Vol 14 No 8, August 2005
1009-1963/2005/14(08)/1544-05

Chinese Physics

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Nuclear halo effect on nucleon capture reaction rates at stellar energies*

Liu Zu-Hua(刘祖华)[a)b)] and Zhou Hong-Yu(周宏余)[a)†]

[a)] *Key Laboratory of Beam Technology and Material Modification of Ministry of Education, Beijing Normal University, Beijing* 100875, *China*

[b)] *China Institute of Atomic Energy, Beijing* 102413, *China*

(Received 5 November 2004; revised manuscript received 14 January 2005)

The capture cross sections at stellar energies are very difficult to measure directly. Hence, data are usually evaluated by using indirect methods or extrapolations from the experimental data obtained at the lowest possible energies. The asymptotic normalization coefficient (ANC) approach of the transfer reactions provides a reliable way for the determination of the capture cross sections at stellar energies. By virtue of its reliability, we have calculated the capture cross sections of the ^{10}Be(n,γ)^{11}Be reaction by using nuclear ANC method. ^{11}Be is a well-known neutron halo nucleus with two weakly bound states. As a typical example, we have shown that the radiative cross sections for a nucleon captured into a halo state are obviously enhanced. The enormous enhancement of the capture cross section is just due to the large overlap of the incident neutron wave with the extended tail of the halo. The ^{10}Be(n,γ)^{11}Be capture reaction is involved in the inhomogeneous big-bang nucleosynthesis. We have evaluated its reaction rates at stellar energies with the nuclear ANC method.

Keywords: nuclear halo, ANC method, direct capture, halo effects
PACC: 2100, 2540L, 2720

1. Introduction

The thermonuclear reactions of astrophysical interest concern mainly the capture of nucleons or α-particles. The experimental determination of their cross sections and the evaluation of the corresponding stellar reaction rates face enormous problems and represent a real challenge. The experimental difficulties relate directly to the fact that the energies of astrophysical interest are very low, much lower than the Coulomb barrier for charged particle induced reactions. As a consequence, the cross sections can dive into the nanobarn to picobarn abysses. In many cases, one cannot succeed experimentally in reaching the energy region of the astrophysical relevance. Thus theorists are requested to supply reliable extrapolations from the experimental data obtained at the lowest possible energies as well as to look for reliable indirect methods.

Xu *et al*[1] proposed an indirect method to obtain the direct capture cross sections at stellar energies from the asymptotic normalization coefficient (ANC) of the wavefunction in peripheral nucleon transfer reactions. The cross sections of the peripheral nucleon transfer at the energies above the Coulomb barrier are several orders of magnitude larger than those of direct capture. Hence they should provide an easy way to determine the capture cross sections of astrophysical interest. In this work, we calculate the capture cross sections of the ^{10}Be(n,γ)^{11}Be reaction by using the nuclear ANC method. ^{11}Be is a well-known neutron halo nucleus with two weakly bound states. The low neutron separation energy (S_n) and the short range of nuclear force can allow the neutron to tunnel into the space surrounding the nuclear core so that the neutron is present with appreciable probability at distances much larger than the normal nuclear radius. The nuclear halo is known to be one of the most prominent and exotic phenomena extensively investigated experimentally as well as theoretically in nuclear physics. However, the appearance of a halo should be a gen-

*Project supported by the National Natural Science Foundation of China (Grant Nos 10075077, 10105016 and 10275092) and the National Key Basic Research of China (Grant No G200007400).

†Corresponding author.

eral phenomenon common to loosely bound particles. In addition to the activity in nuclear physics, other subfields of physics, in particular atomic and molecular physics, have also started looking for halos, one example being the atomic helium trimers.[2,3] The extended spatial distribution of a halo particle can affect a number of observations. As a typical example, we show that the radiative cross sections for a nucleon captured into a halo state are obviously enhanced. The significant enhancement of the capture cross section is just due to the large overlap of the incident neutron wave with the extended tail of the halo. The reaction chain[4]

$$\begin{aligned} &{}^8\mathrm{Li}(\mathrm{n},\gamma)^9\mathrm{Li}(\beta^-\nu)^9\mathrm{Be}(\mathrm{n},\gamma) \\ &{}^{10}\mathrm{Be}(\mathrm{n},\gamma)^{11}\mathrm{Be}(\beta^-\nu)^{11}\mathrm{B}(\mathrm{n},\gamma)... \end{aligned} \quad (1)$$

proceeds through the ^{10}Be(n,γ)^{11}Be capture reaction which is involved in the inhomogeneous big-bang nucleosynthesis. We evaluate its reaction rates at stellar energies with the nuclear ANC method.

