coefficients of ODE for $\mathrm{ns}\xi$ are $P_2 = 1$, $Q_2 = -1 - m^2$. Substituting these into Eq.(22) and using Eq.(14) yields

$$\begin{cases} u_{12} = -\dfrac{6h^2k^2r}{h^2q+4ps^2}(\mathrm{cs}^2\xi \pm \mathrm{cs}\xi\mathrm{ns}\xi) - \dfrac{h^2\omega+4ps\omega+h^2k^3r(5-4m^2)}{k(h^2q+4ps^2)} \\ v_{12} = \dfrac{12hk^2rs}{h^2q+4ps^2}(\,\mathrm{cs}^2\xi \pm \mathrm{cs}\xi\mathrm{ns}\xi) + \dfrac{2h[s\omega-q\omega+k^3rs(5-4m^2)]}{k(h^2q+4ps^2)} \end{cases},$$

where $\xi = kx + \omega t + \xi_0$.

Choose $\mathrm{ns}^2\xi = \mathrm{ds}^2\xi + m^2$, and set $F(\xi) = \mathrm{ds}\xi$, $G(\xi) = \mathrm{ns}\xi$, $\mu = 1$, and $\nu = m^2$, from Appendix B, we find that the respective coefficients of ODE for $\mathrm{ns}\xi$ are $P_2 = 1$, $Q_2 = -1 - m^2$. Substituting these into Eq.(22) and using Eq.(14) yields

$$\begin{cases} u_{13} = -\dfrac{6h^2k^2r}{h^2q+4ps^2}(\mathrm{ds}^2\xi \pm \mathrm{ds}\xi\mathrm{ns}\xi) - \dfrac{h^2\omega+4ps\omega+h^2k^3r(5m^2-4)}{k(h^2q+4ps^2)} \\ v_{13} = \dfrac{12hk^2rs}{h^2q+4ps^2}(\mathrm{ds}^2\xi \pm \mathrm{ds}\xi\mathrm{ns}\xi) + \dfrac{2h[s\omega-q\omega+k^3rs(5m^2-4)]}{k(h^2q+4ps^2)} \end{cases},$$

where $\xi = kx + \omega t + \xi_0$.

In the same way we can obtain other solutions:

$$\begin{cases} u_{14} = -\dfrac{6h^2k^2r}{h^2q+4ps^2}[\mathrm{cs}^2\xi \pm \mathrm{cs}\xi\mathrm{ds}\xi] - \dfrac{h^2\omega+4ps\omega+h^2k^3r(5-m^2)}{k(h^2q+4ps^2)} \\ v_{14} = \dfrac{12hk^2rs}{h^2q+4ps^2}[\,\mathrm{cs}^2\xi \pm \mathrm{cs}\xi\mathrm{ds}\xi] + \dfrac{2h[s\omega-q\omega+k^3rs(5-m^2)]}{k(h^2q+4ps^2)} \end{cases},$$

$$\begin{cases} u_{15} = -\dfrac{6h^2k^2r(1-m^2)}{h^2q+4ps^2}(\mathrm{sc}^2\xi \pm \mathrm{sc}\xi\mathrm{nc}\xi) - \dfrac{h^2\omega+4ps\omega+h^2k^3r(5-m^2)}{k(h^2q+4ps^2)} \\ v_{15} = \dfrac{12hk^2rs(1-m^2)}{h^2q+4ps^2}(\mathrm{sc}^2\xi \pm \mathrm{sc}\xi\mathrm{nc}\xi) + \dfrac{2h[s\omega-q\omega+k^3rs(5-m^2)]}{k(h^2q+4ps^2)} \end{cases},$$

