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Aggregate growth driven by monomer transfer*

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We propose a solvable aggregate growth model with the rate kernel $K(i;j;l) \propto i^{\mu}j^{\nu}l^{\varpi}$ ($\varpi \geq 0$), at which a monomer transfers from the A aggregate of size i to the A aggregate of size j only with the help of a B aggregate of size l. By means of the mean-field rate equation approach, we obtain the analytical solutions of the aggregate size distributions in several different cases. For a symmetrical system with $\mu = \nu$, the aggregate size distribution $a_k(t)$ approaches the conventional scaling form in the $\mu < 3/2$ case; while for an asymmetrical system with $\mu \neq \nu$, $a_k(t)$ takes the scaling form only in the case of $\mu < \nu$ and $\mu + \nu < 2$. As for the case with $\mu + \nu > 2$ ($\mu \neq \nu$) or $\mu > 3/2$ ($\mu = \nu$), the system may have a gelationlike transition. Moreover, we also study the kinetic scaling behaviour of the model with the generalized kernel $K(i;j;l) \sim (i^{\mu}j^{\nu} + i^{\nu}j^{\mu})l^{\varpi}$. The aggregate size distribution obeys a scaling law only in the $\mu + \nu < 3$ case; while in other cases, the system may undergo a gelationlike transition after a sufficiently long time.

Keywords: kinetic behaviour, aggregate growth, scaling law, rate equation
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1. Introduction

Irreversible aggregate growth processes occur in many basic and applied fields such as atmospheric aerosols,[1] star formation[2] and crystal growth.[3] In the last few decades, the kinetics of aggregate growth has been extensively studied.[4−10] The most interesting things that much effort has been devoted to understanding include the explicit time-dependent distribution of the aggregate size and the asymptotic scaling behaviour of the system. Many theoretical studies exhibited that the aggregate size distribution could approach a scaling form in the long-time limit.[5,6] Most of these studies have focused attention on the binary self-coalescence models, in which any two aggregates, A_i of size i and A_j of size j, can bond spontaneously to produce a larger aggregate A_{i+j}. Additionally, there also exist some other classes of universal aggregation mechanisms. Ispolatov *et al* [11] and Leyvraz *et al* [12] introduced a migration-driven aggregate growth mechanism which can provide a natural description for the evolution of city populations as well as the wealth distribution in economic interactions, and we proposed a catalysis-driven aggregation model to investigate the kinetic evolution of a chemically stable species under the action of the catalytic substance.[13] The results show that different aggregation mechanisms may lead to distinct kinetic behaviours of the aggregate growth.[11−15] It is thus of great theoretic and practical significance to seek a variety of universal aggregate growth mechanisms in physics and in other sciences.

In this work, we propose an interesting transfer-driven aggregate growth model on the basis of the studies in Refs.[11,13]. There are two types of aggregates in the system, A and B aggregates. The system evolves according to the following catalysis-like monomer transfer scheme. The first elementary reaction step is a reversible monomer transfer reaction between A and B aggregates, $A_i + B_l \rightleftharpoons A_{i-1} + AB_l$, where AB_l represents an aggregate B_l containing a "temporary " monomer of type A. The second step is an irreversible monomer transfer only between an A aggregate and the B aggregate with a "temporary" A monomer, i.e. $A_j + AB_l \rightarrow A_{j+1} + B_l$. Meanwhile, an irreversible self-coagulation reaction occurs between any two B aggregates, $B_i + B_j \xrightarrow{J(i;j)} B_{i+j}$, with a rate kernel $J(i;j)$. In many situations, it is found that the reversible reaction may reach its steady state more rapidly than the corresponding irreversible re-

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action. Thus, the monomer transfer reaction may be controlled by the second step. The whole monomer transfer reaction then reads $A_i + A_j + B_l \xrightarrow{K(i;j;l)} A_{i-1} + A_{j+1} + B_l$, with a size-dependent reaction rate kernel $K(i;j;l)$. Here each B aggregate plays a role analogous to a catalyst for the monomer transfer between two A aggregates. Our model may mimic many physical processes such as droplet growth,[16] island growth[17] and phase ordering.[18] This model can also be used to study some social phenomena.[11,12] On the other hand, it is also believed that our system is of interest to exhibit rich scaling properties of the aggregate growth.

