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Entanglement dynamics of Gaussian continuous states in correlated noisy channels*

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The evolution of the entanglement of an initial two-mode squeezed vacuum (TMSV) in correlated noisy channels is investigated. It is shown that the correlated environment is helpful for preserving the entanglement of the initial TMSV. The entanglement period of the initial TMSV depends on the correlation property of the correlated environment.

Keywords: entanglement, Gaussian state, noisy channels
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1. Introduction

A number of possible practical applications of entangled states have been proposed, including quantum computation,[1] quantum communication,[2] and quantum cryptography.[3] In recent years, it has been increasingly realized that Gaussian continuous states are essential ingredients of continuous variable (CV) quantum information. Any attempt to exploit these entangled states for quantum information must, however, face the obvious difficulty that pure states are unavoidably corrupted by the interaction with the environment. Therefore, the analysis of the evolution of quantum decoherence of Gaussian CV states in noisy channels is of great importance. Recently, many researches in this field have been carried out. For example, the nonlocality for an initial two-mode squeezed vacuum (TMSV) in a thermal environment was discussed by Jeong *et al.*[4] They found that the larger the degree of initial squeezing, the more rapidly the squeezed state loses its nonlocality. An initial TMSV in an uncorrelated squeezed environment was discussed in Refs.[5–7], and it was shown that squeezing the environment does not obtain lengthened lifetime of the entanglement of the two-mode squeezed state. Serafini *et al*[8] addressed the evolution of cat-like states in general uncorrelated noisy channels, and found that squeezing the environment to protect a non-squeezed cat-state against decoherence is equivalent to orthogonally squeezing the initial cat-state when letting the environment be phase insensitive. Also Li *et al*[9,10] investigated the generation and decoherence of entanglements between two atoms and between excitons in two quantum dots in a cavity coupled to a broadband squeezed vacuum, and found that the decoherence is closely related to the correlation property of the broadband squeezed vacuum. McAneney *et al* [11] studied decoherence by considering the entanglement between a system and its environment. They pointed out that many-body entanglement between a system and its environment may play a crucial role in the process of disentangling the system, and there are two distinct routes to disentangling process: two-way entanglement and Greenberger–Horne–Zeiglinger (GHZ) entanglement. Zhou and Fang[12] discussed the entanglement properties for a two-mode output state from a beam splitter with binomial state input, and obtained a method for gaining the best entanglement. Wu *et al*[13] proposed a scheme to achieve multiparty entanglement with perfectly efficient, ultraslow, and multichannel pairwise four-wave mixing.

Here we consider the evolution of an initial TMSV in the correlated noisy channels. We use the Peres–Simom separability[14] and the Bures distance[15] to discuss the entanglement of such an initial TMSV

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in the correlated noisy channels. The dynamical behaviour of the entanglement is obtained analytically. It is shown that the correlated environment is helpful for preserving the entanglement of the initial TMSV. This paper is organized as follows: In Section 2, we solve the master equation by use of the method of characteristic function and analyze the time evolution of the entanglement of the initial state in the correlated noisy channels with the Peres–Simom separability criterion. In Section 3, we demonstrate that the state of the initial TMSV evolving in the correlated noisy channels is a two-mode squeezed thermal state. With the Bures distance, we obtain the same results of the entanglement evolution of the initial TMSV. In Section 4, we give a summary.

2. Characteristic function and the evolution of entanglement of an initial TMSV in correlated channels

Consider the non-unitary evolution of an initial TMSV in the correlated Gaussian noisy channels. Assume that the two channels have the same thermal number of photons. The density operator of the initial state in the correlated noisy channels is governed by

$$\begin{aligned}\frac{\mathrm{d}\rho}{\mathrm{d}t} =&\gamma(N+1)(2a_1\rho a_1^+ - a_1^+a_1\rho - \rho a_1^+a_1\\ &+2a_2\rho a_2^+ - a_2^+a_2\rho - \rho a_2^+a_2)\\ &+\gamma N(2a_1^+\rho a_1 - a_1a_1^+\rho - \rho a_1a_1^+\\ &+2a_2^+\rho a_2 - a_2a_2^+\rho - \rho a_2a_2^+)\\ &+2\gamma M(a_1\rho a_2 + a_2\rho a_1 - a_1a_2\rho - \rho a_1a_2)\\ &+2\gamma M^*(a_1^+\rho a_2^+ + a_2^+\rho a_1^+ - a_1^+a_2^+\rho - \rho a_1^+a_2^+),\end{aligned} \tag{1}$$

