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Stabilizing effect of ion pressure gradient on magnetic curvature-driven drift modes located at rational surface of tokamak plasma*

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In the fluid model, we derive a dispersion relation for the toroidal drift modes of tokamak plasmas, including the ion pressure gradient and the magnetic field gradient and curvature. It is shown that the magnetic field gradient and curvature (MFGC) can cause instabilities at the rational surface, which are of toroidicity-induced (TI) modes. On the other hand, it is discovered that the ion pressure gradient can stabilize the present MFGC instabilities. The critical threshold of ion pressure gradient, which makes the growth rate reduced to zero, is obtained both analytically and numerically.

Keywords: magnetic field gradient and curvature, drift mode, stabilization
PACC: 5255G, 5235

1. Introduction

The transport of energy and particles across the magnetic field is still an unexplained phenomenon. The observed transport coefficients for energy and particles generally exceed neoclassical predictions by more than two orders of magnitude. Understanding and controlling such anomalous transport is one of the major challenges for magnetic confinement fusion research. As the potential causes for such anomaly, the ion temperature gradient (ITG) and electron temperature gradient (ETG)-driven instabilities have been extensively studied over the past years.[1−6]

In addition, the toroidicity-induced (TI) modes have also been extensively studied.[7−13] It has been shown that TI and ITG modes can exist simultaneously.[7,8] Furthermore, the damping of negative magnetic shear on the TI instabilities with the participation of ITG effect has also been studied.[11] The results indicated that the negative magnetic shear in the typical tokamak parameter regime can damp the growth rate of TI instabilities but not completely suppress them. Thus, it is important to find a mechanism that can completely suppress TI instabilities. In this work, we derive a simple dispersion relation for the TI modes driven by the ions' and electrons' magnetic field gradient and curvature drift frequency ($\omega_{\rm Di}$ and $\omega_{\rm De}$) and study the effects of ion pressure gradient on the instabilities. It is found that the ion pressure gradient can suppress the growth rate of the TI instabilities to zero. Here it should be pointed out that the toroidicity in Refs.[7–12] means the toroidal coupling of several modes due to $\omega_{\rm Di}$ and $\omega_{\rm De}$. However, the present modes[13] are directly driven by $\omega_{\rm Di}$ and $\omega_{\rm De}$ and their coupling. So, in the following we call the present drift mode the MFGC mode.

2. Basic equations

We now study an electrostatic perturbation in the usual circular cross-section tokamak with $B_z = B_0/(1+\varepsilon\cos\theta)$ and $B_\theta = (\varepsilon/q)B_z$, which varies as

$$\begin{aligned}\delta f =& \tilde{f}\exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r} - \mathrm{i}\omega t)\\ =& \tilde{f}\exp[\mathrm{i}(k_r r + k_\theta r\theta + k_z z - \omega t)],\end{aligned} \tag{1}$$

where $\boldsymbol{k} = \hat{r}k_r + \hat{\theta}k_\theta + \hat{z}k_z$ is the wave vector. Here the dependence of $\tilde{f}$ on r is omitted for the radial local mode, which is valid when the radial variations of equilibrium physical quantities over the mode width can be neglected.[14] In the sheared coordinate system

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$\hat{e}_\perp = \hat{b} \times \hat{r}$, this mode gives rise to $k_\perp = [k_\theta - (\varepsilon/q)k_z]$ and $k_{//} = [k_z + (\varepsilon/q)k_\theta]$. In this case, the wave vector is expressed as $\boldsymbol{k} = \hat{b}k_{//} + \hat{r}k_r + \hat{e}_\perp k_\perp$, where $\hat{b} = [\hat{z} + (\varepsilon/q)\hat{\theta}]/[1 + (\varepsilon/q)^2]^{1/2}$ is unit vector of magnetic field and $\hat{e}_\perp = [\hat{\theta} - (\varepsilon/q)\hat{z}]/[1 + (\varepsilon/q)^2]^{1/2}$ is that of the perpendicular direction to magnetic field. Accordingly, the derivative $\nabla = \nabla_r + \nabla_\perp + \nabla_{//}$ with $\nabla_r = \hat{r}\hat{r} \cdot \nabla$, $\nabla_\perp = \hat{e}_\perp \hat{e}_\perp \cdot \nabla$, and $\nabla_{//} = \hat{b}\hat{b} \cdot \nabla$