The paper is organized as follows. In Section 2, we introduce a transfer-driven aggregate growth model and investigate the evolution properties of the aggregate size distributions by means of the rate equation. In Section 3, we then study the kinetic scaling behaviour of another model with a generalized kernel. A brief summary is given in Section 4.

2. Model with the rate kernel $K(i;j;l) \sim i^\mu j^\nu l^\varpi$

In our model, we assume that the system is of spatial homogeneity, so that the fluctuations in reactant densities are ignored and the aggregates are considered to be homogeneously distributed in the space throughout the processes. Thus, theoretical investigations of the kinetics of this aggregate growth model can be based on the mean-field rate equation, which assumes that the reaction proceeds at a rate proportional to the reactant concentrations. The concentrations of A and B aggregates of k-mers are denoted by a_k and b_k, respectively. Then the mean-field rate equations for our model read

$$\begin{aligned}\frac{\mathrm{d}a_k}{\mathrm{d}t} =& \sum_{i,j,l=1}^{\infty} K(i;j;l)a_i a_j b_l \\ & \times [\delta_{i,k+1} + \delta_{j,k-1} - \delta_{i,k} - \delta_{j,k}], \\ \frac{\mathrm{d}b_k}{\mathrm{d}t} =& \frac{1}{2}\sum_{i+j=k} J(i;j)b_i b_j - b_k \sum_{j=1}^{\infty} J(k;j)b_j, \end{aligned} \quad (1)$$

where we impose the boundary condition $a_0(t) \equiv 0$. In the first equation of Eqs.(1), the gain terms of $a_k(t)$ results from the reaction $\{A_{k+1}; A_j\} \to \{A_k; A_{j+1}\}$ through the A_{k+1} aggregate's losing a monomer and $\{A_j; A_{k-1}\} \to \{A_{j-1}; A_k\}$ through the A_{k-1} aggregate's gaining a monomer, respectively; while the last two loss terms of $a_k(t)$ are due to the A_k aggregate's gaining or losing a monomer.

We investigate a simple system with a size-dependent rate kernel, in which the rate of the A_j aggregate gaining a monomer is proportional to the values j^ν, i^μ of its opposite aggregate A_i that loses a monomer and l^ϖ of the B_l aggregate, i.e. $K(i;j;l) = Ii^\mu j^\nu l^\varpi$ (here I is a constant). The indexes (μ, ν and ϖ) may interpret the degrees of the migration possibilities of an aggregate. μ and ν denote respectively the "emigration susceptibility" and "immigration acceptability" of an A aggregate, while ϖ represents the degree of "catalysis ability" of a B aggregate. When the values of the indexes increase, the aggregates become much generous in emigration reactions and much greedy in immigration reactions. It is also argued that the overall transfer rate varies as a power law in some scale factors.[12] Here we investigate a little more realistic system with $\varpi \geq 0$. Obviously, $\varpi > 1$ implies that the cooperative merging of two B aggregates may benefit to their competitive abilities, while $1 > \varpi \geq 0$ implies that the cooperative merging of B aggregates is bad for their competitive abilities. Moreover, the rate of the self-coagulation between any two B aggregates is assumed to be a constant J, i.e. $J(i;j) = J$. Then Eqs. (1) are rewritten as

$$\begin{aligned}\frac{\mathrm{d}a_k}{\mathrm{d}t} =& IM_{B\varpi}[(k+1)^\mu a_{k+1}M_{A\nu} + (k-1)^\nu a_{k-1}M_{A\mu} \\ & - k^\mu a_k M_{A\nu} - k^\nu a_k M_{A\mu}], \end{aligned} \quad (2)$$

$$\frac{\mathrm{d}b_k}{\mathrm{d}t} = \frac{J}{2}\sum_{i+j=k} b_i b_j - Jb_k \sum_{j=1}^{\infty} b_j, \quad (3)$$

with the shorthand notations

$$M_{A\mu}(t) = \sum\nolimits_{i=1}^{\infty} i^\mu a_i(t),$$

$$M_{A\nu}(t) = \sum\nolimits_{j=1}^{\infty} j^\nu a_j(t)$$

and

$$M_{B\varpi}(t) = \sum\nolimits_{l=1}^{\infty} l^\varpi b_l(t).$$

From Eqs.(2) and (3), one can readily find that

$$\dot{M}_{A1}(t) = \sum\nolimits_{k=1}^{\infty} k\dot{a}_k(t) \equiv 0$$

and

$$\dot{M}_{B1}(t) = \sum\nolimits_{k=1}^{\infty} k\dot{b}_k(t) \equiv 0.$$

Hence, both A and B aggregates obey the mass conservation law, which is independent of the details of the reaction events.