$$\begin{cases} u_{16} = -\dfrac{6h^2k^2r}{h^2q+4ps^2}[(1-m^2)\mathrm{sc}^2\xi \pm \sqrt{1-m^2}\mathrm{sc}\xi\mathrm{dc}\xi] - \dfrac{h^2\omega+4ps\omega+h^2k^3r(5-4m^2)}{k(h^2q+4ps^2)} \\ v_{16} = \dfrac{12hk^2rs}{h^2q+4ps^2}[(1-m^2)\mathrm{sc}^2\xi \pm \sqrt{1-m^2}\mathrm{sc}\xi\mathrm{dc}\xi] + \dfrac{2h[s\omega-q\omega+k^3rs(5-4m^2)]}{k(h^2q+4ps^2)} \end{cases},$$

$$\begin{cases} u_{17} = -\dfrac{6h^2k^2r}{h^2q+4ps^2}[(1-m^2)\mathrm{nc}^2\xi \pm \sqrt{1-m^2}\mathrm{nc}\xi\mathrm{dc}\xi] - \dfrac{h^2\omega+4ps\omega+h^2k^3r(5m^2-4)}{k(h^2q+4ps^2)} \\ v_{17} = \dfrac{12hk^2rs}{h^2q+4ps^2}[(1-m^2)\mathrm{nc}^2\xi \pm \sqrt{1-m^2}\mathrm{nc}\xi\mathrm{dc}\xi] + \dfrac{2h[s\omega-q\omega+k^3rs(5m^2-4)]}{k(h^2q+4ps^2)} \end{cases},$$

$$\begin{cases} u_{18} = -\dfrac{6h^2k^2r(m^2-1)}{h^2q+4ps^2}(m^2\mathrm{sd}^2\xi \pm m\mathrm{sd}\xi\mathrm{nd}\xi) - \dfrac{h^2\omega+4ps\omega+h^2k^3r(5m^2-1)}{k(h^2q+4ps^2)} \\ v_{18} = \dfrac{12hk^2rs(m^2-1)}{h^2q+4ps^2}(m^2\mathrm{sd}^2\xi \pm m\mathrm{sd}\xi\mathrm{nd}\xi) + \dfrac{2h[s\omega-q\omega+k^3rs(5m^2-1)]}{k(h^2q+4ps^2)} \end{cases},$$

where $\xi = kx + \omega t + \xi_0$.

These solutions expressed by Jacobi elliptic functions describe a wave phenomenon with a double period, and the periods can be a real and a complex as well as an imaginary.

5. Solitary wave solutions and trigonometric function solutions

In the limit case when $m \to 1$, some solitary wave solutions can be obtained, for example:

$$u_1, v_1 \to \begin{cases} u_{19} = -\dfrac{12h^2k^2r}{h^2q+4ps^2}\tanh^2\xi - \dfrac{h^2\omega+4ps\omega-8h^2k^3r}{k(h^2q+4ps^2)} \\ v_{19} = \dfrac{24hk^2rs}{h^2q+4ps^2}\tanh^2\xi + \dfrac{2h(s\omega-q\omega-8k^3rs)}{k(h^2q+4ps^2)} \end{cases},$$

where $\xi = kx + \omega t + \xi_0$.

$$u_2, v_2 \to \begin{cases} u_{20} = -\dfrac{12h^2k^2r}{h^2q+4ps^2}\coth^2\xi - \dfrac{h^2\omega+4ps\omega-8h^2k^3r}{k(h^2q+4ps^2)} \\ v_{20} = \dfrac{24hk^2rs}{h^2q+4ps^2}\coth^2\xi + \dfrac{2h(s\omega-q\omega-8k^3rs)}{k(h^2q+4ps^2)} \end{cases},$$

where $\xi = kx + \omega t + \xi_0$.

$$u_5, v_5 \to \begin{cases} u_{21} = -\dfrac{12h^2k^2r}{h^2q+4ps^2}(\coth^2\xi + \tanh^2\xi) - \dfrac{h^2\omega+4ps\omega-8h^2k^3r}{k(h^2q+4ps^2)} \\ v_{21} = \dfrac{24hk^2rs}{h^2q+4ps^2}(\coth^2\xi + \tanh^2\xi) + \dfrac{2h(s\omega-q\omega-8k^3rs)}{k(h^2q+4ps^2)} \end{cases},$$

where $\xi = kx + \omega t + \xi_0$.

