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Quantum teleportation and Kerr–Newman spacetime*

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We consider the teleportation in the background of Kerr–Newman spacetime. Because of the Hawking effect, the fidelity of the teleportation is reduced. The results also show the fidelity is closely related to the mass, charge and rotating velocity of the black hole: high fidelity can be reached for massive, slowly rotating Kerr–Newman black holes.

Keywords: quantum teleportation, entanglement, Kerr–Newman spacetime
PACC: 0365, 0460, 9530S, 9620J

1. Introduction

Quantum information science is based on the discovery that there are physical states of a quantum system which enable tasks that cannot be accomplished in a classical world. Ever since quantum teleportation was first proposed by Bennett *et al*,[1] quantum information theory has undergone a rapid development.[2−4] The relationship between quantum information theory and relativity theory is discussed by several authors.[5−12] Czachor[10] studied a version in which the electron is in paramagnetic resonance with relativistic particles, and Peres *et al*[6] demonstrated that the spin of an electron is not covariant under Lorentz transformation. Moreover, Alsing and Milburn[8] studied the effect of Lorentz transformation on maximally spin-entangled Bell states in momentum eigenstates, and Gingrich and Adami[7] derived the general transformation rules for the spin-momentum entanglement of two qubits. Recent work of Alsing and Milburn extended the results to situations where one observer is accelerated.[8] In our former work,[13] we have extended the teleportation scheme to Schwarzschild spacetime. In this paper, we consider the teleportation in the background of a rotating and charged spacetime, say Kerr–Newman space time. Different from the standard teleportation protocol, our scheme proposes that the observer Alice is stationary and stays at a place far from a Kerr–Newman black hole, which can be regarded as an asymptotically flat region, and the other observer Bob passes near her. They coincide at a point in the flat region, where they instantaneously share an entangled Bell state. Then Bob picks his qubit travelling to the surface of a Kerr–Newman black hole. In order to ensure that Bob's trajectory does not influence his description of the qubit state, Bob can access the black hole by free falling. Before he crosses the horizon, he should accelerate and stop free falling outside the horizon. Thus, teleportation can be performed between the two observers. The results show that, for a Kerr–Newman black hole, the fidelity of teleportation is closely related to the mass, the charge and the angular momentum per unit of the back hole.

2. Vacuum structure of the Kerr–Newman spacetime

Before we discuss the nature of entanglement, we will review the essential features of quantum field theory for Kerr–Newman spacetime. It is well known that the Minkowski vacuum state and the vacuum of a curved spacetime are inequivalent.[14−20] In Rindler frames, it was found that an accelerated detector becomes spontaneously excited while moving through the Minkowski vacuum, a result called Unruh effect. On the other hand, in the area near the event horizon of a Kerr–Newman black hole, the vacuum is popu-

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lated by particles and antiparticles as the results of Hawking effect. In the following, we use two-photon states of the electromagnetic field, which are modelled by the massless modes of a scalar field, and ignore the polarization.[8,9] The metric for Kerr–Newman spacetime is given by (Plank units $\hbar = G = c = k = 1$ are used hereafter)

$$\begin{aligned} \mathrm{d}s^2 =& \left(1 - \frac{2Mr - Q^2}{\rho^2}\right)\mathrm{d}t^2 - \frac{\rho^2}{\Delta}\mathrm{d}r^2 - \rho^2\mathrm{d}\theta^2 \\ & - \left[(r^2 + a^2)\sin\theta^2 + \frac{(2Mr - Q^2)a^2\sin\theta^4}{\rho^2}\right]\mathrm{d}\varphi^2 \\ & + \frac{2(2Mr - Q^2)a\sin\theta^2}{\rho^2}\mathrm{d}t\mathrm{d}\varphi, \end{aligned} \tag{1}$$

where ρ^2 and Δ are defined as

$$\begin{aligned} \rho^2 &= r^2 + a^2\cos\theta^2, \\ \Delta &= r^2 - 2Mr + a^2 + Q^2, \end{aligned} \tag{2}$$

and M, Q and a are the mass, the charge and the angular momentum per unit mass respectively. The two horizons of the Kerr–Newman black holes are at

$$\begin{aligned} r_+ &= M + \sqrt{M^2 - a^2 - Q^2}, \\ r_- &= M - \sqrt{M^2 - a^2 - Q^2}, \end{aligned} \tag{3}$$

where r_+ and r_- are the event horizon and the inner horizon respectively. The Klein–Gordon equation of a massless scalar field is given by