Consider a simplest but important case in which there only exist monomer aggregates of A and B

species at $t = 0$ and their concentrations both equal to unity, namely

$$a_k(0) = \delta_{k1}, \quad b_k(0) = \delta_{k1}. \tag{4}$$

Under the above monodisperse initial condition (4), with the help of ansatz[19]

$$b_k(t) = B(t)[b(t)]^{k-1}, \tag{5}$$

we can transform the rate equation (3) into two differential equations

$$\frac{\mathrm{d}b}{\mathrm{d}t} = \frac{JB}{2}, \quad \frac{\mathrm{d}B}{\mathrm{d}t} = -\frac{JB^2}{1-b}, \tag{6}$$

with the corresponding initial condition, $b(0) = 0$ and $B(0) = 1$. Equations (6) can be exactly solved to yield $b(t) = Jt(Jt+2)^{-1}$ and $B(t) = 4(Jt+2)^{-2}$. Thus we obtain the following exact solution for B aggregates:

$$b_k(t) = \frac{4}{(Jt+2)^2}\left(1 - \frac{2}{Jt+2}\right)^{k-1}. \tag{7}$$

Equation (7) shows that in the scaling region of $k \gg 1$ and $t \gg 1$, the aggregate size distribution $b_k(t)$ of species B approaches the well-known Smoluchovski scaling form (see e.g. Refs.[1,2,19,20])

$$b_k(t) \approx 4J^{-2}t^{-2}\exp(-y), \quad y = (2/J)kt^{-1}. \tag{8}$$

The result shows that B aggregates are scaled as in the irreversible single-species aggregation case.

We then turn to discussing the size distribution of A aggregates. Introducing a new time variable,

$$T(t) = I\int_0^t \mathrm{d}t' M_{B\varpi}(t'), \tag{9}$$

we recast Eq.(2) into

$$\begin{aligned}\frac{\mathrm{d}a_k(T)}{\mathrm{d}T} =& [(k+1)^\mu a_{k+1}(T) - k^\mu a_k(T)]M_{A\nu}(T) \\ &+ [(k-1)^\nu a_{k-1}(T) - k^\nu a_k(T)]M_{A\mu}(T).\end{aligned} \tag{10}$$

In the long-time limit, it is not difficult to determine the asymptotic solution of $T(t)$ as follows:

$$T(t) \approx \begin{cases} C_1 t^{\varpi} & \varpi > 0, \\ C_2 \ln t & \varpi = 0, \end{cases} \tag{11}$$

where $C_1 = I\Gamma(\varpi)(J/2)^{\varpi-1}$ and $C_2 = 2I/J$. Equation (11) indicates that $T(t) \to \infty$ at $t \to \infty$. Thus, the problem is reduced to deriving the analytical solution of $a_k(T)$ from Eq.(10). Equation (10) is the same as the corresponding governing equation in Ref.[21] (except for the scaling time denotation). By employing a technique similar to that in Ref.[21] one can easily derive the solution $a_k(T)$ of Eq.(10) and then analyze the scaling properties of the aggregate size distribution $a_k(t)$. Here we simply review the derivation of $a_k(T)$ from Eq.(10).