To derive the present dispersion relation for MFGC modes, we use the two-fluid description in the sheared coordinate system. In addition, for simplicity the electrons are assumed to be adiabatic in an electrostatic case. Then, our model consists of the continuity equation of ions and adiabatic electrons and the energy equation of ions, which can be expressed as, respectively,

$$\begin{aligned}
&[(\boldsymbol{v}_{\mathrm{i}//} - \boldsymbol{v}_{\mathrm{e}//}) + (\boldsymbol{v}_{*pi} - \boldsymbol{v}_{*\mathrm{pe}})] \cdot \nabla n \\
&+ n\nabla \cdot [(\boldsymbol{v}_{\mathrm{i}//} - \boldsymbol{v}_{\mathrm{e}//}) + (\boldsymbol{v}_{*pi} - \boldsymbol{v}_{*\mathrm{pe}})] \\
&+ \nabla \cdot (n\boldsymbol{v}_{\mathrm{pi}}) = 0, \qquad (2) \\
&\frac{\partial}{\partial t}p_\mathrm{i} + (\boldsymbol{v}_{\mathrm{i}//} + \boldsymbol{v}_\mathrm{E}) \cdot \nabla p_\mathrm{i} + \frac{5}{3}p_\mathrm{i}\nabla \cdot (\boldsymbol{v}_{\mathrm{i}//} + \boldsymbol{v}_\mathrm{E}) \\
&+ \frac{5}{3}\boldsymbol{v}_{\mathrm{Di}} \cdot \nabla p_\mathrm{i} + \frac{5}{3}n_\mathrm{i}\boldsymbol{v}_{\mathrm{Di}} \cdot \nabla T_\mathrm{i} + \frac{5}{3}p_\mathrm{i}\nabla \cdot \boldsymbol{v}_{\mathrm{pi}} = 0, \qquad (3)
\end{aligned}$$

where $\boldsymbol{v}_{\mathrm{D}j}$ $(j = i, e)$ is magnetic field gradient and curvature drift velocity, $\boldsymbol{v}_{*\mathrm{p}j}$ is diamagnetic drift, $\boldsymbol{v}_{j//}$ is parallel fluid velocity, $\boldsymbol{v}_\mathrm{E}$ is $\boldsymbol{E} \times \boldsymbol{B}$ velocity, and $\boldsymbol{v}_{\mathrm{pi}}$ is ion polarization drift,

$$\begin{aligned}
\boldsymbol{v}_{\mathrm{pi}} = &-(B\omega_{ci})^{-1}\frac{d}{dt}\phi = -(B\omega_{ci})^{-1} \\
&\times \left[\frac{\partial}{\partial t}(\nabla_r + \nabla_\perp)\phi + (\boldsymbol{v}_\mathrm{i} \cdot \nabla)(\nabla_r + \nabla_\perp)\phi\right]. \qquad (4)
\end{aligned}$$

Here the polarization drift of electrons has been omitted, being small compared with that of ions. Equation (1) is obtained by subtracting the continuity equation of electrons from that of ions with the quasi-neutrality condition $n = n_e = n_\mathrm{i}$. The approach makes our model include the electron drift effects, e.g. $\boldsymbol{v}_{*\mathrm{pe}}$, even for the present adiabatic electrons in the electrostatic case, which is remarkably different from the conventional fluid approach of studying ITG modes.[15] This is a key point leading to the present results. In Eq.(3), the cancellation relation between the convective diamagnetic drift and diamagnetic heat flow of ions has been considered.

