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Uncertainty relation and black hole entropy of Kerr spacetime*

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The properties of thermal radiation are discussed by using a new equation of state density, which is motivated by the generalized uncertainty relation in the quantum gravity. There is no burst at the last stage of the emission of Kerr black hole. When the new equation of state density is utilized to investigate the entropy of a Bosonic field and Fermionic field outside the horizon of a static Kerr black hole, the divergence appearing in the brick wall model is removed, without any cutoff. The entropy proportional to the horizon area is derived from the contribution of the vicinity of the horizon.

Keywords: membrane model, entropy of black hole, quantum statistics, generalized uncertainty relation

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1. Introduction

A growing interest in the thermodynamics of a black hole is indicated by a vast existing literature on the subject. The thermodynamics of a black hole is a junction at the intersection of general relativity, quantum mechanics and statistic physics. Since Bekenstein and Hawking put forward the theory that the entropy of a black hole was proportional to the area of its event horizon,[1−3] the research into black-hole thermodynamics has greatly improved. In particular, the proof of the Hawking radiation has stimulated much enthusiasm for study of the thermal property of the black hole.

It has been found that if the surface gravity of the black hole, κ, is considered to be temperature and the outer horizon area, A, is considered to be the entropy, four black-hole thermodynamics laws, which correspond to the four laws of elementary thermodynamics, can be established. The statistical origin of black hole has been probed and many methods of calculating entropy have emerged.[4−9] A method frequently used is the brick-wall method advanced by 't Hooft.[6] This method is used to study the statistical properties of the scalar field and Dirac fields in various black holes[5−15] and it is found that the general expression of the black-hole entropy consists of a term which is proportional to the area of its event horizon and a divergent logarithmic term which is not. However, it is doubtful whether

(1) the entropy of the scalar or Dirac field outside the event horizon is the entropy of the black hole;

(2) the state density near the event horizon is divergent;

(3) the cutoff can be introduced, the logarithmic term left out, and L^3 is considered as the contribution of distant vacuum surrounding the system, to obtain the entropy of a black hole being proportional to the area of its event horizon.

The problems associated with the original brick-wall method listed above are unnatural. Recently Li, Zhao and He took the thin-layer outside the event horizon of a black hole as an integral region,[14,15] and thus developed a thin-layer method. Using this method they discussed the entropy of a dynamic black hole and found that its entropy was proportional to the area of its event horizon. But they did not essentially overcome the difficulty in original brick-wall

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method.

Recently, Refs.[16–18] derived the partition functions of various particles in spherically symmetric space–time directly by using a new equation of state density, which was motivated by the generalized uncertainty relation in the quantum gravity. They obtained the entropy of the black hole via the relation between partition function and entropy. As a result, the term left out in the original brick-wall method no longer existed. The state density near the event horizon is not divergent. Using the quantum statistical method, Ref.[17] avoided the difficulty in solving the wave equation in the original brick-wall method.

In this paper, we generalize the method in Ref.[16–19] and calculate the statistical entropy of the spinning axially symmetric Kerr black hole. We obtain the factor λ introduced in correction of density of states as a function of polar angle. However, the entropy is proportional to the area of its horizon. In the whole process, the idea is clear in physics, calculation is simple and the result is reasonable. It offers a neat way of studying black hole entropy. In this paper, we take the simplest functional form of the temperature ($C = \hbar = G = K_{\rm B} = 1$).

2. Bosonic entropy

Recently, many efforts have been devoted to the generalized uncertainty relation[20]

$$\Delta x \Delta p \geq \hbar + \frac{\lambda}{\hbar}(\Delta p)^2 \tag{1}$$

and its consequences,[20–26] especially the effect on the density of states.[20,21] Here $\hbar$ is the Planck constant, λ has dimensions of the Planck length squared. Equation (1) means that there is a minimal length, $2\sqrt{\lambda}$. As is well known, the number of quantum states in a volume element in phase space $\mathrm{d}V\mathrm{d}^3p$ is given by

$$\frac{\mathrm{d}V\mathrm{d}^3p}{(2\pi\hbar)^3}, \tag{2}$$

which can be understood as follows: since the uncertainty relation $\Delta x \Delta p \sim 2\pi\hbar$, one quantum state corresponds to a "cell" of volume $(2\pi\hbar)^3$ in the phase space. Based on the Liouville theorem, the authors of Ref.[20] argued that the number of quantum states should be changed to the following form:

$$\frac{\mathrm{d}V\mathrm{d}^3p}{(2\pi\hbar)^3(1+\lambda p^2)^3}, \tag{3}$$

where $p^2 = p_i p^i$, $i = 1, 2, 3$. The equation (3) seriously deforms the Plank spectrum of the black body radiation at the Planck temperature, $T_\lambda = \sqrt{1/\lambda}$ (see Fig.2 in Ref.[24]).