$$u_8, v_8; u_{11}, v_{11} \to \begin{cases} u_{22} = \dfrac{12h^2k^2r}{h^2q+4ps^2}\mathrm{sech}^2\xi - \dfrac{h^2\omega+4ps\omega+4h^2k^3r}{k(h^2q+4ps^2)} \\ v_{22} = -\dfrac{24hk^2rs}{h^2q+4ps^2}\mathrm{sech}^2\xi + \dfrac{2h(s\omega-q\omega+4k^3rs)}{k(h^2q+4ps^2)} \end{cases},$$

where $\xi = kx + \omega t + \xi_0$.

$$u_9, v_9; u_{10}, v_{10}; u_{14}, v_{14} \to \begin{cases} u_{23} = -\dfrac{12h^2k^2r}{h^2q+4ps^2}\mathrm{csch}^2\xi - \dfrac{h^2\omega+4ps\omega+4h^2k^3r}{k(h^2q+4ps^2)} \\ v_{23} = \dfrac{24hk^2rs}{h^2q+4ps^2}\mathrm{csch}^2\xi + \dfrac{2h(s\omega-q\omega+4k^3rs)}{k(h^2q+4ps^2)} \end{cases},$$

where $\xi = kx + \omega t + \xi_0$.

$$u_{12}, v_{12}; u_{13}, v_{13} \to \begin{cases} u_{24} = -\dfrac{6h^2k^2r}{h^2q + 4ps^2}(\mathrm{csch}^2\xi \pm \mathrm{csch}\xi\coth\xi) - \dfrac{h^2\omega + 4ps\omega + h^2k^3r}{k(h^2q + 4ps^2)} \\ v_{24} = \dfrac{12hk^2rs}{h^2q + 4ps^2}(\ \mathrm{csch}^2\xi \pm \mathrm{csch}\xi\coth\xi) + \dfrac{2h(s\omega - q\omega + k^3rs)}{k(h^2q + 4ps^2)} \end{cases},$$

where $\xi = kx + \omega t + \xi_0$.

In the limit case when $m \to 0$, some trigonometric function solutions (single periodic wave solutions) can be obtained, for example:

$$u_2, v_2; u_5, v_5; u_{10}, v_{10}; u_{13}, v_{13} \to \begin{cases} u_{25} = -\dfrac{12h^2k^2r}{h^2q + 4ps^2}\csc^2\xi - \dfrac{h^2\omega + 4ps\omega - 4h^2k^3r}{k(h^2q + 4ps^2)} \\ v_{25} = \dfrac{24hk^2rs}{h^2q + 4ps^2}\csc^2\xi + \dfrac{2h(s\omega - q\omega - 4k^3rs)}{k(h^2q + 4ps^2)} \end{cases},$$

where $\xi = kx + \omega t + \xi_0$.

$$u_3, v_3; u_6, v_6; u_7, v_7; u_{17}, v_{17} \to \begin{cases} u_{26} = -\dfrac{12h^2k^2r}{h^2q + 4ps^2}\sec^2(\xi) - \dfrac{h^2\omega + 4ps\omega - 4h^2k^3r}{k(h^2q + 4ps^2)} \\ v_{26} = \dfrac{24hk^2rs}{h^2q + 4ps^2}\sec^2(\xi) + \dfrac{2h(s\omega - q\omega - 4k^3rs)}{k(h^2q + 4ps^2)} \end{cases},$$

where $\xi = kx + \omega t + \xi_0$.