For an arbitrary vector, e.g. the total fluid velocity $\boldsymbol{v}_j$, we use $\delta\boldsymbol{v}_j$ and $\delta(\nabla \cdot \boldsymbol{v}_j)$ to describe the linearization of $\boldsymbol{v}_j$ and its divergence while the equilibrium fluid velocity is denoted by $\boldsymbol{V}_j$. Then, we obtain, respectively, from linearizing Eqs.(2)–(4),

$$\begin{aligned}
&(\boldsymbol{V}_{\mathrm{i}//} - \boldsymbol{V}_{\mathrm{e}//}) \cdot \nabla\tilde{n} \\
&+ \boldsymbol{V}_{\mathrm{Di}} \cdot \nabla\tilde{p}_\mathrm{i} - \boldsymbol{V}_{\mathrm{De}} \cdot \nabla\tilde{p}_\mathrm{e} + \delta(\nabla \cdot \boldsymbol{v}_{\mathrm{pi}}) \\
&+ \nabla_{//} \cdot (\delta\boldsymbol{v}_{\mathrm{i}//} - \delta\boldsymbol{v}_{\mathrm{e}//}) = 0, \qquad (5) \\
&\frac{\partial\tilde{p}_\mathrm{i}}{\partial t} + (\boldsymbol{V}_{\mathrm{i}//} + \boldsymbol{V}_\mathrm{E}) \cdot \nabla\tilde{p}_\mathrm{i} + \tau^{-1}\left(\frac{5}{3}\boldsymbol{V}_{\mathrm{Di}} - \boldsymbol{V}_{*pi}\right) \cdot \nabla\tilde{\phi} \\
&+ \frac{10}{3}\boldsymbol{V}_{\mathrm{Di}} \cdot \nabla\tilde{p}_\mathrm{i} - \frac{5}{3}\boldsymbol{V}_{\mathrm{Di}} \cdot \nabla\tilde{n} + \frac{5}{3}\delta(\nabla \cdot \boldsymbol{v}_{\mathrm{pi}}) = 0, \qquad (6)
\end{aligned}$$

$$\delta(\nabla \cdot \boldsymbol{v}_{\mathrm{pi}}) = -\mathrm{i}b_\mathrm{i}(\omega - \boldsymbol{k} \cdot \boldsymbol{V}_\mathrm{i})\tilde{\phi}, \qquad (7)$$

where $\tilde{p}_j = \delta p_j/P_j$, $\tilde{n} = \delta n/n$, $\tilde{\phi} = e\delta\phi/T_\mathrm{e}$, $\tau = T_\mathrm{i}/T_\mathrm{e}$, $\rho = (T_\mathrm{e}m_\mathrm{i})^{1/2}/eB$, and $b_\mathrm{i} = (k_\perp^2 + k_r^2)\rho^2$. For simplicity, here we study the mode at the ration surface, i.e. its parallel wavenumber $k_{//} = 0$. At the rational surface, it is physically required that every physical quantity should be finite. Thus, for an arbitrary fluid velocity $\boldsymbol{v}_j$ at the rational surface, we have

$$\boldsymbol{k}_{//} \cdot \boldsymbol{v}_j = 0. \qquad (8)$$

Subsequently, we note that for the adiabatic electrons, $\tilde{T}_\mathrm{e} = \delta T_\mathrm{e}/T_\mathrm{e} = 0$ and thus[16]

$$\tilde{p}_\mathrm{e} = \tilde{n}_\mathrm{e} = \tilde{\phi}. \qquad (9)$$

Then, the combination of Eqs.(5)–(9) yields a simple dispersion relation,

$$\begin{aligned}
&b_\mathrm{i}\Omega^2 + [\omega_{*pi} + \omega_{\mathrm{De}} - \frac{5}{3}\omega_{\mathrm{Di}}(2 - b_\mathrm{i})]\Omega \\
&- \frac{10}{3}\omega_{\mathrm{De}}\omega_{\mathrm{Di}} + \frac{5}{3}(1 - \tau^{-1})\omega_{\mathrm{Di}}^2 = 0, \qquad (10)
\end{aligned}$$

