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Analysis of pulse broadening induced by the second-order PMD*

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We propose a new conception of depolarization vector to describe the effect of depolarization induced by the second-order polarization mode dispersion (PMD). Deriving the formula of pulse broadening induced by the second-order PMD, we find that the polarization-dependent chromatic dispersion (PCD) always enhances the pulse broadening. However the depolarization vector decreases the pulse broadening. The pulse broadening is correlated with the bit-rate of a transmission system. By adjusting the directions of the Stokes vector of initial state of polarization, initial first-order polarization dispersion vector and depolarization vector to be parallel to each other, one can obtain an optimum dispersion compensation. But when the PCD is not equal to zero, the optimum dispersion cannot achieve a complete compensation, and the minimum pulse broadening is equal to $\sigma = \left(\sqrt{2}/4\right)(\mathrm{DCF}/T_0)$.

Keywords: second-order PMD, pulse broadening, PMD compensation, polarization-dependent chromatic dispersion (PCD), depolarization vector

PACC: 4280M, 4281C, 4281F

1. Introduction

The polarization mode dispersion (PMD) is considered as an ultimate limitation to a high bit-rate fibre communication system.[1] Due to the strong frequency dependence of PMD, the need for the high-order PMD compensation grows rapidly as the channel bit rate increases.

It is suggested in some earlier papers that the PMD vector $\boldsymbol{p}$ can be extended to the Taylor series

$$\boldsymbol{p}(\omega) = \boldsymbol{p}(\omega_0) + \left.\frac{\partial \boldsymbol{p}}{\partial \omega}\right|_{\omega=\omega_0}(\omega-\omega_0) + \frac{1}{2}\left.\frac{\partial^2 \boldsymbol{p}}{\partial \omega^2}\right|_{\omega=\omega_0}(\omega-\omega_0)^2 \qquad (1)$$

where $\left.\frac{\partial \boldsymbol{p}}{\partial \omega}\right|_{\omega=\omega_0}$ is the second-order PMD vector. $\boldsymbol{p} = p\hat{p}$, where p is the amplitude of $\boldsymbol{p}$, and $\hat{p}$ is a unit direction vector of $\boldsymbol{p}$, therefore $\frac{\partial \boldsymbol{p}}{\partial \omega} = \frac{\partial p}{\partial \omega}\hat{p} + p\frac{\partial \hat{p}}{\partial \omega}$. In many papers $q_0 = \frac{\partial p}{\partial \omega}$ is defined as the polarization-dependent chromatic dispersion (PCD), and $\frac{\partial \hat{p}}{\partial \omega}$ express the variation of direction of PMD vector and is called depolarization component. Here q_0 and $\frac{\partial \hat{p}}{\partial \omega}$ are both random variables.[2,3]

The effects on the pulse broadening induced by the high-order PMD have been studied in many papers. The first analytic way is using $p(\omega) = p(\omega_0) + q_0(\omega-\omega_0)$, and then inversely transforming it into a time domain and obtaining a complex formula. This method is rough approximation, and cannot be used to calculate the pulse broadening directly.[4,5] The other way is using moments, and finding the autocorrelation function, and then calculating the broadening factor.[6,7] In this paper, we suggest a novel method to study this effect. First we put forward a depolarization vector to describe the depolarization characteristics, then analyse the pulse broadening induced by the PCD and depolarization vector together, and finally analyse how to tune up all these vectors to their minimum pulse broadenings.

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2. The depolarization vector and its effect on PMD vector

Although the second-order PMD is a random variable, it can be considered as an invariable in several wavelength ranges. For example, the depolarization component $\frac{\partial\hat{p}}{\partial\omega}$ as well as $\hat{p}$ varies along with a circle in the Poincaré sphere in a range of 0.3–0.4 nm, when the rms differential group delay (DGD) is 14.7 ps.[2] And when rms DGD is 2 ps, the range is about 3 nm.[8] Now the fibre's PMD already decreases down to 0.1 ps/$\sqrt{\text{km}}$, so the fluctuation is more decreased, and the stable range for the second-order PMD is enlarged. The $\hat{p}$ runs around a circle in unit sphere and $\frac{\partial\hat{p}}{\partial\omega}$ is perpendicular to $\hat{p}$, so there certainly exists such a vector $\boldsymbol{q}$ that $\frac{\partial\hat{p}}{\partial\omega} = \boldsymbol{q}\times\hat{p}$ holds. $\boldsymbol{q}$ is called the depolarization vector, whose amplitude expresses the extent of depolarization effect, and whose direction expresses a running plane of $\hat{p}$. Thus

$$\frac{\partial\boldsymbol{p}}{\partial\omega} = q_0\hat{p} + p\boldsymbol{q}\times\hat{p} = \left(\frac{q_0}{p} + \boldsymbol{q}\times\right)\boldsymbol{p}. \tag{2}$$