The generalized commutation relation is[19]

$$[\hat{x},\ \hat{p}] = \mathrm{i}\hbar(1 + \lambda\hat{p}). \tag{4}$$

In the generalized uncertainty relation (1), the expression of the density of states (3), and the generalized commutation relation (4), the Planck constant, $\hbar$ in the standard Heisenberg principle, has been replaced by a new parameter $\hbar' = \hbar(1 + \lambda p^2)$.

The linear element in Kerr spacetime is given by

$$\begin{aligned}\mathrm{d}s^2 =& -\left(1 - \frac{2Mr}{\rho^2}\right)\mathrm{d}t^2 + \frac{\rho^2}{\Delta}\mathrm{d}r^2 + \rho^2\mathrm{d}\theta^2 \\ &+ \left[(r^2+a^2)\sin^2\theta + \frac{2Mra^2\sin^4\theta}{\rho^2}\right]\mathrm{d}\varphi^2 \\ &- \frac{4Mra\sin^2\theta}{\rho^2}\mathrm{d}t\mathrm{d}\varphi,\end{aligned} \tag{5}$$

where $\rho^2 = r^2 + a^2\cos^2\theta$, and $\Delta = r^2 - 2Mr + a^2$, M represents the black hole mass and $a(= J/M)$ is the black hole angular momentum per unit mass. The radiation temperature of the black hole is

$$T_+ = \frac{r_+ - r_-}{4\pi(r_+^2 + a^2)},$$

where $r_\pm = M \pm \sqrt{M^2 - a^2}$ are the locations of outer and inner horizons respectively.

$$A_+ = 4\pi(r_+^2 + a^2). \tag{6}$$

Based on the theory of general relativity, an observer at rest at an infinite point finds that the frequency shift of particles from the surface of a star is

$$\nu = \nu_0\sqrt{-g'_{tt}}, \tag{7}$$

where ν_0 is the natural frequency shift of the atoms on the surface of the star. ν is the natural frequency shift of particles from the surface of the star at an arbitrary point r, obtained by the observer at rest at an infinite point.

The natural radiation temperature [27–29] measured by the observer at rest at an infinite point is

$$T = \frac{T_+}{\sqrt{-g'_{tt}}}, \tag{8}$$

where T_+ is the equilibrium temperature and

$$\begin{aligned}g'_{tt} =& \frac{g_{tt}g_{\varphi\varphi} - g_{t\varphi}^2}{g_{\varphi\varphi}} \\ =& -\frac{(r-r_+)(r-r_-)(r^2+a^2\cos^2\theta)}{(r^2+a^2)^2 - (r-r_+)(r-r_-)a^2\sin^2\theta}.\end{aligned} \tag{9}$$

For the Bosonic system, we calculate the normal partition function as follows:

$$\ln Z = -\sum_i g_i \ln(1 - \mathrm{e}^{-\beta\varepsilon_i}). \tag{10}$$

In unit volume, from Eq.(3), the number of quantum states with energy less than ν is given by

$$g(\nu) = j\frac{4\pi p^3}{3(2\pi\hbar)^3(1+\lambda p^2)^3} = j\frac{4\pi\nu^3}{3(1+\lambda 4\pi^2\nu^2)^3}, \tag{11}$$

where j is the spinning degeneracy of particles, $p = 2\pi\hbar\nu$, and ν is given by the expression (7). In the spacetime (5), the area of two-dimensional curved surface at an arbitrary point r is

$$A(r) = \int \mathrm{d}A = \int \sqrt{g}\mathrm{d}\theta\mathrm{d}\varphi, \tag{12}$$

where $g = \begin{vmatrix} g_{\theta\theta} & g_{\theta\varphi} \\ g_{\varphi\theta} & g_{\varphi\varphi} \end{vmatrix} = g_{\theta\theta}g_{\varphi\varphi}$. The volume of the lamella at an arbitrary point r outside the horizon is

$$\mathrm{d}V = A(r)\sqrt{g_{rr}}\mathrm{d}r. \tag{13}$$

So, the partition function of the system at the lamella with an arbitrary thickness at the point r outside the horizon is

