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Doubly coupled vibration band gaps in periodic thin-walled open cross-section beams*

Yu Dian-Long(郁殿龙)†, Liu Yao-Zong(刘耀宗), Qiu Jing(邱 静), Wang Gang(王 刚), and Wen Ji-Hong(温激鸿)

Institute of Mechatronical Engineering, National University of Defense Technology, Changsha 410073, *China*

PBG Research Center, National University of Defense Technology, Changsha 410073, *China*

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This paper studies flexural-torsional doubly coupled vibration through periodic thin-walled open cross-section beams composed of two kinds of materials. Based on the doubly coupled vibration equation, plane wave expansion method for the thin-walled beams is provided. If the filling fraction keeps constant, the lattice is one of the factors that affect the normalized gap width. If the lattice and filling fraction keeps constant, the Young modulus contrast plays a fundamental role for the band gap width, but not density contrast. Finally, the frequency response of a finite periodic binary beam is simulated by finite element method, which provides an attenuation of about 40dB in the frequency range of the band gaps. The findings will be significant in the application of phononic crystals.

Keywords: phononic crystals, doubly coupled vibration, band gaps, thin-walled beam
PACC: 0340K, 4600, 0260, 8160H

1. Introduction

The study of wave propagation in periodic structures has originated many novel discoveries in physics. Analogies from such subfields in physics have also opened new fruitful avenues in the researches on the propagations of classical waves in periodic composite materials, for example "photonic crystals" and "phononic crystals". Photonic crystals, made of a periodic repletion of dielectric or metallic inclusions in a homogeneous dielectric background, have attracted much attention during the last decade, in relation with their ability to control the propagation of light and the possibility of new optical devices. [1]

The analogy between electromagnetic waves and vibration stimulated research on the propagation of elastic waves in phononic crystals constituted of periodic composite materials. The emphasis of these studies was laid on the existence of complete elastic band gaps within which sound and vibration are all forbidden.[2−8] This is of interest for applications such as frequency filters, vibrationless environments for high-precision mechanical systems or design of new transducers, as well as for pure physics concerned with the Anderson localization of sound and vibration.[4,5]

Vibration band gaps in phononic crystals have been found both theoretically and experimentally.[6−8] Recently, band gaps in thin-walled structure phononic crystals have been researched theoretically,[9] in which propagation of flexural waves in a periodic plate and axisymmetric waves in a periodic cylindrical shell with and without heavy fluid loading was considered.

Beams of thin-walled open cross-sections are widely used in structural design. The most common application areas are aerospace engineering, civil engineering and related sites. In general, the centroid and the shear centre of cross-section are not coincident. This leads to the phenomenon that the flexural vibration is coupled with the torsional vibration. The flexural–torsional vibration problem has attracted scientists for a long time.[10−13] Most of those works concern with natural vibration frequencies of the thin-walled structures. To our best knowledge, coupled vibration band gaps in periodic thin-walled open cross-section beams have not been reported.

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†E-mail: dianlongyu@yahoo.com.cn

2. Equations of motion and plane wave expansion method

Figure 1(a) shows a periodic binary constant thin-walled section beam. The system consists of an infinite repetition of alternating thin-walled beam of material A with length a_1 and material B with length a_2. Thus the phononic crystal's lattice constant is $a = a_1 + a_2$. The cross-section of the beam is symmetric with respect to the y-axis, which is illustrated in Fig.1(b). A transverse force applied through the centroid C results in a transverse force through the shear centre O and a twisting torque about O.[10] In this case, the flexural vibration in the z direction are coupled with the torsional vibration whereas the flexural vibration in the y direction occurs independently[10,11]