where $\omega_{\mathrm{De}} = \boldsymbol{k} \cdot \boldsymbol{V}_{\mathrm{De}}$, $\omega_{\mathrm{Di}} = \boldsymbol{k} \cdot \boldsymbol{V}_{\mathrm{Di}}$, $\omega_{*\mathrm{pi}} = \boldsymbol{k} \cdot \boldsymbol{V}_{*\mathrm{pi}}$, and $\Omega = \omega - \boldsymbol{k} \cdot \boldsymbol{V}_\mathrm{i}$. In deriving Eq.(10), the terms $\tau^{-1}\omega_{\mathrm{Di}}\omega_{*\mathrm{pi}}$ and $\omega_{\mathrm{De}}\omega_{*\mathrm{pi}}$ have cancelled each other. In addition, we have used $\boldsymbol{V}_{\mathrm{i}//} + \boldsymbol{V}_\mathrm{E} = \boldsymbol{V}_\mathrm{i} - \boldsymbol{V}_{*\mathrm{pi}}$. From Eq.(10), we obtain the real frequency and growth rate of mode,

$$\Omega_r = \left[\frac{\tau}{2}\left(\frac{R}{L_{P_\mathrm{i}}}\right) - \frac{5}{3}\tau(2 - b_\mathrm{i}) - 1\right]/(2b_\mathrm{i}), \qquad (11)$$

$$\gamma = \sqrt{20\tau(1 + \tau)/3 - 4b_\mathrm{i}^2\Omega_r^2}/2b_\mathrm{i}. \qquad (12)$$

Here the frequencies are normalized to ω_{De}, e.g. the normalized $\omega_{*\mathrm{pi}} = -\tau R/2L_{\mathrm{pi}}$, where R/L_{pi} is the normalized ion pressure gradient.

3. Numerical results

When the ion pressure gradient tends to zero, e.g. in the core regime of tokamaks, the instabilities, described by Eqs.(11) and (12), are driven completely by magnetic field gradient and curvature drifts of ions

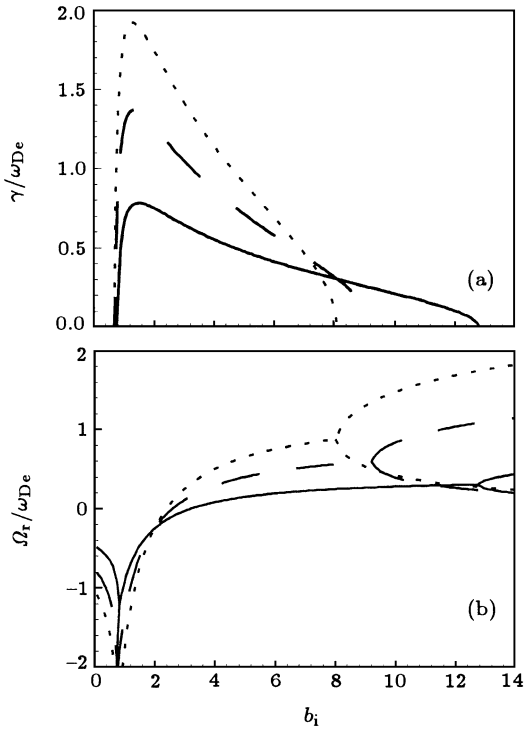


Fig.1. Growth rate and real frequency versus b_i for $\tau = 0.5$ (solid line), 1 (dashed line), and 1.5 (dotted line), where $R/L_{P_i} = 0$.