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\left(\sqrt{-g}g^{\mu\nu}\frac{\partial\Phi}{\partial x^\nu}\right) = 0. \tag{4}$$

The normal-mode solutions to this equation may be written as

$$\Phi = \mathrm{e}^{-\mathrm{i}\omega t}f(r)Y(\theta,\psi). \tag{5}$$

Then we have the radial equation

$$\begin{aligned} &\left(r^2 + a^2\right)^2\frac{\mathrm{d}^2 f}{\mathrm{d}r_*} + 2r\Delta\frac{\mathrm{d}f}{\mathrm{d}r_*} \\ =&\lambda\Delta f - [(r^2 + a^2)\omega - ma]^2 f, \end{aligned} \tag{6}$$

where $r^* = r + (1 + 1/2\kappa_+)\ln[(r - r_+)/r_+] - (1/2\kappa_-)\ln[(r - r_-)/r_-]$, $\kappa_\pm = (r_+ - r_-)/2(r_\pm^2 + a^2)$ is the surface gravity, and λ is the separation constant. The solution of Eq.(6) can be formulated as

$$f_\pm = \mathrm{e}^{\pm\mathrm{i}(\omega - \omega_0)r_*}, \tag{7}$$

where $\omega_0 = m\Omega_+$, and ($\Omega_+ = a/(r_+^2 + a^2)$) is the angular velocity. We obtain the incoming and outgoing modes near the horizon, which read

$$R_{\mathrm{in}}(r > r_+) = \mathrm{e}^{-\mathrm{i}\omega t - \mathrm{i}(\omega - \omega_0)r_*} = \mathrm{e}^{-\mathrm{i}(\omega - \omega_0)v}, \tag{8}$$

$$R_{\mathrm{out}}(r > r_+) = \mathrm{e}^{\mathrm{i}\omega t + \mathrm{i}(\omega - \omega_0)r_*} = \mathrm{e}^{-\mathrm{i}(\omega - \omega_0)u}, \tag{9}$$

$$R_{\mathrm{out}}(r < r_+) = \mathrm{e}^{\mathrm{i}\omega t + \mathrm{i}(\omega - \omega_0)r_*} = \mathrm{e}^{\mathrm{i}(\omega - \omega_0)u}, \tag{10}$$

where $v = [\omega/(\omega - \omega_0)]\,t + r_*$, $u = [\omega/(\omega - \omega_0)]\,t - r_*$. R_{in} is analytic everywhere in the spacetime manifold. The incoming solution Φ_{in} can be expressed in the form of R_{in}, which is a complete orthonomal family of the incoming wavefunction

$$\Phi_{\mathrm{in}} = \int \mathrm{d}\omega(a_\omega R_{\mathrm{in}} + a_\omega^+ R_{\mathrm{in}}^*). \tag{11}$$