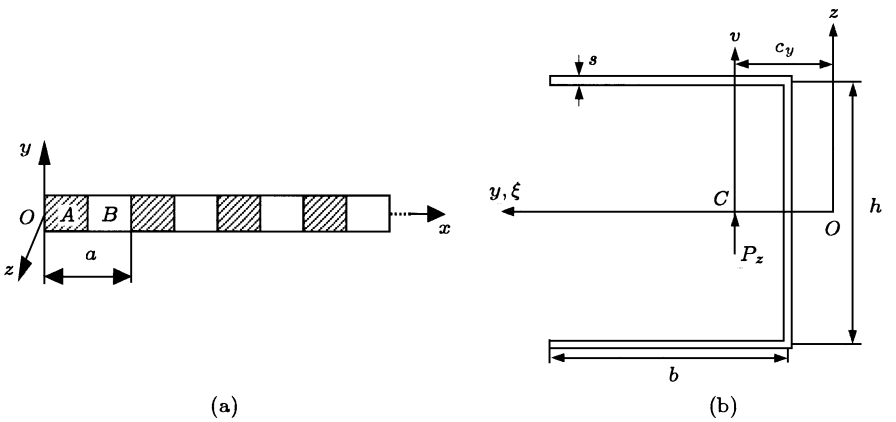


Fig.1. The sketch map of periodic binary thin-walled beam. (a) Infinite extended periodic structure. (b) Cross-section of the beam.

The equations of motion for these types of doubly coupled free vibration are known to be[10,11]

$$\frac{\partial^2}{\partial x^2}\left(E(x)I_\xi\frac{\partial^2 w}{\partial x^2}\right)+\rho(x)A\frac{\partial^2 w}{\partial^2 t}+\rho(x)Ac_y\frac{\partial^2\phi}{\partial t^2}=0, \tag{1a}$$

$$\frac{\partial^2}{\partial x^2}\left[E(x)\Gamma_0\frac{\partial^2\phi}{\partial x^2}\right]-\frac{\partial}{\partial x}\left[\mu(x)J\frac{\partial\phi}{\partial x}\right]+\rho(x)Ac_y\frac{\partial^2 w}{\partial t^2}+\rho(x)I_0\frac{\partial^2\phi}{\partial t^2}=0, \tag{1b}$$

where w is the flexural displacement in the z direction, ϕ is the torsional displacement. E, μ, ρ are Young's modulus, shear modulus and density of material respectively, which are periodic function of x. EI_ξ is the flexural rigidity in the z direction, μJ is the torsional stiffness, $E\Gamma_0$ is the warping stiffness with respect to the shear centre. I_0 is the second polar moment of area with respect to the shear centre, A is the cross-sectional area, and c_y is the eccentricity between the centroid and the shear centre in the y direction. I_ξ, J, Γ_0, I_0, A, c_y are constant along x.

Following the plane wave expansion methodology[14], we can derive Eq.(1) in the form of a standard eigenvalue problem at hand:

$$\omega^2\left(\sum_{G'}\rho(G-G')Aw_k(G')+\sum_{G'}\rho(G-G')Ac_y\phi_k(G')\right)=\sum_{G'}((G+k)^2(G'+k)^2E(G-G')I_\xi)w_k(G'), \tag{2a}$$

$$\omega^2\left(\sum_{G'}\rho(G-G')c_yw_k(G')+\sum_{G'}\rho(G-G')I_0\phi_k(G')\right)=\sum_{G'}((G+k)^2(G'+k)^2E(G-G')\Gamma_0+(G+k)(G'+k)\mu(G-G')J)\phi_k(G'). \tag{2b}$$

where k is the Bloch wave vector restricted within the first Brillouin zone $[-\pi/a\ ,\ \pi/a]$, ω is the circular frequency of the wave, G and G' are reciprocal vectors. The summation extends over all the reciprocal vectors. In this paper, we write all the one-dimension vectors as scalars.

Then one can formally write Eqs.(2) in matrix form:

$$\omega^2\boldsymbol{M}X=\boldsymbol{N}X, \tag{3}$$

where $\boldsymbol{M} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$, $\boldsymbol{N} = \begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{bmatrix}$ and

$$m_{11} = m_{21} = \rho(G - G')A,$$
$$m_{12} = \rho(G - G')Ac_y,$$
$$m_{22} = \rho(G - G')I_0,$$

$$n_{11} = (G + k)^2(G' + k)^2 E(G - G')I_\xi,$$
$$n_{22} = (G + k)(G' + k)\mu(G - G')J + (G + k)^2(G' + k)^2 E(G - G')\Gamma_0,$$
$$\boldsymbol{X} = [w_k(G') \quad \phi_k(G')]^{\mathrm{T}}.$$

Equation (3) is an infinite set of linear equations. In practice, only a finite number of G vectors are taken into account for the numerical calculation. If the infinite series is approximated by a sum of N reciprocal vectors, Eq.(3) is reduced to $2N \times 2N$ matrix eigenvalue equations.