2.1. The symmetrical system with $\mu = \nu$

For a symmetrical system with $\mu = \nu$, Eq.(10) reduces to the discrete diffusionlike equation

$$\begin{aligned}\frac{\mathrm{d}a_k(T)}{\mathrm{d}T} =& M_{A\mu}(T)[(k+1)^\mu a_{k+1}(T) \\ &+ (k-1)^\mu a_{k-1}(T) - 2k^\mu a_k(T)].\end{aligned} \tag{12}$$

Under the monodisperse initial condition, we make the conventional scaling ansatz for the large-time behaviour of $a_k(T)$,[5]

$$a_k(T) \approx k^{-\tau}T^{-w}\Phi[k/S(T)], \quad S(T) \propto T^z. \tag{13}$$

Here $S(T)$ denotes the typical aggregate size of the system, which plays a role analogous to that of the correlation length in critical phenomena, and the scaling function $\Phi(x) \approx 1$ for $x \ll 1$ and $\Phi(x) \approx 0$ for $x \gg 1$. It follows from the mass conservation law that $w = (2-\tau)z$. The scaling ansatz (13) can be rewritten as

$$a_k(T) \approx k^{-\tau}[S(T)]^{\tau-2}\Phi[k/S(T)], \quad S(T) \propto T^z. \tag{14}$$

We then turn to determining the index τ, the typical aggregate size $S(T)$ and the scaling function $\Phi(x)$.

We first investigate the typical aggregate size $S(T)$. By summing up Eq.(12) we obtain

$$\begin{aligned}\frac{\mathrm{d}M_2(T)}{\mathrm{d}T} &= 2[M_\mu(T)]^2, \\ \frac{\mathrm{d}M_0(T)}{\mathrm{d}T} &= -a_1(T)M_\mu(T).\end{aligned} \tag{15}$$

Substituting Eq.(14) into Eqs.(15) yields two distinct sets of asymptotic behaviours of $\dot{S}(T)$ in different cases. By making a comparison between the two equation sets we find that for the $\mu < 3/2$ case, $\tau = \mu - 1$ and the typical size $S(T)$ satisfies[21]

$$S(T) \propto [(3-2\mu)T]^{1/(3-2\mu)}. \tag{16}$$

Equation (16) is in agreement with the statement of the mean aggregate size $S(T) \propto T^{1/(3-\lambda)}$ for a system with the symmetric migration rate kernel $K(ck;cl) = K(cl;ck) = c^\lambda K(k;l)$ (see Ref.[12]).

In the $\mu \geq 3/2$ case, it is found that the scaling form of Eq.(13) is invalid and the system may undergo a gelationlike transition after a sufficiently long time (see, e.g. Refs.[22–24]). In this work, we focus only on the more interesting case of $\mu < 3/2$, in which the aggregate size distribution evolves infinitely according to a conventional scaling regime.

Finally, we derive the scaling function $\Phi(x)$. Substituting the scaling ansatz (14) into Eq.(12), we deduce the differential equation for the large A aggregates,

$$x\Phi''(x)+(2+px^{2-\mu})\Phi'(x) \approx p(\mu-3)x^{1-\mu}\Phi(x), \quad (17)$$

where p is such a separation constant for the x and T dependence that $[S(T)]^{2-2\mu}\dot{S}(T) = p$. Equation (17) yields[21]

$$\Phi(x) \approx C_3 \exp(-C_4 x^{2-\mu}), \quad (18)$$

where C_3 is an integration constant and $C_4 = p/(2-\mu)$. Since our system obeys the mass conservation law, C_3 and p are related as

$$\int_0^\infty \mathrm{d}x\{C_3 x^{2-\mu}\exp[-px^{2-\mu}/(2-\mu)]\} \approx 1.$$

For simplicity, one may choose the constant p so that $C_3 = 1$. It is thus verified that $\Phi(x) \approx 1$ for $x \ll 1$ and $\Phi(x) \approx 0$ for $x \gg 1$.

Inserting Eqs.(11), (16) and (18) into Eq.(13), one can then obtain the t-dependent evolution behaviour of this symmetrical system. When $\varpi > 0$, the size distribution $a_k(t)$ approaches the following scaling form in the $\mu < 3/2$ case:

$$a_k(t) \sim k^{-\tau}t^{-w}\exp(-C_4 x^{2-\mu}), \quad x \propto kt^{-z}, \quad (19)$$

with the nonuniversal exponents $\tau = \mu - 1$, $w = \varpi(3-\mu)/(3-2\mu)$ and $z = \varpi/(3-2\mu)$. Moreover, the result shows that the typical aggregate size $S(t)$ in the $\mu < 3/2$ case grows as $t^{\varpi/(3-2\mu)}$, which is dependent on the indices μ and ϖ of the rate kernel.