Here a_ω^+ and a_ω are the creation and annihilation operators respectively. The vacuum state that corresponds to the incoming modes is defined as $a_\omega|0\rangle_{\mathrm{in}} = 0$. Since R_{out} is not analytic at r_+, according to the suggestion of Domour-Ruffini,[18] we can analytically extend R_{out} to the region $r < r_+$ from the second half of the complex r plane $(r - r_+) \to |r - r_+|\mathrm{e}^{-\mathrm{i}\pi} = (r_+ - r)\mathrm{e}^{-\mathrm{i}\pi}$, then we have $\tilde{R}_{\mathrm{out}}(r < r_+) = \mathrm{e}^{4\pi(\omega - \omega_0)/\kappa_+}\mathrm{e}^{2\mathrm{i}(\omega - \omega_0)r_* - \mathrm{i}(\omega - \omega_0)v}$. It is considered in Eq.(8) that inside the horizon r_* is the time coordinate and t is the space coordinate. Equations (9) and (10) are analytic outside and inside the event horizon respectively, but they are not analytic at $r = r_+$. They form a complete orthogonal family. In second-quantizing the field Φ_{out} in the exterior of the Kerr–Newman black hole, we expand it as follows:

$$\begin{aligned} \Phi_{\mathrm{out}} = \int \mathrm{d}\omega[&b_\omega^{(1)}R_{\mathrm{out}}(r < r_+) + b_{\tilde{\omega}}^{(2)}R_{\mathrm{out}}(r > r_+) \\ &+ b_\omega^{(1)+}R_{\mathrm{out}}^*(r < r_+) \\ &+ b_{\tilde{\omega}}^{(2)+}R_{\mathrm{out}}^*(r > r_+)], \end{aligned} \tag{12}$$

where $b_\omega^{(1)}, b_{\tilde{\omega}}^{(2)}, b_\omega^{(1)+}$ and $b_{\tilde{\omega}}^{(2)+}$ are the annihilation and creation operators inside the horizon and outside the horizon respectively, with $\tilde{\omega} = -\omega$. The Fock vacuum state is defined as

$$b_\omega^{(1)}|0\rangle_A = b_{\tilde{\omega}}^{(2)}|0\rangle_B = 0. \tag{13}$$

When we use Kruskal coordinates, we can obtain an alternative expansion of Φ in both halves of the full Kruskal extension. Introducing the general light-like Kruskal coordinates

$$\begin{aligned} U &= -\frac{1}{\kappa_+}\mathrm{e}^{-\kappa_+ u}, \quad V = \frac{1}{\kappa_+}\mathrm{e}^{\kappa_+ v}, (r > r_+), \\ U &= \frac{1}{\kappa_+}\mathrm{e}^{-\kappa_+ u}, \quad V = \frac{1}{\kappa_+}\mathrm{e}^{\kappa_+ v}, (r < r_+), \end{aligned} \tag{14}$$

Eqs.(8), (9) and (10)can be rewritten as

$$R_{\mathrm{out}}(r > r_+) = \exp\left[\mathrm{i}(\omega - \omega_0)/\kappa_+ \ln\left(-\kappa_+ U\right)\right], \tag{15}$$

$$R_{\mathrm{out}}(r < r_+) = \exp\left[-\mathrm{i}(\omega - \omega_0)/\kappa_+ \ln\left(\kappa_+ U\right)\right], \tag{16}$$

$$\tilde{R}_{\mathrm{out}}(r < r_+) = \exp\left[\mathrm{i}(\omega - \omega_0)/\kappa_+ \ln\left(-\kappa_+ U\right)\right]. \tag{17}$$

Apparently, Eqs.(15) and (16) are different in their form. However, Eqs.(15) and (17) have the same form, and the Kruskal coordinate U is analytic inside and outside the horizon. It is natural for one to define a second quantization by forming the wave operator $\Phi_{\rm out}$ in the normal modes (15) and (17). We first define

$$\begin{aligned} f_1 =& {\rm e}^{-\pi(\omega-\omega_0)/2\kappa_+} R^*_{\rm out}(r>r_+) \\ &+ {\rm e}^{\pi(\omega-\omega_0)/2\kappa_+} R_{\rm out}(r<r_+), \qquad (18) \\ f_2 =& {\rm e}^{\pi(\omega-\omega_0)/2\kappa_+} R_{\rm out}(r>r_+) \\ &+ {\rm e}^{-\pi(\omega-\omega_0)/2\kappa_+} R^*_{\rm out}(r<r_+). \qquad (19) \end{aligned}$$