When we neglect the third- or higher-order PMD, the q_0 and $\boldsymbol{q}$ are all considered to be constant. Due to $p(\omega) = p_0 + q_0\Omega$, where p_0 is the amplitude of initial first-order PMD vector $\boldsymbol{p}_0$ when $\omega = \omega_0$ and $\Omega = \omega - \omega_0$, we can obtain

$$\frac{\partial\boldsymbol{p}}{\partial\omega} = \left(\frac{q_0}{p_0 + q_0\Omega} + \boldsymbol{q}\times\right)\boldsymbol{p}. \tag{3}$$

The equation (3) can be solved as follows:

$$\begin{aligned}\boldsymbol{p}(\Omega) =&(p_0 + q_0\Omega)\\ &\times\{\hat{q}(\hat{q}\cdot\hat{p}_0) + [\hat{p}_0 - \hat{q}(\hat{q}\cdot\hat{p}_0)]\cos q\Omega\\ &+ \hat{B}\sin2\alpha\sin q\Omega\},\end{aligned} \tag{4}$$

where $\hat{B}$ is the unit vector of $\hat{q}\times\hat{p}_0$, and 2α is the angle between $\boldsymbol{p}_0$ and $\boldsymbol{q}$.

3. The pulse broadening induced by the second-order PMD

In 1998 Karlsson obtained a formula to calculate the pulse broadening due to any-order PMD.[9,10] By neglecting the group velocity dispersion in the fibre and chirp of an optical source, it can be written as

$$\tau^2 = \tau_0^2 + \frac{1}{4}\left[\frac{\langle p^2\rangle}{P_{\text{total}}} - \left(\frac{\boldsymbol{S}_0\cdot\langle\boldsymbol{p}\rangle}{P_{\text{total}}}\right)^2\right], \tag{5}$$

where τ_0 is the initial rms width of a pulse, P_{total} is its total power, $\boldsymbol{S}_0$ is the Stokes vector of initial SOP when $\omega = \omega_0$, $\langle a\rangle = \frac{1}{2\pi}\int_{-\infty}^{+\infty} a|F(\Omega)|^2\mathrm{d}\Omega$, $F(\Omega)$ is the frequency spectrum of signal. When the input signal is a Gaussian pulse $f(t) = \exp\left(-\frac{t^2}{2T_0^2}\right)$, $P_{\text{total}} = \sqrt{\pi}T_0$, we can obtain

$$\begin{aligned}\frac{\boldsymbol{S}_0\cdot\langle\boldsymbol{p}\rangle}{P_{\text{total}}} =&p_0\cos2\beta\exp\left(-\frac{q^2}{4T_0^2}\right)\\ &+ p_0\cos2\alpha\cos2\gamma\left[1 - \exp\left(-\frac{q^2}{4T_0^2}\right)\right]\\ &+ \frac{q_0 q}{2T_0^2}\sin2\alpha\cos2\delta\exp\left(-\frac{q^2}{4T_0^2}\right),\end{aligned} \tag{6}$$

where 2β, 2γ and 2δ are the angles included between the Stocks vector of initial state of polarization $\boldsymbol{S}_0$ and initial PMD vector $\boldsymbol{p}_0$, depolarization vector $\boldsymbol{q}$ and $\vec{B} = \hat{q}\times\hat{p}_0$ respectively. In addition, we have

$$\frac{\langle p^2\rangle}{P_{\text{total}}} = p_0^2 + \frac{q_0^2}{2T_0^2}. \tag{7}$$

Thus the rms pulse broadening σ is

$$\begin{aligned}\sigma^2 =&\frac{p_0^2}{4}\left\{1 + \frac{q_0^2}{2p_0^2T_0^2} - \left[\cos2\beta\exp\left(-\frac{q^2}{4T_0^2}\right)\right.\right.\\ &+\cos2\alpha\cos2\gamma\left[1 - \exp\left(-\frac{q^2}{4T_0^2}\right)\right]\\ &\left.\left.+\frac{q_0 q}{2p_0T_0^2}\sin2\alpha\cos2\delta\exp\left(-\frac{q^2}{4T_0^2}\right)\right]^2\right\}.\end{aligned} \tag{8}$$

If we let $x = \frac{q_0}{p_0T_0}$, $y = \frac{q}{2T_0}$, then

$$\begin{aligned}\sigma^2 =&\frac{p_0^2}{4}\left\{1 + \frac{x^2}{2} - \{\cos2\beta\exp(-y^2)\right.\\ &+\cos2\alpha\cos2\gamma[1 - \exp(-y^2)]\\ &\left.+xy\sin2\alpha\cos2\delta\exp(-y^2)\}^2\right\}.\end{aligned} \tag{9}$$

Formula (9) is important, which describes the relationship between the pulse broadening and all parameters.