In the case of $\varpi = 0$ and $\mu < 3/2$, the typical aggregate size grows as $(\ln t)^{1/(3-2\mu)}$, and the aggregate size distribution is scaled according to the logarithm-correction form $a_k(t) \sim k^{-\tau}(\ln t)^{-w}\Phi[k/(\ln t)^z]$, with the exponents $\tau = \mu - 1$, $w = (3-\mu)/(3-2\mu)$ and $z = 1/(3-2\mu)$.

2.2. The asymmetrical system with $\mu \neq \nu$

Following a spirit similar to the above-mentioned symmetrical case, we also make the conventional scaling ansatz (13) for the asymmetrical system with $\mu \neq \nu$. Summing up Eq.(10), one can obtain

$$\begin{aligned}\frac{\mathrm{d}M_2(T)}{\mathrm{d}T} =& 2[M_\mu(T)M_\nu(T) + M_\mu(T)M_{\nu+1}(T) \\ & - M_{\mu+1}(T)M_\nu(T)], \\ \frac{\mathrm{d}M_0(T)}{\mathrm{d}T} =& -a_1(T)M_\nu(T). \end{aligned} \quad (20)$$

Substituting Eq.(14) into Eqs.(20), we obtain two sets of evolution equations of the typical size $S(T)$ in several different cases (such as the $\tau > \mu+1$, $\tau = \mu+1$, and $\tau < \mu+1$ cases) and then make a detailed comparison between these resulting equations. It is found that the modified scaling ansatz (14) only holds in the system with $\mu < \nu$ and $\mu < 1$, while for all other cases the scaling form (14) is invalid and a gelationlike transition of the system arises.

In this work, we focus only on the kinetic scaling behaviour of the nongelling system with $\mu < \nu$ and $\mu < 1$. Making a comparison between the evolution equations of the typical size $S(T)$ derived from Eqs.(20), we find that the scaling exponent is $\tau = \mu$ and the asymptotic behaviour of $S(T)$ is[21]

$$\frac{\mathrm{d}S(T)}{\mathrm{d}T} \approx [S(T)]^{\mu+\nu-1}. \quad (21)$$

Equation (21) can be straightforwardly solved to yield the asymptotic solution of $S(T)$ for $\mu+\nu < 2$,

$$S(T) \propto [(2-\mu-\nu)T]^{1/(2-\mu-\nu)}. \quad (22)$$

Equation (22) indicates that the evolution behaviour of the typical size $S(T)$ in the transfer-driven aggregate growth system is identical with the irreversible migration case (see Ref.[12]). As for the system with $\mu+\nu > 2$, the scaling ansatz (13) of the aggregate size distribution is invalid and the system will also undergo a gelationlike transition after a long time, which is similar to the kinetic behaviour of the above symmetrical system with $\mu > 3/2$.

We then derive the scaling function $\Phi(x)$ in the nongelling system. Substituting the scaling ansatz (14) into Eq.(10), we find the following differential equation for the large A aggregates:[21]

$$\begin{aligned}&(C_5 x^\nu - C_6 x^\mu - qx)\Phi'(x) \\ \approx& [q(2-\mu) - C_5(\nu-\mu)x^{\nu-1}]\Phi(x), \end{aligned} \quad (23)$$

where $C_5 = \int_0^\infty \mathrm{d}x\,\Phi(x)$, $C_6 = \int_0^\infty \mathrm{d}x[x^{\nu-\mu}\Phi(x)]$ and q is a separation constant for the x and T dependence which satisfies $[S(T)]^{1-\mu-\nu}\dot{S}(T) = q$. From Eq.(23) one can analyze the important feature of $\Phi(x)$.