Here f_1 and f_2 denote the complete orthogonal family of the outgoing modes. Compared with Eq.(8), f_1 and f_2 are also of negative frequency. One can express Φ in the form

$$\begin{aligned} \Phi_{\rm out} =& \int {\rm d}\omega \left[2\sinh(\pi(\omega-\omega_0)/\kappa_+)\right]^{-1/2} \\ & \left(d^{(1)}_\omega f_1 + d^{(2)}_\omega f_2 + d^{(1)+}_\omega f^*_1 + d^{(2)+}_\omega f^*_2\right). \qquad (20) \end{aligned}$$

The state which contains no scalar particles at this condition, is defined as

$$d^{(1)}_\omega |0\rangle'_A = d^{(2)}_{\bar\omega} |0\rangle'_B = 0. \qquad (21)$$

One would find out that the incoming solution (8) and the outgoing modes f_1 and f_2 are identical in the nature of manifolds: they are both analytic everywhere in the Kerr–Newman spacetime. Therefore, they should correspond to the same vacuum state, i.e.

$$|0\rangle'_{AB} = |0\rangle_{\rm in}. \qquad (22)$$

In fact, they can be related by introducing the Unruh modes[15]

$$d^{(\sigma)}_\omega = \int_{-\infty}^{\infty} {\rm d}\boldsymbol{k} p^{(\sigma)}_\Omega(\boldsymbol{k}) a_\omega, \qquad (23)$$

where $\{p^{(\sigma)}_\Omega(\boldsymbol{k})\}$ is the complete set of orthogonal functions, $\sigma = 1, 2$. The essential feature of a scalar field in Kerr–Newman spacetime allows the following Bogoliubov transformation:[19,20]

$$\begin{aligned} b^{(1)}_\omega =& \left[2\sinh(\pi(\omega-\omega_0)/\kappa_+)\right]^{1/2} \\ & \times \left[{\rm e}^{\pi(\omega-\omega_0)/2\kappa_+} d^{(1)}_\omega + {\rm e}^{-\pi(\omega-\omega_0)/2\kappa_+} d^{(2)+}_\omega\right], \qquad (24) \\ b^{(2)}_{\bar\omega} =& \left[2\sinh(\pi(\omega-\omega_0)/\kappa_+)\right]^{-1/2} \\ & \times \left[{\rm e}^{\pi(\omega-\omega_0)/2\kappa_+} d^{(2)}_\omega + {\rm e}^{-\pi(\omega-\omega_0)/2\kappa_+} d^{(1)+}_\omega\right]. \qquad (25) \end{aligned}$$

One then obtains[16]

$$|0\rangle_{\rm in} = Z \prod_\omega \exp(\tanh r\, b^{(1)+}_\omega b^{(2)+}_{\bar\omega}) |0\rangle_A \otimes |0\rangle_B, \qquad (26)$$

where Z is a normalization constant

$$Z = \prod_\omega \cosh^{-2} r, \tanh r = {\rm e}^{-\pi(\omega-\omega_0)/2\kappa_+},$$

and

$$\cosh r = (1 - {\rm e}^{-\pi(\omega-\omega_0)/\kappa_+})^{-1/2}.$$

The vacuum of a distant observer Alice outside the black hole can be defined as $d^{(1)}_\omega |0\rangle'_A = d^{(2)}_{\bar\omega} |0\rangle'_B = 0$. On the other hand, the vacuum near the horizon can be written as $b^{(1)}_\omega |0\rangle_A = b^{(2)}_{\bar\omega} |0\rangle_B = 0$. Equation (26) can be rewritten as

$$\begin{aligned} |0\rangle_{\rm in} =& Z \prod_\omega \left(1 + ... + \frac{1}{n!} \tanh^n r b^{n(1)+}_\omega b^{n(2)+}_{\bar\omega} + ...\right) \\ & \times |0\rangle_A \otimes |0\rangle_B. \qquad (27) \end{aligned}$$