where γ denotes the decay rates of the two modes of the initial TMSV in the correlated environment . a_i and a_i^+ $(i=1,2)$ are annihilation and creation operators for these two modes respectively. N denotes the average of thermal photons, and $M=|M|\mathrm{e}^{-\mathrm{i}\theta}$ characterizes the two-photon correlation property of the Gaussian correlated environment. On Eq.(1) positivity of the density matrix imposes the inequality $|M|\leq\sqrt{N(N+1)}$.

The master equation (1) can be solved by the aid of the characteristic function (CF). The CF of a two-mode state with density operator ρ in Wigner representation is defined as[16]

$$\chi(\xi_1,\xi_2)=\mathrm{Tr}[\rho\exp(\xi_1a_1^+-\xi_1^*a_1)\exp(\xi_2a_2^+-\xi_2^*a_2)]. \tag{2}$$

Using the standard operator correspondence, we find that the CF corresponding to Eq.(1) obeys

$$\begin{aligned}\frac{\partial\chi(\xi_1,\xi_2,t)}{\partial t} =&-\gamma[\xi_1\frac{\partial}{\partial\xi_1}+\xi_1^*\frac{\partial}{\partial\xi_1^*}+(2N+1)|\xi_1|^2\\ &+\xi_2\frac{\partial}{\partial\xi_2}+\xi_2^*\frac{\partial}{\partial\xi_2^*}+(2N+1)|\xi_2|^2\\ &+2(M\xi_1\xi_2+M^*\xi_1^*\xi_2^*)]\chi(\xi_1,\xi_2,t).\end{aligned} \tag{3}$$

Under the initial condition that a TMSV has $|\psi\rangle=S(r_\mathrm{s},\varphi)|00\rangle$, where $S(r_\mathrm{s},\varphi)=\exp(-sa_1^+a_2^+ + s^*a_1a_2)$ is a two-mode squeezing operator with squeezing parameter $s=r_\mathrm{s}\mathrm{e}^{\mathrm{i}\varphi}$,[17] we obtain the solution for Eq.(3)

$$\begin{aligned}\chi(\xi_1,\xi_2,t) =&\exp[-D_0(|\xi_1|^2+|\xi_2|^2)\\ &-D_1\xi_1\xi_2-D_1^*\xi_1^*\xi_2^*].\end{aligned} \tag{4}$$

With an appropriate rotating in phase space(which corresponds to the local transformation on the two modes), we can rewrite Eq.(4) in the form:

$$\chi(\xi_1,\xi_2\,t)=\exp\left(-\frac{1}{2}\xi^+V\xi\right)$$

with

$$\xi^+=(\xi_1^*,\xi_1,\xi_2^*,\xi_2).$$

Here V is the correlation matrix expressed as

$$V=\begin{vmatrix} D_0 & 0 & |D_1| & 0\\ 0 & D_0 & 0 & |D_1|\\ |D_1| & 0 & D_0 & 0\\ 0 & |D_1| & 0 & D_0\end{vmatrix}, \tag{5}$$

where

$$D_0=(N+1/2)(1-\mathrm{e}^{-2\gamma t})+\frac{1}{2}\cosh2r_\mathrm{s}\mathrm{e}^{-2\gamma t}, \tag{6}$$

$$\begin{aligned}D_1 =&\mathrm{e}^{-\mathrm{i}\theta}[|M|(1-\mathrm{e}^{-2\gamma t})\\ &+\frac{1}{2}\mathrm{e}^{-2\gamma t}\sinh2r_\mathrm{s}\mathrm{e}^{-\mathrm{i}(\varphi-\theta)}]=|D_1|\mathrm{e}^{2\mathrm{i}\phi}.\end{aligned} \tag{7}$$