3. Results calculated by plane wave expansion method

For doubly coupled vibrations, we choose a thin-walled beam, whose cross-section is given in Fig.1. The geometric properties of the beam are b=h=15×10^{-3}(m), s=1×10^{-3}(m), and A = 4.5×10^{-5}(m^2), I_ξ = 1.97 × 10^{-9}(m^4)[15], I_0 = 8.97×10^{-9} (m^4)[15], c_y = 1.14 × 10^{-2}(m)[16], Γ_0 = 4.52 × 10^{-15}(m^6)[17], J=1.5×10^{-11} (m^4)[17].

As an example, we calculated the band structure of the thin-walled beam with material A being epoxy and material B being steel, the material properties are given in Table 1. In this paper, we use 101 reciprocal vectors. The results show a very good convergence.

Table 1. Physical parameters of the materials.

Material	Density ρ/kg·m^{-3}	Young's modulus $E/10^{10}$Pa	Shear modulus $\mu/10^{10}$Pa
Pb	11600	4.08	1.49
Cu	8950	16.46	7.53
Steel	7780	21.06	8.1
Ti	4540	11.70	4.43
Al_2O_3	3970	39.64	15.98
Al	2730	7.76	2.87
Be	1850	31.72	15.33
C	1750	23.01	8.85
Epoxy	1180	0.435	0.159

Figure 2 shows the doubly coupled vibration band structure of the epoxy/steel beam with the lattice a=300 mm and a_1:a_2=1:1. Since the velocity of the flexural wave is different from that of the torsional wave, hence there are two branches from 0Hz to 200Hz. The branch with the lower frequency is generated by torsional wave, the other branch is generated by flexural wave. The branches with high frequency is generated by coupled vibration of flexural wave and torsional wave. We can find three complete band gaps within 1000Hz, settled as the shadowed regions in Fig.2. The first gap extends from the frequency of 165.15Hz up to 349.15Hz, the second gap occurs between 426.80Hz and 631.80Hz and the third gap frequency range is from 778.26Hz to 885.08Hz. The flexural and torsional wave can't propagate through the thin-walled beam in the band gap frequency range, so the doubly coupled vibration in the beam will be forbidden.

The three band gaps with gap width Δf=184Hz, 225.82Hz and 106.82Hz respectively are shown in Fig.2. The normalized gap width $\Delta f/f_g$=0.7155, 0.4352 and 0.1284, where f_g is the midgap frequency.

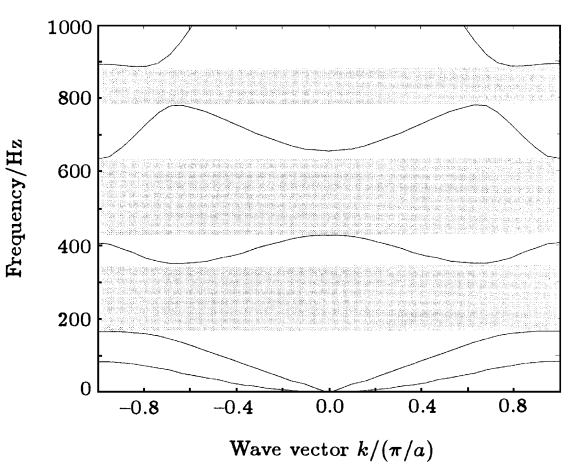


Fig.2. The band structure of doubly coupled vibration with infinite periodic structure epoxy/steel beam, the lattice a=300mm and a_1:a_2=1:1.

The parameters inducing the appearance of a gap in binary systems are the filling fraction, the lattice symmetry, the topology of the scatterers, and the contrast between elastic properties of materials. The density contrast plays a fundamental role, and its high contrast is more possible to obtain wide band gaps.[18]

Now we research the important factors that affect the band gaps width for doubly coupled vibrations. Firstly, we consider the geometric structures. In general, if the filling fraction is constant, the gap width is linear with respect to the lattice a, namely the normalized gap width is constant while the lattice changes linearly.