Assume

$$\begin{aligned} T^{(1)}(r) =& -\sum_\omega (b^{(1)+}_\omega b^{(1)}_\omega \ln\sinh^2 r \\ & - b^{(1)}_\omega b^{(1)+}_\omega \ln\cosh^2 r), \\ T^{(2)}(r) =& -\sum_\omega b^{(2)+}_{\bar\omega} b^{(2)}_{\bar\omega} \ln\sinh^2 r \\ & + \sum_\omega b^{(2)}_{\bar\omega} b^{(2)+}_{\bar\omega} \ln\cosh^2 r. \qquad (28) \end{aligned}$$

The following can be easily proved:

$$\begin{aligned} & {\rm e}^{-T^{(\sigma)}(r)/2} b^{(\sigma)+}_\omega {\rm e}^{-T^{(\sigma)}(r)/2} = b^{(\sigma)+}_{(\omega)} \tanh r, \\ & {\rm e}^{-T^{(\sigma)}(r)/2} |0\rangle_{A,B} = \prod_\omega \cosh^{-2} r |0\rangle_A \otimes |0\rangle_B, \qquad (29) \end{aligned}$$

where $\sigma = 1, 2$. Using Eqs.(29), one can rewrite Eq.(27) as

$$\begin{aligned} |0\rangle_{\rm in} =& \left({\rm e}^{-T^{(\sigma)}/2} {\rm e}^{\sum_\omega b^{(\sigma)+}_\omega b^{(-\sigma)}_\omega} {\rm e}^{T(\sigma)/2}\right) \\ & \times \left({\rm e}^{-T^{(\sigma)}/2} |0\rangle_{A,B}\right) \\ =& {\rm e}^{-T^{(\sigma)}/2} \sum_{n_\omega=0}^{\infty} |n_\omega\rangle_A \otimes |n_{\bar\omega}\rangle_B \\ =& {\rm e}^{\sum_\omega [n_\omega \ln\sinh r - (1+n_\omega)\ln\cosh r]} \\ & \times \sum_{n_\omega=0}^{\infty} |n_\omega\rangle_A \otimes |n_{\bar\omega}\rangle_B \\ =& \sum_{n_\omega=0}^{\infty} \prod_\omega \tanh^{n_\omega} r \cosh^{-1} r \\ & \times |n_\omega\rangle_A \otimes |n_{\bar\omega}\rangle_B, \qquad (30) \end{aligned}$$

where $N_\omega = b^{(\sigma)+}_\omega b^{(\sigma)}_\omega$, $b^{(\sigma)}_\omega b^{(\sigma)+}_\omega = 1 + N_\omega$. From Eq.(23), we see that a given incoming mode of frequency $\omega_{\vec k}$ is spread over all positive outgoing frequencies Ω, as a result of the Fourier transform relationship between a_ω and $d^{(\sigma)}_\omega$. One can simplify the

analysis by considering the effect of teleportation of the state $|\varphi\rangle_{\rm in} = \alpha|0\rangle_{\rm in} + \beta|1\rangle_{\rm in}$ by the Minkowski observer Alice to a single Kerr–Newman mode of the observer Bob. Thus, one can only consider the mode ω in region I which is distinct from the mode $\tilde{\omega}$ in the same region.[8] From Eq.(26), one can find that the Minkowski vacuum appears as an entangled state of Hawking particles with nonlocal Einstein–Podolsky–Rosen-type correlations, i.e.

$$|0\rangle_{\rm in} = Z\Big[|0\rangle_A \otimes |0\rangle_B + \sum_p \tanh r(|1_\omega\rangle_A^{(1)} \otimes |1_{\tilde{\omega}}\rangle_B^{(2)} + |1_{\tilde{\omega}}\rangle_A^{(1)} \otimes |1_\omega\rangle_B^{(2)}) + ...\Big]. \quad (31)$$