Based on the fact that the partial transpose of a bipartite density operator should be positive if it is separable, Simon[14] found the sufficient and necessary condition for the separability of a two-mode Gaussian state. In their work, Englert and Wodkiewicz[18] proposed an equivalent criterion in the frame of the operator formalism for the density operator. They noted that if a two-mode Gaussian state is P-representable,

it is separable if and only if $V - \frac{1}{2}I \geq 0$, where V is a 4×4 correlation matrix for the Weyl–Wigner CF defined by Eq.(2). Therefore, the two-mode Gaussian state with the correlation matrix described by Eq.(5) is separable if and only if all the eigenvalues of $V - \frac{1}{2}I$ are nonnegative. The four eigenvalues of $V - \frac{1}{2}I$ are

$$\lambda_{1,2} = D_0 + |D_1| - 1/2,$$

$$\lambda_{3,4} = \lambda_- = D_0 - |D_1| - 1/2. \tag{8}$$

The positivity of $\lambda_{1,2}$ can be easily verified. So they are not important for establishing the inseparability of the state. But the eigenvalue λ_- may be negative for determining the inseparability of the state. We can see that the negativity of λ_- depends on the relative phase $\varphi - \theta$ between the initial TMSV and the environment. When $\varphi = \theta$, λ_- takes its minimum value, which means the maximum degree of entanglement. So in the following discussion, the relation $\varphi = \theta$ is always taken into account. Under this condition, λ_- can be expressed as

$$\lambda_- = (N - |M|)(1 - \mathrm{e}^{-2\gamma t}) - \frac{1}{2}(1 - \mathrm{e}^{-2r_\mathrm{s}})\mathrm{e}^{-2\gamma t}. \tag{9}$$

If $M = 0$, which means that the initial TMSV evolves in a thermal environment, the system will disentangle after an entanglement period $t_\mathrm{en} = \frac{1}{2\gamma}\ln\left(1 + \frac{1 - \mathrm{e}^{-2r_\mathrm{s}}}{2N}\right)$. This shows obviously that the rapidity of the loss of the entanglement of the initial state is related to the average of thermal photons N of the environment and the decay rate γ of the initial state in the environment. The larger the degree of initial squeezing, the longer the entanglement of the system. In the long-time limit, i.e, $t \to \infty$, we have $\lambda_- = N \geq 0$, which means that the initial TMSV eventually assimilates with the thermal environment and loses its entanglement completely. When $0 < |M| < N$, i.e. the environment has classical correlation, the entanglement will disappear after a longer entanglement time $t'_\mathrm{en} = \frac{1}{2\gamma}\ln\left[1 + \frac{1 - \mathrm{e}^{-2r_\mathrm{s}}}{2(N - |M|)}\right]$. In this case, the entanglement time increases with increase of the value of $|M|$. When $\sqrt{N(N+1)} \geq |M| \geq N$, $\lambda_- < 0$ holds always. That is to say, if the environment has nonclassical correlation, the initial TMSV in the correlated noisy channels will entangle forever. In the long-time limit we have $\lambda_- = N - |M|$, which means that the entanglement depends only on the property of the environment, and the nonclassically correlated environment can entangle any initial two-mode states. From these discussions we can conclude that the correlated environment can preserve the entanglement of an initial TMSV in the correlated noisy channels.

3. Comparison with the Bures distance as a measure of entanglement

Bures distance is originally introduced to measure entanglement by Vedral and Plenio.[19] For the first time, Marian *et al*[15] used it to quantify entanglement in the case of continuous variables, i.e., two-mode squeezed thermal states(STS). Firstly, we demonstrate that an initial TMSV evolving in the correlated noisy channels is a two-mode STS. Performing the unitary transformation, defined as

$$\tilde{\rho} = S^+ \rho S, \tag{10}$$

where $S(r_\mathrm{s}, \varphi)$ is a two-mode squeeze operator, to the master equation (1), we have

$$\begin{aligned}\frac{\mathrm{d}\tilde{\rho}}{\mathrm{d}t} =& \sum_{i=1}^{2} \gamma(N_1 + 1)(2a_i\rho a_i^+ - a_i^+ a_i\rho - \rho a_i^+ a_i) \\ &+ \gamma N_1(2a_i^+\rho a_i - a_i a_i^+\rho - \rho a_i a_i^+),\end{aligned} \tag{11}$$