But it is not so for coupled vibrations. Figure 3 shows the band structure with the lattice a=600 mm and a_1:a_2=1:1. The normalized gap width $\Delta f/f_g$ = 0.8170, 0.2106 and 0.4502 respectively. The normalized gap width with lattice a=600 mm is quite different from that with lattice a=300 mm. The reason is, for different lattice, the pure flexural vibration band gap and the pure torsional vibration band gap couple non-linearly in different frequency range. Then the normalized gap width is not linear with respect to the lattice a. Further, we plot the normalized gap width of the first three band gap as only a increases monotonically from 300 mm to 1000 mm in Fig.4, while the filling fraction keeps constant a_1:a_2=1:1. In Fig.4, the continuous line, dash line and dash-dot line represent the normalized gap width of the first, second and third band gap respectively.

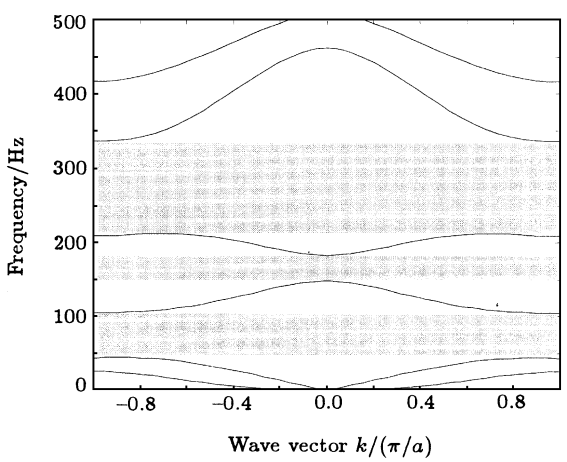


Fig.3. The band structure with the lattice a=600 mm and a_1:a_2=1:1.

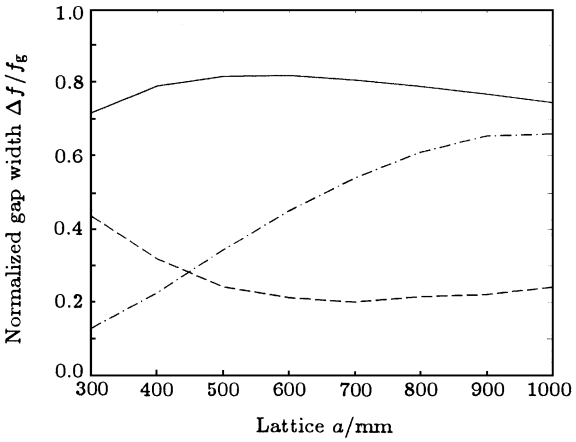


Fig.4. The normalized gap width curve of the first three band gap with changing lattice a. The continuous line, dash line and dash-dot line represent the normalized gap width of the first, second and third band gap respectively.

As to the next step, we consider the effect of material properties on the band gap width. Keeping lattice a=300 mm, a_1:a_2=1:1, we calculated the band structure of epoxy and some kind of materials, whose elastic constant are given in Table 1. In Fig.5, we plot the normalized gap width of the first band gap as Young's modulus increases monotonically from 4×10^{10} Pa to 40×10^{10}Pa. The material B corresponding to its Young's modulus is labelled as square cross-section, we keep material A being epoxy. The normalized gap width of the first gap increases monotonically as the Young modulus increases. For the band gap width, the Young modulus contrast plays a fundamental role, but not density contrast.

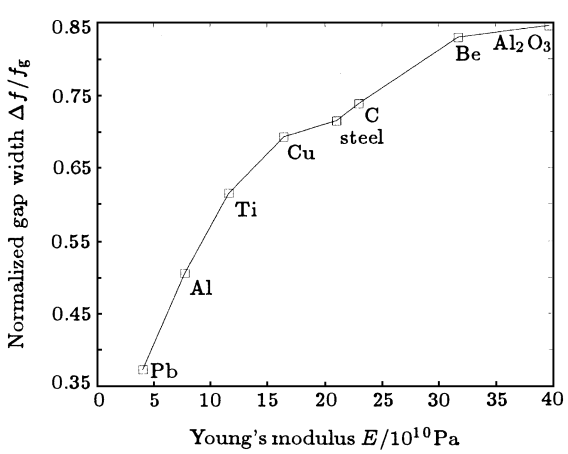


Fig.5. The dependency of the first band gap normalized width related to the changing of Young's modulus.