Thus we select the terms with $\sigma = 1$ in Eq.(31), which corresponds to the correlated mode pair in the first set of parentheses, and drop the unessential phase factors $\{p_\Omega^{(\sigma)}(\boldsymbol{k})\}$. Therefore, the single mode component of the Minkowski vacuum state, namely the two-mode squeezed state, is given by

$$|0\rangle_{\rm in} = \frac{1}{\cosh r}\sum_{n=0}^{\infty}\tanh^n r|n\rangle_A \otimes |n\rangle_B. \quad (32)$$

3. Quantum teleportation in the Kerr–Newman spacetime background

We suppose that Alice and Bob each hold an optical cavity and they coincide at a point. Then, Alice travels to an asymptotically flat region and Bob is situated just outside the event horizon. One can assume that prior to their coincidence, Alice and Bob have no photons in their cavities. Suppose that each cavity supports two orthogonal modes, with the same frequency, labelled A_i and R_i with $i = 1, 2$, which are each excited to a single photon Fock state at the coincidence point. The state held by Alice and Bob is then the entangled Bell state

$$|\varphi\rangle_{\rm in} = \frac{1}{\sqrt{2}}\left(|0\rangle_{\rm in}|0\rangle_{\rm in} + |1\rangle_{\rm in}|1\rangle_{\rm in}\right), \quad (33)$$

where the first qubit in each term refers to cavity Alice, the second qubit cavity Bob. The states $|0\rangle_{\rm in}$, $|1\rangle_{\rm in}$ are defined in terms of the physical Fock states for Alice's cavity by the dual-rail basis states as suggested by Ref[9]: $|0\rangle_{\rm in} = |1\rangle_{A_1}|0\rangle_{A_2}$, $|1\rangle_{\rm in} = |0\rangle_{A_1}|1\rangle_{A_2}$, and the similar expressions for Bob's cavity. In order to teleport the unknown state $|\varphi\rangle_{\rm in} = \alpha|0\rangle_{\rm in} + \beta|1\rangle_{\rm in}$ to Bob, we should assume that Alice has an additional cavity, which contains a single qubit with dual-rail encoding by a photon excitation of a two-mode incoming (Minkowski) vacuum state. This will allow Alice to perform a joint measurement on the two orthogonal modes of each cavity. After Alice's measurement, Bob's photon will be projected according to the measurement outcome. The final state Bob received can be given by $|\varphi_{ij}\rangle = x_{ij}|0\rangle + y_{ij}|1\rangle$, where there are four possible conditional state amplitudes expressed as $(x_{00}, y_{00}) = (\alpha, \beta), (x_{01}, y_{01}) = (\beta, \alpha), (x_{10}, y_{10}) = (\alpha, -\beta)$, and $(x_{11}, y_{11}) = (-\beta, \alpha)$. Once receiving Alice's results of measurement, Bob can apply a unitary transformation to verify the protocol in his local frame. However, Bob must be confronted with the fact that his cavity will become teemed with thermally excited photons because of the Hawking effect.

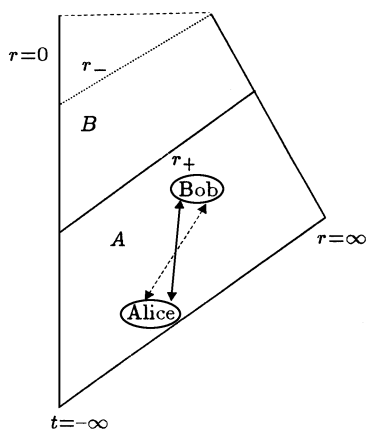


Fig.1. Penrose diagram for quantum teleportation in the background of Kerr–Newman spacetime. Alice is stationary in the asymptotically flat region, while Bob is located near the event horizon. With Alice sending the result of her measurement to Bob, say by photons, Bob will eventually receive them, and be able to rotate his half of the shared entangled state into the state $|\varphi\rangle_A = \alpha|0\rangle_A + \beta|1\rangle_A$.