where $N_1 = (N+1)\sinh^2 2r_\mathrm{s} + N\cosh^2 r_\mathrm{s} - |M|\sinh 2r_\mathrm{s}$, and the parameters r_s and φ are chosen to be $\tanh 2r_\mathrm{s} = \frac{2|M|}{2N+1}$ and $M = |M|\mathrm{e}^{-\mathrm{i}\varphi}$. Defining superoperators as

$$\begin{aligned} J_i\tilde{\rho} &= 2\gamma(N_1 + 1)a_i\tilde{\rho}a_i^+ = \gamma_1 a_i\tilde{\rho}a_i^+, \\ L_i\tilde{\rho} &= 2\gamma N_1 a_i^+\tilde{\rho}a_i^+ = \gamma_2 a_i^+\tilde{\rho}a_i^+, \\ O_i\tilde{\rho} &= -(\gamma_1 + \gamma_2)(a_i^+ a_i\tilde{\rho} + \tilde{\rho}a_i^+ a_i) \quad (i = 1, 2), \end{aligned} \tag{12}$$

we have the formal solution of Eq.(11) as follows:

$$\begin{aligned}\tilde{\rho}(t) =& \exp(-4\gamma_2 t)\exp(J_1 + L_1 + O_1) \\ &\times \exp(J_2 + L_2 + O_2)\tilde{\rho}(0).\end{aligned} \tag{13}$$

By disentangling, Eq.(13) reduces to

$$\begin{aligned}\tilde{\rho}(t) =& \exp(-4\gamma_2 t)\exp(2g_3)\exp[g_2(L_1 + L_2)] \\ &\times \exp[g_0(O_1 + O_2)]\exp[g_1(J_1 + J_2)]\tilde{\rho}(0),\end{aligned} \tag{14}$$

with

$$g_0 = \frac{\kappa}{r_1 + \gamma_2}t + \frac{1}{\gamma_1 + \gamma_2}\ln\left[\frac{\kappa + G}{\kappa}\right],$$

$$g_1 = g_2 = \frac{1}{2} \times \frac{G}{\kappa + G},$$

$$g_3 = 2\gamma_2 t - \ln\left(\frac{\kappa + G}{\kappa}\right), \qquad (15)$$

where $G = [1 - \exp(-2\kappa t)]\gamma_2$. Since the density operator of the initial state can be expressed as $S(r_s, \varphi)|00\rangle\langle 00|S^+(r_s, \varphi)$, we finally obtain

$$\rho(t) = \left(\frac{\kappa}{\kappa + G}\right)^2 \sum_{m,n=0}^{\infty} \left(\frac{G}{G + \kappa}\right)^{m+n} \times S(r_s, \varphi)|m, n\rangle\langle m, n|S^+(r_s, \varphi). \qquad (16)$$

Obviously the above equation is just a two-mode STS. So we can study the evolution of the initial TMSV in the correlated environment using the Bures distance.

Here we review in brief the notation of fidelity and Bures distance. A good quantum communication channel must be able to transfer the output quantum states which are close to the input states. To quantify this closeness, the idea of fidelity is introduced,[20] i.e. $F(\rho_1, \rho_2) = [\mathrm{Tr}\sqrt{\sqrt{\rho_1}\rho_2\sqrt{\rho_1}}]^2$. Here ρ_1 and ρ_2 are two quantum states. A fidelity of unity implies two identical states whereas a fidelity of zero implies two orthogonal states. The Bures distance $d_B(\rho_1, \rho_2)$, which is related to the fidelity, is defined as $d_B(\rho_1, \rho_2) = [2 - 2\sqrt{F(\rho_1, \rho_2)}]^{\frac{1}{2}}$.[20] It is proved to be a reliable measure of the entanglement of a two-mode STS.[15] For a given two-mode STS with a density operator ρ, its amount of entanglement is

$$E(\rho) = \min\frac{1}{2}d_B^2(\rho, \rho') = 1 - \max\sqrt{F(\rho, \rho')}, \qquad (17)$$

where $\rho' \in R_0$, and R_0 is the reference set of all separable two-mode STS. If the state ρ coincides with the reference state ρ', we have $F(\rho, \rho') = 1$ and $E(\rho) = 0$. For a symmetric two-mode STS such as Eq.(5), the amount of entanglement[15]

$$E = 1 - \frac{\sqrt{2\sigma}}{\sigma + 1/2}, \qquad (\sigma = D_0 - |D_1|), \qquad (18)$$

where D_0 and D_1 are defined by Eqs.(6) and (7), respectively.