4. Frequency response simulated by finite element method

For the infinite periodic system, the double-coupled vibration band structure is given in section 3, but we do not know the effect of vibration reduction for the finite periodic structure. So we simulated a finite periodic binary epoxy/steel thin-walled beam with the same geometric parameters as in Section 3 by the finite element method. We calculate the beam made of eight periodic sections of epoxy-steel.

In the calculation, the acceleration bandwidth white noise in y direction about centroid from 0Hz to 1000Hz is excited in one end of the beam with lattice a=300 mm and a_1:a_2=1:1. The frequency response of the other end of the beam is shown in Fig.6. There are three sharp drops in the frequency response curve. The first ranges from 164.15Hz to 362.35Hz, the second from 447.22Hz to 625.15Hz and the third from723.61Hz to 896.76Hz. It is in very good agreement with the band structure. Every band gap has a response drop-off about 40dB compared with the response outside the band gaps.

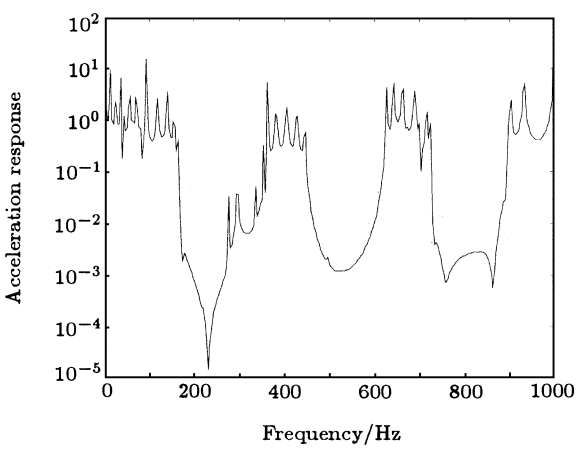


Fig.6. Response curves showing the acceleration response at the end of the epoxy/steel beam as a function of the excitation frequency, the lattice a=300 mm and a_1:a_2=1:1.

In order to analyse the influence of the lattice for the normalized band gap width, we calculated the frequency response of the beam with lattice a=600 mm in Fig.7. The frequency of band gaps is not linear with respect to the lattice.

The result calculated by finite element method is in very good agreement with the band structure. But we can find the frequency error between the band structure and frequency response by means of the convergence of plane wave expansion method. The better convergence for plane wave expansion method is easily satisfied, but the value is still far from the accurate eigenvalue.[19]

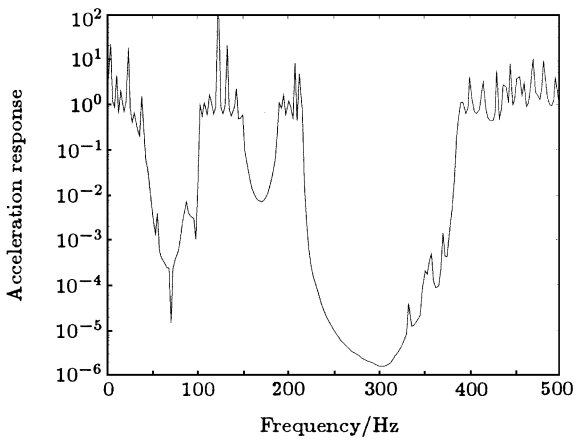


Fig.7. Frequency response curve with the lattice a=600 mm and a_1:a_2=1:1.

5. Conclusion

In conclusion, the flexural–torsional coupled vibration for a periodic binary thin-walled beam with open cross-sections is studied theoretically in this paper. The plane wave expansion method for doubly coupled vibration is provided. The doubly coupled vibration band gaps exist in the periodic structure.

Further, we consider the effect of geometric and material properties on the band gap width. The lattice is one of the factors that affect the normalized gap width for the same filling fraction. If the lattice and filling fraction keeps constant, the Young's modulus contrast plays a fundamental role for the band gap width, but not density contrast.

Finally, the frequency response of a finite periodic binary thin-walled beam is simulated by finite element method, which provides an attenuation of about 40dB in the frequency range of the band gaps. The existence of doubly coupled vibration gaps in thin-walled beams gives a new idea in vibration control of beams. The findings will be significant in the application of phononic crystals.

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