$$u_9, v_9 \to \begin{cases} u_{27} = -\dfrac{12h^2k^2r}{h^2q + 4ps^2}(\cot^2\xi + \tan^2\xi) - \dfrac{h^2\omega + 4ps\omega + 8h^2k^3r}{k(h^2q + 4ps^2)} \\ v_{27} = \dfrac{24hk^2rs}{h^2q + 4ps^2}(\cot^2\xi + \tan^2\xi) + \dfrac{2h(s\omega - q\omega + 8k^3rs)}{k(h^2q + 4ps^2)} \end{cases},$$

where $\xi = kx + \omega t + \xi_0$.

$$u_{12}, v_{12}; u_{14}, v_{14} \to \begin{cases} u_{28} = -\dfrac{6h^2k^2r}{h^2q + 4ps^2}(\cot^2\xi \pm \cot\xi\csc\xi) - \dfrac{h^2\omega + 4ps\omega + 5h^2k^3r}{k(h^2q + 4ps^2)} \\ v_{28} = \dfrac{12hk^2rs}{h^2q + 4ps^2}(\ \cot^2\xi \pm \cot\xi\csc\xi) + \dfrac{2h(s\omega - q\omega + 5k^3rs)}{k(h^2q + 4ps^2)} \end{cases},$$

where $\xi = kx + \omega t + \xi_0$.

$$u_{15}, v_{15}; u_{16}, v_{16} \to \begin{cases} u_{29} = -\dfrac{6h^2k^2r}{h^2q + 4ps^2}(\tan^2\xi \pm \tan\xi\sec\xi) - \dfrac{h^2\omega + 4ps\omega + 5h^2k^3r}{k(h^2q + 4ps^2)} \\ v_{29} = \dfrac{12hk^2rs}{h^2q + 4ps^2}(\ \tan^2\xi \pm \tan\xi\sec\xi) + \dfrac{2h(s\omega - q\omega + 5k^3rs)}{k(h^2q + 4ps^2)} \end{cases},$$

where $\xi = kx + \omega t + \xi_0$.

6. Conclusion

In this paper, more periodic wave solutions expressed by the Jacobi elliptic functions for the coupled KdV equations are obtained by using the extended F-expansion method, and in the limit cases, the solitary wave solutions and the other types of travelling wave solutions are also obtained. The extended F-expansion includes every one of Jacobi elliptic function expansion, so the former can be regarded as a concentration of the latter. The main advantages of the extended F-expansion are that first of all, one only needs to calculate the functions $F(\xi)$ and $G(\xi)$, which are different solutions satisfying relation (7) of ODEs (5) and (6), to derive the concentration formulae of the solutions to the equations considered, instead of calculating the Jacobi elliptic function one by one; secondly, with the aid of Appendices A and B, from the concentration formulae of the solutions, one can simultaneously obtain many periodic wave solutions expressed

by various Jacobi elliptic functions. It seems that the extended F-expansion is more effective than the F-expansion and Jacobi elliptic function expansion, and it is a useful tool to explore the periodic wave solutions for nonlinear evolution equations. The method used here can be applied to solve other nonlinear evolution equations.

Appendix A Identify relations of Jacobi elliptic functions

Jacobi elliptic functions with modulus $m(0 < m < 1)$ have the identity relations in the form $G^2 = \mu F^2 + \nu$:

$$\begin{aligned}
&\mathrm{cn}^2\xi = -\mathrm{sn}^2\xi + 1,\\
&\mathrm{dn}^2\xi = -m^2\mathrm{sn}^2\xi + 1,\\
&\mathrm{dn}^2\xi = m^2\mathrm{cn}^2\xi + 1 - m^2,\\
&\mathrm{ns}^2\xi = \mathrm{cs}^2\xi + 1,\\
&\mathrm{ns}^2\xi = \mathrm{ds}^2\xi + m^2,\\
&\mathrm{ds}^2\xi = \mathrm{cs}^2\xi + 1 - m^2,\\
&\mathrm{nc}^2\xi = \mathrm{sc}^2\xi + 1,\\
&\mathrm{dc}^2\xi = (1-m^2)\mathrm{sc}^2\xi + 1,\\
&\mathrm{dc}^2\xi = (1-m^2)\mathrm{nc}^2\xi + m^2,\\
&\mathrm{nd}^2\xi = m^2\mathrm{sd}^2\xi + 1,\\
&\mathrm{cd}^2\xi = \frac{m^2-1}{m^2}\mathrm{nd}^2\xi + \frac{1}{m^2},\\
&\mathrm{cd}^2\xi = (m^2-1)\mathrm{sd}^2\xi + 1.
\end{aligned}$$