$$\begin{aligned}
\ln Z =& -\int A(r)\sqrt{g_{rr}}\mathrm{d}r \sum_i g_i \ln(1 - \mathrm{e}^{\beta\varepsilon_i}) \\
=& -\int A(r)\sqrt{g_{rr}}\mathrm{d}r \int_0^\infty dg(\nu)\ln(1 - \mathrm{e}^{-\beta h\nu}) \\
=& j\int A(r)\sqrt{g_{rr}}\mathrm{d}r \\
& \times \int_0^\infty \frac{4\pi\beta h}{3(1+\lambda 4\pi^2\nu^2)^3(\mathrm{e}^{\beta h\nu}-1)}\nu^3\mathrm{d}\nu \\
=& j\beta_0\int A(r)\sqrt{g_{rr}}\mathrm{d}r \\
& \times \int_0^\infty \frac{4\pi h\sqrt{-g'_{tt}}}{3(1+\lambda 4\pi^2\nu^2)^3(\mathrm{e}^{\beta h\nu}-1)}\nu^3\mathrm{d}\nu,
\end{aligned} \tag{14}$$

where $\beta = \beta_0\sqrt{-g'_{tt}}$. The free energy of the system is

$$\begin{aligned}
F =& -\frac{1}{\beta_0}\ln Z \\
=& -j\int A(r)\sqrt{g_{rr}}\mathrm{d}r \\
& \times \int_0^\infty \frac{4\pi h\sqrt{-g'_{tt}}}{3(1+\lambda 4\pi^2\nu^2)^3(\mathrm{e}^{\beta h\nu}-1)}\nu^3\mathrm{d}\nu.
\end{aligned} \tag{15}$$

So the entropy of the system is

$$\begin{aligned}
S_\mathrm{b} =& \beta_0^2\frac{\partial F}{\partial\beta_0} \\
=& j\beta_0\int A(r)\sqrt{g_{rr}}\mathrm{d}r \\
& \times \int_0^\infty \frac{4\pi\beta\nu h^2\sqrt{-g'_{tt}}\mathrm{e}^{\beta h\nu}}{3(1+\lambda 4\pi^2\nu^2)^3(\mathrm{e}^{\beta h\nu}-1)^2}\nu^3\mathrm{d}\nu \\
=& j\frac{1}{6\pi^2\beta_0^3}\int \frac{A(r)\sqrt{g_{rr}}}{(-g'_{tt})^{3/2}}\mathrm{d}r \\
& \times \int_0^\infty \frac{\mathrm{e}^x x^4\mathrm{d}x}{\left(1+\lambda\dfrac{x^2}{\beta_0^2(-g'_{tt})}\right)(\mathrm{e}^x-1)^2},
\end{aligned} \tag{16}$$

where $x = \beta h\nu$, on the supposition that

$$\begin{aligned}
I_1(g'_{tt}) =& \int_0^\infty \frac{\mathrm{e}^x x^4\mathrm{d}x}{\left(1+\lambda\dfrac{x^2}{\beta_0^2(-g'_{tt})}\right)(\mathrm{e}^x-1)^2} \\
\approx& \int_0^\infty \frac{(x^2+x^3)\mathrm{d}x}{\left(1+\lambda\dfrac{x^2}{\beta_0^2(-g'_{tt})}\right)^3} \\
=& \frac{\pi}{16}\beta_0^3\left(\frac{-g'_{tt}}{\lambda}\right)^{3/2} + \frac{1}{4}\beta_0^4\left(\frac{-g'_{tt}}{\lambda}\right)^2.
\end{aligned} \tag{17}$$

In Eq.(16), we perform the integration over x in the integral region $[r_+,\ r_+ + \varepsilon]$ near the horizon of the black hole, where r_+ is the location of horizon, and then obtain

$$\begin{aligned}
S_\mathrm{b} =& \frac{j}{6\pi^2\beta_0^3}\int_{r_+}^{r_+\varepsilon}\frac{A(r)\sqrt{g_{rr}}\mathrm{d}r}{(-g'_{tt})^{3/2}}\left[\frac{\pi}{16}\beta_0^3\left(\frac{-g'_{tt}}{\lambda}\right)^{3/2} + \frac{1}{4}\beta_0^4\left(\frac{-g'_{tt}}{\lambda}\right)^2\right] \\
=& \frac{j}{24\pi^2}\int_{r_+}^{r_+\varepsilon}\frac{1}{\lambda^{3/2}}\left[\frac{\pi}{4}\left(\frac{[(r^2+a^2)^2-\Delta a^2\sin^2\theta][r^2+a^2\cos^2\theta]}{(r-r_+)(r-r_-)}\right)^{1/2}\sin\theta + \frac{\beta_0}{\lambda^{1/2}}(r^2+a^2\cos^2\theta)\sin\theta\right]\mathrm{d}\theta\mathrm{d}\varphi\mathrm{d}r \\
=& \frac{j}{12\pi}\int_0^\pi \frac{1}{\lambda^{3/2}}\left[\frac{\pi}{4}(r_+^2+a^2)\sqrt{\frac{2\varepsilon(r_+^2+a^2\cos^2\theta)}{\kappa(r_+^2+a^2)}} + \beta_0\frac{(r_+^2+a^2\cos^2\theta)\varepsilon}{\lambda^{1/2}}\right]\sin\theta\mathrm{d}\theta.
\end{aligned} \tag{18}$$