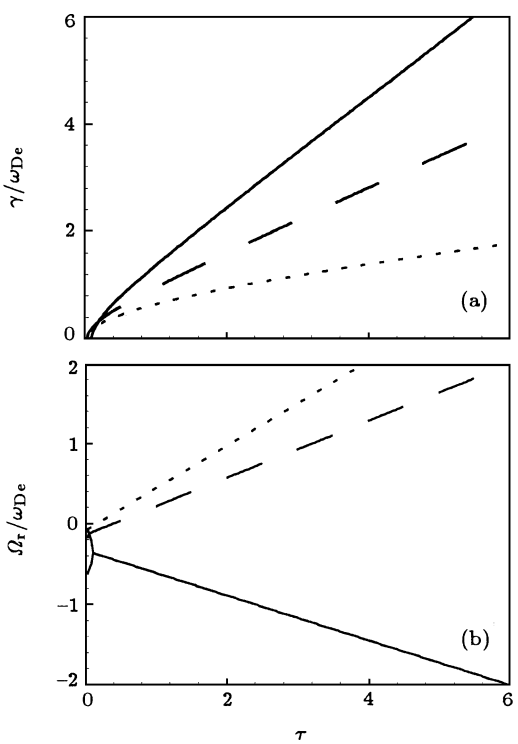


Fig.2. Growth rate and real frequency versus τ for $b_i = 1.5$ (solid line), 3.5 (dashed line), and 5.5 (dotted line), where $R/L_{P_i} = 0$.

and electrons. In that case, the b_i spectrum of growth rate ranges from $b_i = 0.7$ to $b_i = 12.8$, as shown in Fig.1. However, the peak values of growth rate are always near $b_i = 1.2$. The τ spectrum of long-wavelength modes (here $b_i < 6$) is completely different from that of short-wavelength modes. It can be seen from Fig.2 that the long-wavelength modes have almost no stable τ parameter regime. However, for the short-wavelength modes (see Fig.3), the shorter the wavelength is, the larger the stable τ parameter regime is.

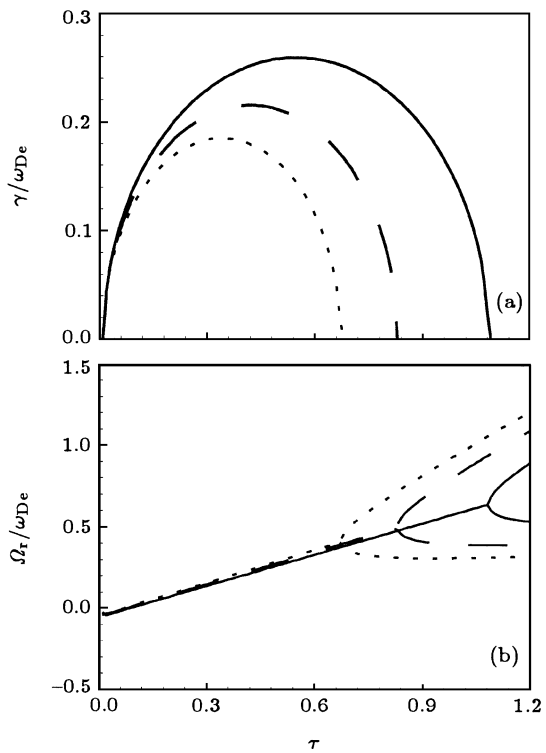


Fig.3. Growth rate and real frequency versus τ for $b_i = 9$ (solid line), 10 (dashed line), and 11 (dotted line), where $R/L_{P_i} = 0$.

Figure 4 shows the b_i spectrum of growth rate and real frequency for different ion pressure gradients. Generally, the growth rate first increases and then decreases to zero with increase of ion pressure gradient. Thus, there exists a critical threshold of ion pressure gradient, which suppresses the growth rate to zero. Taking the growth rate in Eq.(12) to be zero, we have the threshold

$$R/L_{\rm pi} = \frac{[1 + 5\tau(2 - b_i)/3] + \sqrt{20 b_i \tau(1+\tau)/3}}{\tau/2}. \tag{13}$$

The maximum threshold of ion pressure gradient is near $b_i = 1.2$, as displayed in Fig.5. Furthermore, Fig.5, together with Fig.6, shows that the threshold increases with decrease of parameter τ.