When Alice sends the result of her measurement to Bob, Bob' state will be projected into

$$\begin{aligned}\rho_{ij}^{(A)} =& \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} {}_B\langle k, l|\phi_{ij}\rangle\langle\phi_{ij}|k, l\rangle_B \\ =& \frac{1}{\cosh^6 r}\sum_{k=0}^{\infty}\sum_{l=0}^{n}[(\tanh^2 r)^{n-1} \\ & \times |(n-m)|x_{ij}|^2 + m|y_{ij}|^2] \\ & \times |m, n-m\rangle_B\langle m, n-m| + (x_{ij}y_{ij}^*\tanh^{2n} r \\ & \times \sqrt{(m+1)(n-m+1)}) \times |m, n-m+1\rangle_B \\ & \times \langle m+1, n-m| + \text{H.c.})]. \end{aligned} \quad (34)$$

Bob's premeasurement state can be written as[8]

$$\rho_{ij}^{A}=\sum_{n=0}^{\infty}p_n\rho_{ij,n}^{A}, \tag{35}$$

in particular with

$$\rho_{ij,1}^{A}=|\phi_{ij}\rangle\langle\phi_{ij}|, p_0=0, p_1=1/\cosh^6 r. \tag{36}$$

Since we are concerned with to what extent $|\varphi_{ij}\rangle$ might deviate from unitarity, it is reasonable for us to perform a unitary transformation on $|\varphi_{ij}\rangle$ and convert its form into $|\varphi\rangle$. In this way, we may define the fidelity corresponding to the teleportation as

$$\begin{aligned}F^{A}(|\varphi\rangle)=&{}_{A}\langle\varphi|\rho^{A}\hat{U}|\varphi_{ij}\rangle_{A}\\=&{}_{A}\langle\varphi|\rho^{A}|\varphi\rangle_{A}=(1-\mathrm{e}^{-\pi\omega/\kappa_{+}})^{3},\end{aligned} \tag{37}$$

which is identical with the celebrated results of Alsing and Milburn. From Fig.2, we can see that, if ω is fixed, then the fidelity of teleportation is closely related to the radius (or mass), the charge and the angular momentum per unit of the black hole. Thus, we can reasonably deduce that for a Sun-like (with the mass of $M_{\odot}$) or an almost extremal Kerr–Newman black hole, the fidelity of teleportation is incredibly high.

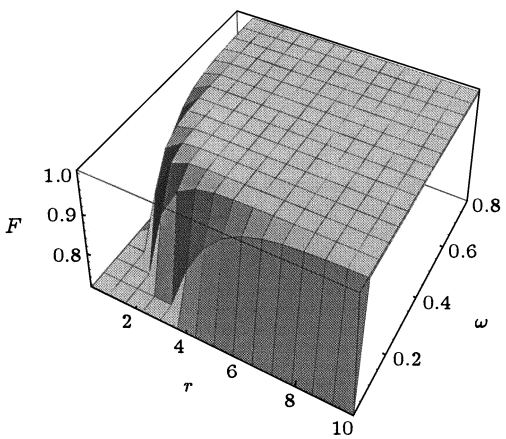


Fig.2. The fidelity of teleportation between Alice and Rob. It is closely related to the radius of the black hole and the value of a and Q. For massive or almost extremal Kerr–Newman black holes, the fidelity is high. This figure shows the fidelity of teleportation for a small black hole, with $a=0.1M$, $Q=0.01M$, $0.2\leq r\leq 10$ (metres) and $0.01\leq\omega\leq 0.8$ (Hz).

4. Conclusion and discussion

We have demonstrated that the fidelity of teleportation is reduced as seen by Bob who is located near the black hole event horizon because of Hawking radiation. For teleportation outside a tiny black hole, all information is lost and what Bob perceives is only thermalized state. But one can avoid worrying about fidelity reduction when teleportation is conducted outside a galaxy-like black hole since the fidelity there is high.

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