4. The optimum compensation of the second-order PMD

The dispersion compensation means adjusting all parameters affecting σ^2 and minimizing them. We discuss this issue in the following cases.

Case 1 $x = 0$ and $y = 0$. This means that the second-order PMD does not exist, so, neither amplitude nor direction of the PSP vector is changed. In this case

$$\sigma^2 = \frac{p_0^2}{4}\{1 - \cos^2 2\beta\} = \frac{p_0^2}{4}\sin^2 2\beta. \tag{10}$$

The result is identical with that for the first-order PMD.

Case 2 $x = 0$ but $y \neq 0$. In this case, the second-order PMD can only change the PSP vector's direction but not the amplitude. Thus

$$\sigma^2 = \frac{p_0^2}{4}\{1 - \{\cos 2\beta \exp(-y^2) + \cos 2\alpha \cos 2\gamma[1 - \exp(-y^2)]\}^2\}. \quad (11)$$

It is clear that the depolarization vector always decreases the pulse broadening. It is not difficult to find that when

$$\cos 2\beta = \cos 2\alpha \cos 2\gamma \quad (12)$$

and $\sigma^2 = 0$, the complete compensation of PMD is achieved.

Case 3 $y = 0$ but $x \neq 0$. In this case, the second-order PMD only changes the PSP vector's amplitude but not its direction. Thus

$$\sigma^2 = \frac{p_0^2}{4}\sin^2 2\beta + \frac{1}{8}\left(\frac{q_0}{T_0}\right)^2. \quad (13)$$

This means that the effect of DCF is always to increase the pulse broadening, and no matter how to tune up the angle included between the PSP vector and the Stokes vector of initial state of polarization, the complete compensation of DSP (referring to $\sigma^2 = 0$) cannot be achieved. In an optimum compensation, the minimum pulse broadening is equal to

$$\sigma = (\sqrt{2}/4)(q_0/T_0). \quad (14)$$

It is noted that in the optimum compensation, the minimum pulse broadening is inversely proportional to width of input pulse. It means that the higher the bit rate is, the greater the broadening is. This result is quite different from that for the first-order PMD, because the pulse broadening due to the first-order PMD has no relationship with the bit rate of a communication system. So for a super-high bit rate system the effect of the second-order PMD is more serious.

Case 4 $x \neq 0$ and $y \neq 0$. In this case, the second-order PMD changes not only the PSP vector's amplitude, but also its direction. Its broadening is expressed as the formula (9).

Generally for the case of $x \neq 0$ and $y \neq 0$, only when $\boldsymbol{S}_0$, $\boldsymbol{p}_0$ and $\boldsymbol{q}$ are parallel to each other can the optimum compensation be achieved. But in the optimum compensation it cannot be fully compensated. If the PCD is equal to 1 ps^2 and $T_0 = 4$ps (for a 40 Gb/s communication system), the minimum pulse broadening is 0.09ps, and for a 100 Gb/s OTDM system ($T_0 = 1.58$ ps) it is 0.22 ps.

5. Conclusion

To the best of our knowledge, this is the first time a new conception of depolarization vector is proposed to describe the effect of the depolarization induced by the second-order PMD. Deriving the formula of pulse broadening induced by the second-order PMD yields the following results: (1) the polarization dependent Chromatic Dispersion (PCD) always enhances the pulse broadening, but the depolarization vector always decreases the pulse broadening. (2) The pulse broadening is correlated with the bit-rate of a transmission system, and the higher the bit-rate, the greater the pulse broadening. (3) By adjusting the directions of the Stocks vector of initial state of polarization, initial first-order polarization dispersion vector and the depolarization vector to be parallel to each other, one can obtain the optimum dispersion compensation. But when the PCD is not equal to zero, the dispersion cannot be completely compensated, and the minimum pulse broadening is equal to $\sigma = (\sqrt{2}/4)(q_0/T_0)$.

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