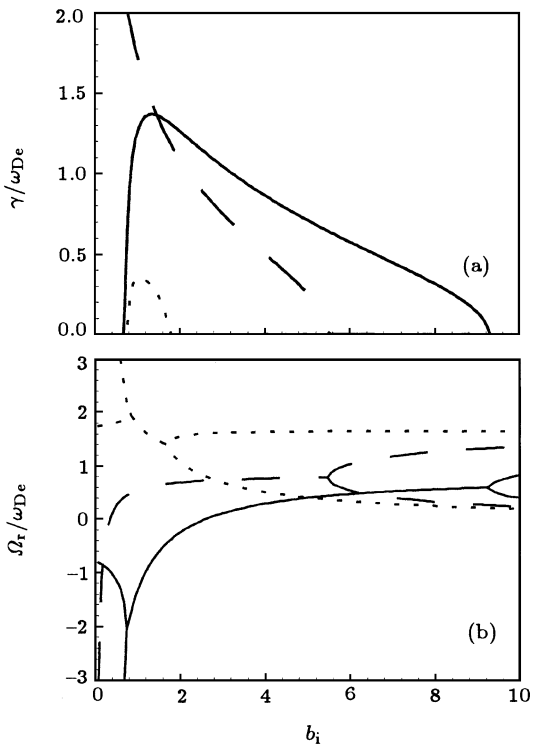


Fig.4. Growth rate and real frequency versus b_i for $R/L_{pi} = 0$ (solid line), 7.5 (dashed line), and 12.5 (dotted line), where $\tau = 1$.

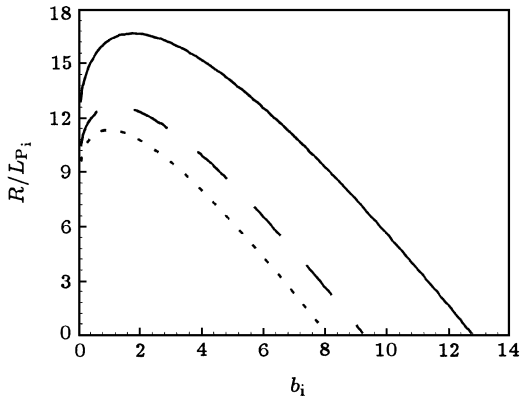


Fig.5. Critical threshold of ion pressure gradient R/L_{pi} versus b_i for $\tau = 0.5$ (solid line), 1(dashed line), and 1.5 (dotted line).

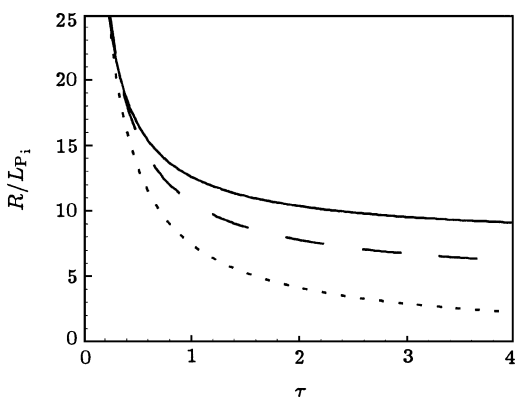


Fig.6. Critical threshold of ion pressure gradient R/L_{pi} versus τ for $b_i = 1.5$ (solid line), 3.5 (dashed line), and 5.5 (dotted line).

4. Summary

In summary, the present paper gives a simple method of building a fluid model for TI modes. In the present model the dominant MFGC instability is near $b_i = 1.2$, which is important in a tokamak core regime, where the plasma pressure gradient is small. It is discovered that the ion pressure gradient stabilizes the MFGC modes. The critical threshold of ion pressure gradient, which makes the growth rate decrease to zero, is obtained both analytically and numerically. The maximum threshold is in the position where the maximum growth rate is, near $b_i = 1.2$. On the other hand, the threshold decreases with increase of parameter τ.

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