2. Capture cross sections extracted with ANC method

The direct radiative capture reaction for $A+n \to B+\gamma$ is essentially determined by the matrix elements

$$\begin{aligned} M =&\langle\phi_B(\zeta_A,\zeta_n,\boldsymbol{r})|\widehat{T}(\boldsymbol{r})|\phi_A(\zeta_A)\phi_n(\zeta_n)\psi_i^+(\boldsymbol{r})\rangle \\ =&\langle I^B_{An}(\boldsymbol{r})|\widehat{T}(\boldsymbol{r})|\psi_i^+(\boldsymbol{r})\rangle, \end{aligned} \quad (2)$$

where ϕ_i, ζ_i and $\boldsymbol{r}$ represent respectively the wavefunction and the internal coordinate for the bound state of particle i, and the relative coordinate between A and n; $\widehat{T}$ is the electromagnetic transition operator; $\psi_i^+(\boldsymbol{r})$ is the distorted wave (DW) in the entrance channel $A+n$ and I^B_{An} is the overlap function for $B \leftrightarrow A+n$:

$$\begin{aligned} I^B_{An}(\boldsymbol{r}) =&\langle\phi_B(\zeta_A,\zeta_n,\boldsymbol{r})\phi_A(\zeta_A)\phi_n(\zeta_n)\rangle \\ =&\sum\langle J_AM_Ajm_j|J_BM_B\rangle \\ &\times\langle J_nM_nlm_l|jm_j\rangle i^lY_{lm_l}(\hat{r})I^B_{Anlj}(r). \end{aligned} \quad (3)$$

Here, J_i (M_i) is the spin (projection) of particle i, $\langle J_1M_1J_2M_2|J_3M_3\rangle$ is the Clebsch–Gordan coefficient, and $I^B_{Anlj}(r)$ is the radial part of the overlap wavefunction. At asymptotic distances where the nuclear forces are vanishingly small, $r > R_N$, the radial overlap function behaves as[1,5]

$$I^B_{Anlj}(r) \to C^B_{Anlj}\frac{W_{-\eta,l+1/2}(2k_nr)}{r}. \quad (4)$$

Here C^B_{Anlj} is the nuclear ANC defining the amplitude of the tail of the overlap function, W is the Whittaker function, $k_n=\sqrt{2\mu S_n/\hbar^2}$ is the wave number, μ is the reduced mass of the system $(A+n)$, and η the Sommerfield parameter for the corresponding bound state. In the case of neutron, $\eta=0$, the Whittaker function reduces to $W_{0,l+1/2}(2k_nr)=\sqrt{k_nr/\pi}K_{l+1/2}(k_nr)$ with K being the modified Bessel function. In the single-particle approach the radial overlap function can be approximated by

$$I^B_{Anlj}(r) \approx [S_{lj}]^{1/2}\phi_{nlj}(r), \quad (5)$$

where S_{lj} is the single-particle spectroscopic factor, ϕ_{nlj} is the normalized single-particle radial wavefunction in the (nlj) bound state. The asymptotic normalization coefficients (ANCs) of $I^B_{Anlj}(r)$ can be extracted from the angular distribution of a peripheral reaction. For example, the nuclear ANCs for the virtual process $^{11}\mathrm{Be} \leftrightarrow ^{10}\mathrm{Be}+n$ are obtained from the ^{10}Be(d, p)^{11}Be reaction at 25MeV.[6] The extracted values of nuclear ANC are $C_{Anl_fj_f}$=0.81±0.05 [7] and 0.18±0.01 fm$^{-1/2}$ [8] for the ground and first excited states, respectively.