We are interested only in the contribution from the vicinity near the horizon, $[r_+,\ r_+ + \varepsilon]$, which corresponds to a proper distance of the order of the minimal length, $2\sqrt{\lambda}$. This is because the entropy approaches the upper bound only in this vicinity. Furthermore, it is just the vicinity that is neglected by the brick wall model. We have

$$\begin{aligned} 2\sqrt{\lambda} =& \int_{r_+}^{r_+ + \varepsilon} \sqrt{g_{rr}}\mathrm{d}r \\ \approx& \int_{r_+}^{r_+ + \varepsilon} \sqrt{\frac{r_+^2 + a^2\cos^2\theta}{2\kappa(r - r_+)(r_+^2 + a^2)}}\mathrm{d}r \\ =& \sqrt{\frac{2\varepsilon(r_+^2 + a^2\cos^2\theta)}{\kappa(r_+^2 + a^2)}}, \end{aligned} \tag{19}$$

where κ is the surface gravity at the horizon of the black hole, and it is identified as $\kappa = 2\pi\beta_0^{-1}$. Thus we naturally derive the entropy proportional to the horizon area

$$\begin{aligned} S_\mathrm{b} =& \frac{j}{12\pi}\int_0^\pi \frac{1}{\lambda^{3/2}}\left[\frac{\pi}{4}(r_+^2 + a^2)\sqrt{\frac{2\varepsilon(r_+^2 + a^2\cos^2\theta)}{\kappa(r_+^2 + a^2)}}\right. \\ & \left. + \beta_0\frac{(r_+^2 + a^2\cos^2\theta)\varepsilon}{\lambda^{1/2}}\right]\sin\theta\mathrm{d}\theta \\ =& \frac{j}{12\pi}\int_0^\pi \frac{1}{\lambda}\left[\frac{\pi}{2} + 2\beta_0\kappa\right](r_+^2 + a^2)\sin\theta\mathrm{d}\theta \\ =& j\frac{3}{8}(r_+^2 + a^2)\int_0^\pi \frac{1}{\lambda}\sin\theta\mathrm{d}\theta \\ =& j\frac{3}{2\varepsilon}(r_+^2 + a^2)\frac{\kappa\alpha}{\sin\alpha\cos\alpha} \\ =& j\frac{3A(r_+)}{8\pi\varepsilon}\frac{\kappa\alpha}{\sin\alpha\cos\alpha} \\ =& j\frac{3A(r_+)}{8\pi\varsigma}, \end{aligned} \tag{20}$$

where $A(r_+) = 4\pi(r_+^2 + a^2)$ is the surface area of the black hole, $\alpha = \mathrm{arctg}\dfrac{a}{r_+}$, $\varsigma = \varepsilon\dfrac{\sin\alpha\cos\alpha}{\kappa\alpha}$.

From Eq.(20), we know that the entropy of the black hole is proportional to the area of its horizon. According to Eq.(19), we derive the entropy related to polar angle λ. When $a = 0$, our results are the same as those given in Refs.[16] and [17]. But the spacetime that we study has a general meaning in physics.

3. Fermionic entropy

For a Fermionic gas, the partition function is

$$\ln Z = \sum_i g_i \ln(1 + \mathrm{e}^{-\beta\varepsilon_i}). \tag{21}$$

From the above calculation, we obtain the entropy of the system is as follows:

$$\begin{aligned} S_\mathrm{f} =& \beta_0^2\frac{\partial F}{\partial\beta_0} \\ =& i\beta_0\int A(r)\sqrt{g_{rr}}\mathrm{d}r \\ & \times\int_0^\infty \frac{4\pi\beta\nu h^2\sqrt{-g'_{tt}}\mathrm{e}^{\beta h\nu}}{3(1 + \lambda 4\pi^2\nu^2)^3(\mathrm{e}^{\beta h\nu} + 1)^2}\nu^3\mathrm{d}\nu \\ =& i\frac{1}{6\pi^2\beta_0^3}\int\frac{A(r)\sqrt{g_{rr}}}{(-g'_{tt})^{3/2}}\mathrm{d}r \\ & \times\int_0^\infty \frac{\mathrm{e}^x x^4\mathrm{d}x}{\left(1 + \lambda\dfrac{x^2}{\beta_0^2(-g'_{tt})}\right)(\mathrm{e}^x + 1)^2}. \end{aligned} \tag{22}$$