Appendix B Table of solutions of the corresponding ODE

Table B-1 Relations between values of (P, Q, R) and corresponding $F(\xi)$ in ODE $F'^2 = PF^4 + QF^2 + R$.

P	Q	R	$F'^2 = PF^4 + QF^2 + R$	$F(\xi)$
m^2	$-(1+m^2)$	1	$F'^2 = (1-F^2)(1-m^2F^2)$	$\mathrm{sn}\xi$, $\mathrm{cd}\xi = \frac{\mathrm{cn}\xi}{\mathrm{dn}\xi}$
$-m^2$	$2m^2-1$	$1-m^2$	$F'^2 = (1-F^2)(F^2+m^2-1)$	$\mathrm{cn}\xi$
-1	$2-m^2$	m^2-1	$F'^2 = (1-F^2)(m^2F^2+m^2-1)$	$\mathrm{dn}\xi$
1	$-(1+m^2)$	m^2	$F'^2 = (1-F^2)(m^2-F^2)$	$\mathrm{ns}\xi = (\mathrm{sn}\xi)^{-1}$, $\mathrm{dc}\xi = \frac{\mathrm{dn}\xi}{\mathrm{cn}\xi}$
$1-m^2$	$2m^2-1$	$-m^2$	$F'^2 = (1-F^2)[(m^2-1)F^2-m^2]$	$\mathrm{nc}\xi = (\mathrm{cn}\xi)^{-1}$
m^2-1	$2-m^2$	-1	$F'^2 = (1-F^2)[(1-m^2)F^2-1]$	$\mathrm{nd}\xi = (\mathrm{dn}\xi)^{-1}$
$1-m^2$	$2-m^2$	1	$F'^2 = (1+F^2)[(1-m^2)F^2+1]$	$\mathrm{sc}\xi = \frac{\mathrm{sn}\xi}{\mathrm{cn}\xi}$
$-m^2(1-m^2)$	$2m^2-1$	1	$F'^2 = (1+m^2F^2)[(m^2-1)F^2+1]$	$\mathrm{sd}\xi = \frac{\mathrm{sn}\xi}{\mathrm{dn}\xi}$
1	$2-m^2$	$1-m^2$	$F'^2 = (1+F^2)(F^2+1-m^2)$	$\mathrm{cs}\xi = (\mathrm{sc}\xi)^{-1}$
1	$2m^2-1$	$-m^2(1-m^2)$	$F'^2 = (m^2+F^2)(F^2+1-m^2)$	$\mathrm{ds}\xi = (\mathrm{sd}\xi)^{-1}$

Appendix C Identities among μ, ν in Appendix A and $P_i, Q_i, R_i (i = 1, 2)$ in Appendix B

If $F(\xi)$ and $G(\xi)$ satisfies the first-order ODEs

$$F'^2(\xi) = P_1F^4(\xi) + Q_1F^2(\xi) + R_1,$$

and

$$G'^2(\xi) = P_2G^4(\xi) + Q_2G^2(\xi) + R_2,$$

respectively, and

$$G^2 = \mu F^2 + \nu,$$

where P_i, Q_i, R_i $(i = 1, 2)$, μ and ν are constants, then the following identities hold,

$$P_1 = \mu P_2; \quad Q_1 = Q_2 + 3\nu P_2,$$

which can be examined by using Appendices A and B. The identities are very useful when solving the algebraic equations obtained by using extended F-expansion method.

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