In the following we consider the case in which the incoming neutron with the orbital angular momentum l_i is captured into a bound state (nl_fj_f) by radiating a γ ray with the electrical multipolarity λ, i.e. by $E\lambda$ γ transition. Here n, l_f, j_f represent the node number of the bound state wavefunction, the orbital and total angular momenta of the final state, respectively. The cross section is calculated by

$$\sigma(E\lambda)=\frac{8\pi(\lambda+1)}{[(2\lambda+1)!!]^2\lambda}\left(\frac{E_\gamma}{\hbar c}\right)^{2\lambda+1}\frac{1}{\hbar\nu}\sum_m|M_{\lambda m}|^2, \quad (6)$$

where E_γ is the γ-ray energy, and ν denotes the relative velocity between the incoming neutron and the target ^{10}Be, m is the magnetic quantum number of the emitted γ photon. For the $E1$ transition ($\lambda=1$), the transition matrix element M_{1m} is given by

$$\begin{aligned} M_{1m} =&\frac{eM_pM_T}{M_p+M_T}\left(\frac{Z_p}{M_p}-\frac{Z_T}{M_T}\right) \\ &\times\langle I^B_{Anl_fj_f}(\boldsymbol{r})|rY_{1m}(\hat{r})|\psi_i^+(\boldsymbol{r})\rangle, \end{aligned} \quad (7)$$

where M_p, M_T, Z_p and Z_T are the incoming particle and target nucleus mass, and their charge numbers respectively, Y_{1m} is the spherical harmonic function. After integrating the angular part and summing over the magnetic quantum number m of the initial channel, one can calculate the capture cross section of the

$E1$ transition by[9,10]

$$\sigma(E1) = \alpha\frac{16\pi}{3}\left(\frac{E_\gamma}{\hbar c}\right)^3\left(\frac{c}{\nu}\right)\frac{1}{(2I_{\rm p}+1)(2I_{\rm T}+1)} \times\left(\frac{Z_{\rm p}M_{\rm T}-Z_{\rm T}M_{\rm p}}{M_{\rm p}+M_{\rm T}}\right)^2 \times\sum_{l_i j_f}\frac{2j_f+1}{2l_f+1}\max(l_i,l_f)R^2_{l_i S_i l_f j_f}, \quad (8)$$

where α is the fine structure constant, $I_{\rm p}$ and $I_{\rm T}$ are the incoming particle and target spins respectively, S_i is the initial channel spin, and

$$R_{l_i S_i l_f j_f} = \frac{1}{k}\int_0^\infty r^2 I_{Anl_f j_f}(r)\chi_{l_i S_i}(kr)\mathrm{d}r, \quad (9)$$

where $\chi_{l_i S_i}(kr)$ is the radial DW function which is calculated in the distorted wave of Born approximation (DWBA). The integration in Eq.(9) can be written as two terms of the interior and asymptotic regions,

$$R_{l_i S_i l_f j_f} = \frac{1}{k}\left[(S_{l_f j_f})^{1/2}\int_0^{R_N} r^2\phi_{nl_f j_f}(r)\chi_{l_i S_i}(kr)\mathrm{d}r + C^B_{Anl_f j_f}\int_{R_N}^\infty rW_{-\eta,l_f+1/2} \times(2k_n r)\chi_{l_i S_i}(kr)\mathrm{d}r\right], \quad (10)$$

where R_N is the nuclear radius. For ^{11}Be, the nuclear radius is R_N=4.0fm. In the energy region of astrophysical interest, the asymptotic part gives the most contribution to the cross section.

Figure 1 shows the calculated cross sections of the ^{10}Be(n,γ)^{11}Be capture reactions. The dashed, dotted, dash-dotted lines represent the calculated results for the different reaction channels in which a p-wave, an s-wave and a d-wave neutron are captured into an s-orbit and a p-orbit by emitting an $E1$ γ-ray, respectively. The solid line illustrates the total capture reaction cross sections of ^{10}Be(n,γ)^{11}Be. In the calculations of the single-particle wavefunction ϕ_{nlj}, the Woods–Saxon (WS) potential geometry parameters r_0=1.19fm and a=0.75 fm are used. The depth of WS potential is adjusted to reproduce the valence neutron separation energy S_n. The optical-model parameters in Ref.[11] are adopted for the calculations of the radial DW function. The cross sections of the ^{10}Be(n,γ)^{11}Be capture reactions are not measured yet. Because the ^{10}Be neutron capture processes are involved in the inhomogeneous big-bang nucleosynthesis, so our results obtained with ANC method should be useful in astrophysics.