$$\begin{aligned} I_2(g'_{tt}) =& \int_0^\infty \frac{\mathrm{e}^x x^4\mathrm{d}x}{\left(1 + \lambda\dfrac{x^2}{\beta_0^2(-g'_{tt})}\right)(\mathrm{e}^x + 1)^2} \\ =& \int_0^\infty \frac{\mathrm{e}^x x^4\mathrm{d}x}{(1 + \mu x^2)^3(\mathrm{e}^x + 1)^2} \\ =& \frac{1}{2}\frac{\partial^2}{\partial\mu^2}\int_0^\infty \frac{\mathrm{e}^x\mathrm{d}x}{(1 + \mu x^2)(\mathrm{e}^x + 1)^2} \\ =& -\frac{\partial^2}{\partial\mu^2}\int_0^\infty \frac{\mu x\mathrm{d}x}{(\mathrm{e}^x + 1)(1 + \mu x^2)^2} \\ \approx& -\frac{\partial^2}{\partial^2\mu}\int_0^\infty \frac{\mu x\mathrm{d}x}{(x + 2)(1 + \mu x^2)} \\ =& -\frac{\partial^2}{\partial\mu^2}\int_0^\infty \left[\frac{\mu}{(1 + \mu x^2)} - \frac{2\mu}{(x + 2)(1 + \mu x^2)}\right]\mathrm{d}x \\ =& -\frac{\partial^2}{\partial\mu^2}\left[\frac{\sqrt{\mu}}{2}\pi - \frac{\mu}{4\mu + 1}(\ln(2 + x)^2\right. \\ & \left. - \ln(1 + \mu x^2) - 4\sqrt{\mu}\mathrm{arctg}x\sqrt{\mu})_0^\infty\right] \\ \approx& \frac{1}{4}\mu^{-2} = \frac{1}{4}\beta_0^4\left(\frac{-g'_{tt}}{\lambda}\right)^2. \end{aligned} \tag{23}$$

So the entropy of the black hole is

$$\begin{aligned} S_\mathrm{f} =& i\frac{1}{6\pi^2\beta_0^3}\int_{r_+}^{r_+ + \varepsilon}\frac{A(r)\sqrt{g_{rr}}}{(-g'_{tt})^{3/2}}\frac{1}{4}\beta_0^4\left(\frac{-g'_{tt}}{\lambda}\right)^2 \\ =& i\frac{A(r_+)}{3\pi\varsigma}, \end{aligned} \tag{24}$$

where i is the spinning degeneracy of particles.

4. Conclusions

Based on the above analysis, the properties of the thermal radiation from an axially symmetric black hole are discussed by using a new equation of state density, which is motivated by the generalized uncertainty relation in the quantum gravity. When the new equation of state density is utilized to investigate the entropy of a Bosonic field and Fermionic field outside the horizon of a static and spherically symmetric black hole, the divergence appearing in a brick wall model is removed, without any cutoff. The entropy proportional to the horizon area is derived from the contribution of the vicinity of the horizon.

As early as 1992, Li and Liu phenomenally proposed that the state equations of the thermal radiation near the horizon should be changed into a series of new formulae, in order to maintain the validity of the generalized second law of thermodynamics.[30] Using the Li–Liu equation, Wang investigated the entropy of a self-gravitational radiation system and obtained the Bekenstein's entropy bound.[31] In this paper, we discuss the entropy of a Bosonic and Fermionic field near the horizon of an axially symmetric black hole. We obtain the entropy proportional to the area of the event horizon. There is no divergence term. This convergence is due to the effect of the generalized uncertainty relation on the quantum states.

In the above calculation, we compute the entropy of a Bosonic and Fermionic field. It is found that the entropy is finite and proportional to the area of the event horizon. Equations (20) and (24) are general expressions of every axially symmetric black hole. Our results are reasonable and practical. In this paper, we directly use the quantum statistical method to calculate the entropy. The difficulty in solving a wave equation is avoided. In the whole process, the idea is clear in its physics, the calculation is simple and the result is reasonable. This gives deeper insight into the understanding of the statistical origin of the entropy of a black hole.

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