For the nongelling system, we have $\mu < 1$ and $\mu < \nu$, and Eq.(23) can be rewritten as

$$\Phi^{-1}(x)\Phi'(x) \approx [(\nu-\mu)C_5/C_6]x^{\nu-\mu-1} - [q(2-\mu)/C_6]x^{-\mu}. \quad (24)$$

From Eq.(24) we obtain the asymptotic solution of $\Phi(x)$ for $x \ll 1$,

$$\Phi(x) \approx \begin{cases} \exp(-C_7 x^{1-\mu}) & \nu \geq 1, \\ \exp[C_5/C_6)x^{\nu-\mu}] & \nu < 1, \end{cases} \quad (25)$$

where $C_7 = q(2-\mu)/(1-\mu)C_6$ for $\nu > 1$ or $C_7 = q(2-\mu)/(1-\mu)C_6 - C_5/C_6$ for $\nu = 1$ (it should be pointed out that all the integration constants in Eq.(25) are simplified as unity). On the other hand, from Eq.(23) one can also derive the asymptotic solution of $\Phi(x)$ for $x \gg 1$,[21]

$$\Phi(x) \approx \begin{cases} x^{-(\nu-\mu)} & \nu > 1, \\ x^{-\gamma} & \nu = 1, \\ x^{-(2-\mu)} & \nu < 1, \end{cases} \quad (26)$$

where $\gamma = 1 - \mu + q/(q - C_5)$ $(q \neq C_5)$. In particular, if $\nu = 1$ and $q = C_5$, Eq.(24) yields a uniform solution $\Phi(x) \approx \exp[-C_5 x^{1-\mu}/(1-\mu)C_6]$. Equations (25) and (26) accord with the restrictions on the scaling function $\Phi(x)$. However, if $\Phi(x)$ is continuous for all $x \gg 1$, it follows from $\Phi(x) \approx x^{-(\nu-\mu)}$ that C_6 diverges in the $\nu > 1$ case. Moreover, since our system obeys the mass conservation law, $\int_0^\infty \mathrm{d}x[x^{1-\mu}\Phi(x)]$ should be a finite constant. In the $\nu < 1$ case, $\Phi(x) \approx x^{-(2-\mu)}$ for all $x \gg 1$ implies that the total mass of aggregates also diverges. Hence, $\Phi(x)$ may be discontinuous in most cases and can be consistently set to zero for all $x > x_c$ (x_c is the value of the critical point).

So far, the asymptotic scaling behaviour of the asymmetrical system is acquired. The aggregate size distribution $a_k(t)$ obeys a scaling law only in the case of $\mu < \nu$ and $\mu + \nu < 2$; while in other cases, a gelationlike transition will occur after a sufficiently long time. When $\varpi > 0$, we find that in the case of $\mu < \nu$ and $\mu + \nu < 2$, $a_k(t)$ approaches the scaling form (13), where the scaling function $\Phi(x)$ satisfies Eq.(25) for $x \ll 1$ and Eq.(26) for $x \gg 1$ and the governing exponents are

$$\tau = \mu, \quad w = \varpi(2-\mu)/(2-\mu-\nu), \quad z = \varpi/(2-\mu-\nu). \quad (27)$$

When $\varpi = 0$, $a_k(t)$ satisfies the logarithm-correction scaling form of Eq.(13) with the exponents

$$\tau = \mu, \quad w = (2-\mu)/(2-\mu-\nu), \quad z = 1/(2-\mu-\nu). \quad (28)$$

It is obvious that the scaling exponents for the symmetrical system differ from those for the asymmetrical system.

The results exhibit that for this asymmetrical system, the kinetic behaviour of the aggregate growth depends crucially on the reaction details such as the indices of the rate kernel and the initial concentrations.

3. Model with the generalized rate kernel $K(i;j;l) \sim (i^\mu j^\nu + i^\nu j^\mu)l^\varpi$

In order to thoroughly understand the kinetics of the transfer-driven aggregate growth, we also investigate the model with the generalized symmetrical kernel $K(i;j;l) = I(i^\mu j^\nu + i^\nu j^\mu)l^\varpi$. Introducing the scaling time $T(t)$, we then obtain the following rate equation for this system:

$$\begin{aligned} \frac{\mathrm{d}a_k(T)}{\mathrm{d}T} =& M_{A\nu}(T)[(k+1)^\mu a_{k+1}(T) \\ &+ (k-1)^\mu a_{k-1}(T) - 2k^\mu a_k(T)] \\ &+ M_{A\mu}(T)[(k+1)^\nu a_{k+1}(T) \\ &+ (k-1)^\nu a_{k-1}(T) - 2k^\nu a_k(T)], \end{aligned} \quad (29)$$

which is similar to the governing rate equation for an exchange-driven growth model in Ref.[15]. Here Eq.(29) is analytically solved by employing the technique used in the above section 2. Obviously, if $\mu = \nu$, this generalized model reduces to the simple one discussed in the section 2. In this section, we only focus on the system with $\mu \neq \nu$. Without any loss of generality, we set $\nu > \mu$.