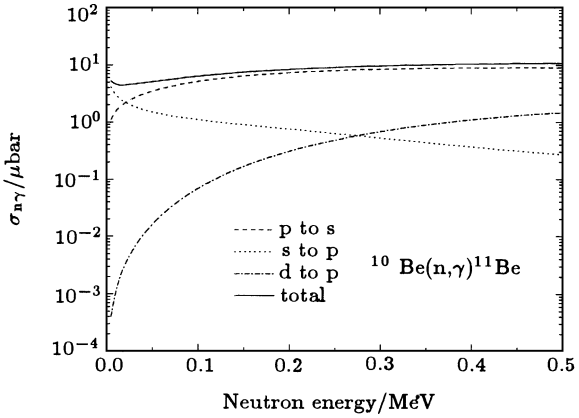


Fig.1. The radiative capture cross sections of the ^{10}Be(n,γ)^{11}Be reactions. The dashed, dotted, dash-dotted lines represent the calculated results for the different reaction channels in which a p-wave, an s-wave and a d-wave neutron are captured into an s-orbit and a p-orbit by emitting an $E1$ γ-ray, respectively. The solid line illustrates the total capture reaction cross sections of ^{10}Be(n,γ)^{11}Be.

3. Capture reaction rate

For a two-body reaction, the Maxwellian-averaged reaction rate $N_A\langle\sigma\nu\rangle$ is computed from

$$N_A\langle\sigma\nu\rangle = N_A\frac{8^{1/2}}{(\pi\mu)^{1/2}(k_{\rm B}T)^{3/2}} \times\int_0^\infty\sigma E\exp\left(-\frac{E}{k_{\rm B}T}\right)\mathrm{d}E, \quad (11)$$

where N_A is the Avogadro number, $k_{\rm B}$ the Boltzmann constant, T the temperature, and E the energy in the center-of-mass system. $N_A\langle\sigma\nu\rangle$ is expressed in cm^{3}mol^{-1}s^{-1}, the energy E and $k_{\rm B}T$ in MeV, and the cross section σ in barn. Figure 2 shows the ^{10}Be(n,γ)^{11}Be reaction rates for the different reaction channels. The solid line in the figure is the total direct capture rate. It can be seen from the figure that the s→p transition leading to the first excited state in ^{11}Be dominates the reaction rate in the energy region of astrophysical interest. In Fig.3, we compare our reaction rate with the results of Rauscher *et al*[12] and Mengoni *et al*.[13] As well as the direct capture rate, a resonant contribution coming from the level at E_x=1.778MeV in ^{11}Be is included. For this resonant contribution, only $E1$ transition (with a gamma width Γ_γ=1.04eV and a neutron width Γ_n=98.96keV) from the resonant state (E_x=1.778MeV, $J^\pi = 5/2^+$) to first excited state in ^{11}Be is considered. From Fig.3

one may find that our reaction rates are at least 20% less than the values of Mengoni *et al.*[13] The difference may be due to the radial single-particle wavefunctions used. Mengoni *et al*[13] calculated the radial wavefunction with a potential model. We apply the ANC method as described in Section 2.

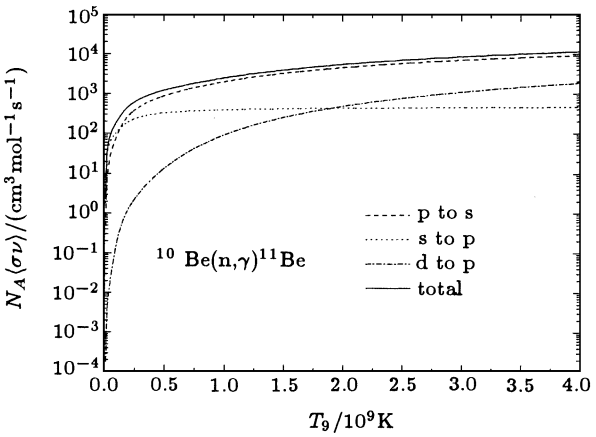


Fig.2. The ^{10}Be(n,γ)^{11}Be reaction rate. The dashed, dotted, dash-dotted lines illustrate the reaction rates for the different reaction channels as mentioned in Fig.1. The solid line shows the total capture reaction rate of ^{10}Be(n,γ)^{11}Be.