Figure 1(a) shows that when $|M| < N$, E is a decreasing function of time t. After going through an entanglement period t_{en}, the initial TMSV will disentangle, i.e., $E = 0$. According to Eq.(18), we can obtain the entanglement period $t_{en} = \frac{1}{2\gamma}\ln\left[1 + \frac{1 - e^{-2r_s}}{2(N - |M|)}\right]$, which is equal to t'_{en} that obtained in Section 2. Obviously, t_{en} gets longer with the increase of $|M|$. Figure 1(b) shows that when $|M| \geq N$, $E > 0$ holds all the time. Moreover, when $N + (1 - e^{-2r_s})/2 < |M| \leq \sqrt{N(N+1)}$, E is not only positive all the time but also increases as time evolutes.

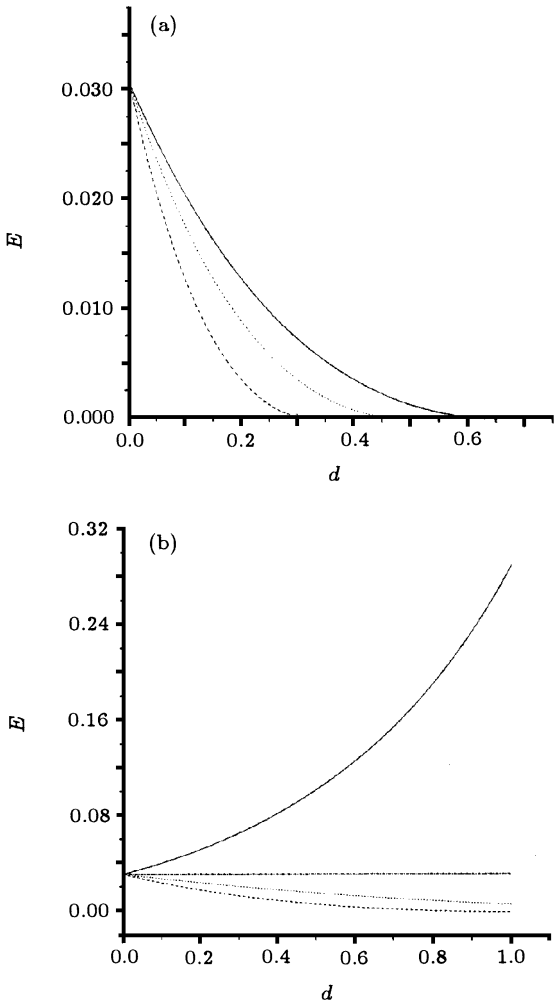


Fig.1. (a) Time evolution of E versus $d(\equiv 1 - \exp(-2\gamma t))$ when $|M| < N$ for $N = 1.0$, $r_s = 0.5$, $|M| = 0.7$ (dashed), $|M| = 0.8$ (dotted), $|M| = 0.9$ (solid). (b) Time evolution of E versus $d(\equiv 1 - \exp(-2\gamma t))$ when $|M| \geq N$ for $N = 1.0$, $r_s = 0.5$, $|M| = 1.0$ (dashed), $|M| = 1.2$ (dotted), $|M| = 1.30$ (dot-dashed), and $|M| = \sqrt{2}$ (solid).

4. Conclusion

We have studied consider the time evolution of an initial TMSV in the correlated noisy channels. By using the Peres–Simom separability and the Bures distance to discuss the entanglement, the dynamical behaviour of the entanglement in the initial TMSV in the noisy channels is obtained analytically. The results obtained in the above two ways are in agreement with each other. It shows that the correlated environment is helpful for preserving the entanglement in the initial TMSV. The entanglement period of the initial state in the correlated noisy channels depends on the correlation property of such a correlated environment.

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