We assume that the conventional scaling ansatz (13) also holds in this generalized system. By summing up Eq.(29) one can then obtain

$$\begin{aligned} \frac{\mathrm{d}M_2(T)}{\mathrm{d}T} &= 2M_\mu(T)M_\nu(T), \\ \frac{\mathrm{d}M_0(T)}{\mathrm{d}T} &= -\frac{a_1(T)}{2}[M_\mu(T) + M_\nu(T)]. \end{aligned} \quad (30)$$

Substituting Eq.(14) into Eqs.(30), we deduce two sets of evolution equations of the typical size $S(T)$ and then make a detailed comparison between these resulting equations. It is found that the modified scaling

ansatz (14) holds only in the system with $\mu+\nu<3$, while for the $\mu+\nu>3$ case the scaling form (14) is invalid and a gelationlike transition arises.

In this work, we also investigate only the kinetic scaling behaviour of the nongelling system with $\mu+\nu<3$. From Eqs.(30) we deduce that the scaling exponent is $\tau=\mu-1$ and the asymptotic behaviour of $S(T)$ is

$$\frac{\mathrm{d}S(T)}{\mathrm{d}T}\approx[S(T)]^{\mu+\nu-2}. \tag{31}$$

Equation (31) directly yields the asymptotic solution of $S(T)$ for the $\mu+\nu<3$ case,

$$S(T)\propto[(3-\mu-\nu)T]^{1/(3-\mu-\nu)}. \tag{32}$$

We then determine the scaling function $\Phi(x)$ in the nongelling system. Substituting the scaling ansatz (14) into Eq.(29), we find the following differential equation for large A aggregates:

$$\begin{aligned}&(C_8x+C_9x^{1+\nu-\mu})\Phi''(x)\\&+[rx^{2-\mu}+2C_8+2C_9(1+\nu-\mu)x^{\nu-\mu}]\Phi'(x)\\\approx&[r(\mu-3)x^{1-\mu}-C_9(1+\nu-\mu)\\&\times(\nu-\mu)x^{\nu-\mu-1}]\Phi(x),\end{aligned} \tag{33}$$

where

$$C_8=\int_0^\infty \mathrm{d}x[x^{1+\nu-\mu}\Phi(x)],$$

$$C_9=\int_0^\infty \mathrm{d}x[x\,\Phi(x)]$$

and r is a separation constant for the x and T dependence which satisfies $[S(T)]^{2-\mu-\nu}\dot{S}(T)=r$. Since we have $\mu<\nu$, from Eq.(33) we then obtain the asymptotic solution of $\Phi(x)$ for $x\ll1$,

$$\Phi(x)\approx\begin{cases}\exp(-C_{10}x^{1-\mu}) & \nu\geq2,\\ \exp[-(C_9/C_8)x^{\nu-\mu}] & \nu<2,\end{cases} \tag{34}$$

where $C_{10}=r/(2-\mu)C_8$ for $\nu>2$ or $C_{10}=[r+C_9(2-\mu)]/(2-\mu)C_8$ for $\nu=2$ (here all the integration constants in Eq.(34) are simplified as unity). On the other hand, from Eq.(33) one can also derive the asymptotic solution of $\Phi(x)$ for $x\gg1$,

$$\Phi(x)\approx\begin{cases}x^{-(\nu-\mu)} & \nu>2,\\ x^{-\eta} & \nu=2,\\ \exp(-C_{11}x^{2-\nu}) & \nu<2,\end{cases} \tag{35}$$

where $\eta=3-\mu$ for $r\geq C_9$ or $\eta=2-\mu+r/C_9$ for $r<C_9$ and $C_{11}=r/(2-\nu)C_9$. Equations (34) and (35) accord with the restrictions on the scaling function $\Phi(x)$. However, it follows from Eq.(35) that C_8 diverges for $\nu\geq2$. Thus for the $\nu\geq2$ case, $\Phi(x)$ may be discontinuous and can be consistently set to zero for all x greater than a critical point x_c'; otherwise, a gelationlike transition may arise in the system.