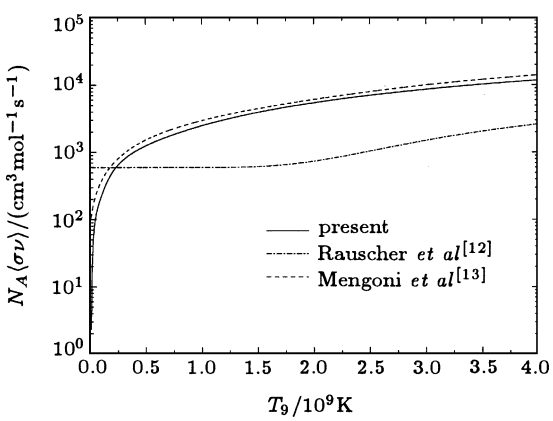


Fig.3. Comparison of ^{10}Be(n,γ)^{11}Be reaction rate between present result and the calculations in Refs.[12,13].

Our reaction rate can be expressed as

$$N_A\langle\sigma\nu\rangle = 38.1 + 2555T_9 + 4.62\times10^5 T_9^{-3/2}\mathrm{e}^{-13.866/T_9}. \quad (12)$$

Here T_9 is the temperature in units of 10^9K. The first term comes from the s→p transition, and the third term is the resonant contribution. The s-wave contribution to the reaction rate can be directly related with the thermal cross section $\sigma^{\mathrm{th}}_{n,\gamma}$[14] by

$$N_A\langle\sigma\nu\rangle_s = N_A\sigma^{\mathrm{th}}_{n,\gamma}v_{\mathrm{th}}. \quad (13)$$

With this relationship, we obtain $\sigma^{\mathrm{th}}_{n,\gamma} = (0.29\pm 0.09)$mb for the ^{10}Be(n,$\gamma$)^{11}Be capture reaction.

4. Halo effect on direct capture process

In order to show nuclear halo effect on direct capture process at stellar energies, we first calculate and compare the cross section captured into a halo state with that into a normal state. To this end, we arbitrarily increase the neutron separation energy of the ^{11}Be first excited state by a factor of ten, and calculate the single-particle wavefunctions with S_n=0.184MeV and 1.84MeV in a WS potential, respectively. They are shown in Fig.4(a) as solid and dashed lines. One may notice from Fig.4(a) that the wavefunction of a halo state is much extended, specially compared with that of a normal state. In order to illustrate the halo effect on the capture reaction rate clearly, we display the reduced transition amplitude $Q_{l_iS_il_fj_f}(r) = |r^2 I^B_{Anl_fj_f}\chi_{l_iS_i}(kr)|$ in Fig.4(b) for the ^{11}Be first excited state with S_n=0.184 and 1.84MeV at the energy of neutron in entrance channel E_n=20keV. This comparison very clearly demonstrates the origin of the halo effect on the capture reaction rate. It is

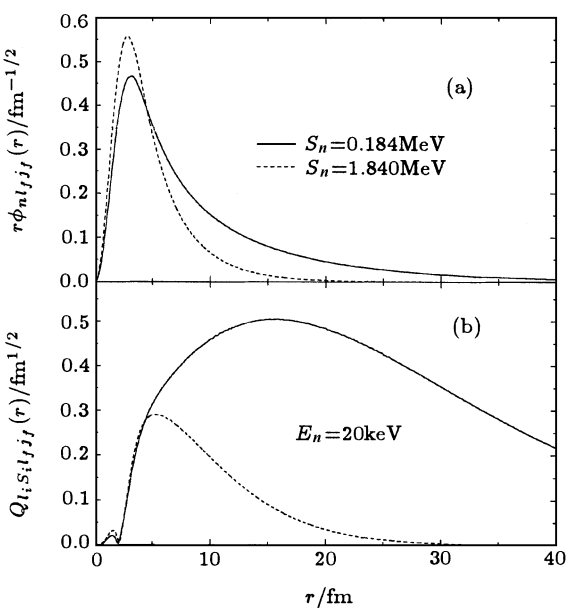


Fig.4. The wavefunction $r\phi_{nl_fj_f}(r)$ (panel a), and the reduced transition amplitude $Q_{l_iS_il_fj_f}(r) = |r^2 I^B_{Anl_fj_f}\chi_{l_iS_i}(kr)|$ at the neutron energy in entrance channel E_n=20keV (panel b) for the ^{11}Be first excited state with S_n=0.184 and 1.84MeV, respectively.

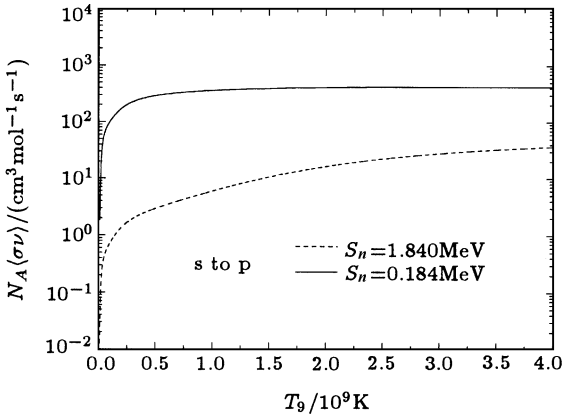


Fig.5. The ^{10}Be(n,γ)^{11}Be reaction rate for an s-wave neutron captured into the p-state of ^{11}Be. The solid and dashed lines represent the cases with S_n=0.184MeV and 1.84MeV, respectively.

just due to the large overlap of the incident neutron wave with the extended tail of the halo state wavefunction of ^{11}Be which makes the reaction rate of ^{10}Be(n,γ)^{11}Be enormously enhanced. With the wavefunctions shown in Fig.4(a), we have calculated the reaction rate of the $^{10}\mathrm{Be}(n,\gamma)^{11}\mathrm{Be}\left(\frac{1}{2}^-, 0.320\mathrm{MeV}\right)$ in which an s-wave neutron is captured into a p-state of ^{11}Be by emitting an $E1$ γ-ray. The results are shown in Fig.5 with solid and dashed lines for the cases with S_n=0.184MeV and 1.84MeV, respectively. Figure 5 clearly shows that the capture rate for a nucleon captured into a halo state at stellar energies is two orders of magnitude larger than that of a nucleon captured into a normal state.

5. Summary

The capture cross sections at stellar energies are very difficult to measure directly. Hence, in the case where the direct measurement is not available, the required data are usually evaluated by using indirect methods. Recently, it is proved[1,5,7,8] that the ANC approach of the transfer reactions provides a reliable way for the determination of the capture cross sections at stellar energies. By virtue of its reliability, we have calculated the capture rates of the ^{10}Be(n,γ)^{11}Be reaction by using the nuclear ANC method. Our reaction ^{10}Be(n,γ)^{11}Be rate can be expressed as

$$N_A\langle\sigma\nu\rangle = 38.1 + 2555T_9 + 4.62\times10^5 T_9^{-3/2}\mathrm{e}^{-13.866/T_9}.$$

By the use of the relationship (13), we obtain its capture cross section of thermal neutron, $\sigma^{\mathrm{th}}_{n,\gamma} = (0.29\pm0.09)$mb.^{11}Be is a well-known neutron halo nucleus with two weakly bound states. As a typical example, we have shown that the reaction rate for a nucleon captured into a halo state is obviously enhanced. The enormous enhancement of the capture rate is just due to the large overlap of the incident neutron wave with the extended tail of the halo, which is clearly demonstrated by the reduced transition amplitude function $Q_{l_iS_il_fj_f}(r) = |r^2 I^B_{Anl_fj_f}\chi_{l_iS_i}(kr)|$.

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