Thus we know that for the system with the generalized rate kernel, the aggregate size distribution $a_k(t)$ obeys a scaling law only in the $\mu+\nu<3$ case. In other cases, the system may undergo a gelationlike transition after a sufficiently long time.

When $\varpi>0$ and $\mu+\nu<3$, we find that the aggregate size distribution $a_k(t)$ approaches the scaling form (13) with the time variable t. The scaling function $\Phi(x)$ satisfies Eq.(34) for $x\ll1$ and Eq. (35) for $x\gg1$, and the exponents are

$$\begin{aligned}&\tau=\mu-1,\quad w=\varpi(3-\mu)/(3-\mu-\nu),\\&z=\varpi/(3-\mu-\nu).\end{aligned} \tag{36}$$

When $\varpi=0$ and $\mu+\nu<3$, $a_k(t)$ satisfies the logarithm-correction scaling form of Eq.(13) with the exponents

$$\begin{aligned}&\tau=\mu-1,\quad w=(3-\mu)/(3-\mu-\nu),\\&z=1/(3-\mu-\nu).\end{aligned} \tag{37}$$

The results show that for this generalized system, the kinetic behaviour of the aggregate growth is also crucially dependent on the reaction details such as the indexes of the rate kernel and the initial concentrations.

4. Summary

We have introduced a general transfer-driven aggregate growth model with the rate kernel $K(i;j;l)\sim i^\mu j^\nu l^\varpi$ ($\varpi\geq0$), in which the monomer transfers between two A aggregates proceed only in the presence of a B aggregate. Meanwhile, the B aggregates can spontaneously coagulate each other with a constant reaction rate kernel. Based on the mean-field rate equations, we have analyzed the evolution behaviours of the aggregate size distributions in different cases with indexes μ, ν, and ϖ varying. We have found that the size distribution of B aggregates approaches the well-known Smoluchovski scaling form (8), while the kinetic evolution of the A aggregates depends crucially on the details of the transfer rate kernel.

For the symmetrical system with $\mu=\nu$, we have found that the scaling behaviour of the system depends strongly on the indexes μ and ϖ. In the case of $\mu<3/2$ and $\varpi>0$, the aggregate size distribution

$a_k(t)$ approaches the conventional scaling form; while for $\mu < 3/2$ and $\varpi = 0$, $a_k(t)$ satisfies the logarithm-correction scaling form. Moreover, the scaling exponents are nonuniversal and dependent on the reaction details (such as the indexes of the rate kernel). On the other hand, the system with $\mu > 3/2$ may undergo a gelationlike transition.

For the asymmetrical system with $\mu \neq \nu$, $a_k(t)$ also satisfies a scaling law in the case of $\mu < \nu$ and $\mu + \nu < 2$. When $\varpi > 0$, $a_k(t)$ approaches the conventional scaling form. When $\varpi = 0$, $a_k(t)$ takes the logarithm-correction scaling form. However, in other cases ($\mu + \nu > 2$ or $\mu > \nu$), a gelationlike transition may arise in the system. It is also found that the scaling exponents are nonuniversal and depend on the details of the reaction events.

In addition, we have also investigated the kinetic scaling behaviour of the model with the generalized kernel $K(i; j; l) \sim (i^\mu j^\nu + i^\nu j^\mu) l^\varpi$. When $\mu = \nu$, this model is equivalent to the above-mentioned symmetrical one. When $\nu \neq \mu$, the aggregate size distribution $a_k(t)$ satisfies a conventional scaling law only in the $\mu + \nu < 3$ case. In other cases, a gelationlike transition of the system may occur after a sufficiently long time. For the nongelling system, $a_k(t)$ approaches the conventional scaling form in the case of $\mu + \nu < 3$ and $\varpi > 0$, while it may take the logarithm-correction scaling form in the borderline case of $\varpi = 0$.

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