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Proposal for quantum entanglement of six photons*

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We propose a different scheme to achieve six-photon entangled states based entirely on the concept of quantum erasure. To begin with, a scheme for making use of a group of four entangled photons to generate six-photon entangled states is presented. Then, with the same technique, the preparation of the six-photon entanglement from five-particle entanglement which is then combined with Bell states is considered. Our experimental methods can be used for the investigations of measurement-based quantum computation and multi-party quantum communication. We find that the success probability is determined by the small coefficients of the entangled states.

Keywords: entanglement, Bell states, polarizing beam splitter (PBS), quantum erasure, the trigger particles

PACC: 0365, 4250

1. Introduction

Entanglement is one of the most striking features of quantum physics. It serves to demonstrate fundamental quantum properties, which are beyond the conceptual framework defined by classical physics. Aside from its theoretical importance, we point out its promising practical applications in diverse areas such as quantum cryptography,[1] quantum teleportation,[2] quantum dense coding,[3] and precise measurements. Entanglements of six or more particles are fascinating quantum systems, especially when the entanglement is maximal. For example, if the polarizations of six particles are maximally entangled, as in Greenberger–Horne–Zeilinger (GHZ) state,[4] then, according to quantum mechanics, each of them is unpolarized. However, there are perfect correlations among the six particles: if the results of arbitrary polarization measurements on five of the particles are given, one can predict with certainty an appropriate measurement on the sixth particle. This feature seems to imply that each particle possesses many realistic Einstein–Podolsky–Rosen (EPR) elements.[5] However, introduction of these realistic elements implies a contradiction.[4] It would be interesting to experimentally exhibit the dance of correlations present in the six-particle entanglement.

Most of the earlier proposals employ interaction between the particles to achieve entanglement.[6−17] Recently, quantum-mechanical entanglement of four or five particles has been achieved experimentally, and used to demonstrate the extreme contradiction between quantum mechanics and local reality. However, the realization of six-particle entanglement remains an experimental challenge. The ability to manipulate the entanglement of six or more particles is required for universal quantum error correction.

In this regard, in this paper we propose a realizable method based entirely on the concept of quantum erasure.[18−24] First of all, we present a general scheme and realizable procedures for generating the six-photon entanglements out of just a group of four entangled photons from emission.[25] The basic idea is to set up such an arrangement that all the information about the source of two of the eight photons is erased. This can entangle the other six particles as they travel to their observation station. Alternatively, with the same technique, we propose a scheme for observing the six-particle entanglement which can be obtained from a five-particle entanglement.

2. Scheme of the six-photon entanglement from a group of four entangled photons

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Let us consider the arrangement in Fig.1. The four-particle sources, A and B, pumped by short pulses, each emit four photons in the superposition

$$\alpha|HHHH\rangle + \beta|VVVV\rangle,$$
$$|\alpha|^2 + |\beta|^2 = 1(\alpha, \beta \in c), \tag{1}$$

where V and H refer to the horizontal and vertical linear polarizations, respectively.

Take 0 or 1 to represent H or V polarizing state, that is

$$\alpha|0000\rangle + \beta|1111\rangle. \tag{2}$$

By using half-wave plates in $\alpha|0000\rangle+\beta|1111\rangle$ and performing the following transformation

$$|0\rangle \to |1\rangle, \quad |1\rangle \to |0\rangle, \tag{3}$$

the state $\beta|0000\rangle + \alpha|1111\rangle$ can be obtained.

Then the initial state of the system is assumed to be in the following form:

$$\begin{aligned}|\varphi\rangle =&(\alpha|0000\rangle + \beta|1111\rangle)_{abcd}\\ &\otimes(\beta|0000\rangle + \alpha|1111\rangle)_{efgh}\\ =&\alpha\beta|00000000\rangle_{abcdefgh} + \alpha^2|00001111\rangle_{abcdefgh}\\ &+ \beta^2|11110000\rangle_{abcdefgh}\\ &+ \alpha\beta|11111111\rangle_{abcdefgh},\end{aligned} \tag{4}$$

where the letters a, b, c, $\cdots$, and h represent the corresponding beams. All beams have the same polarization.[26]

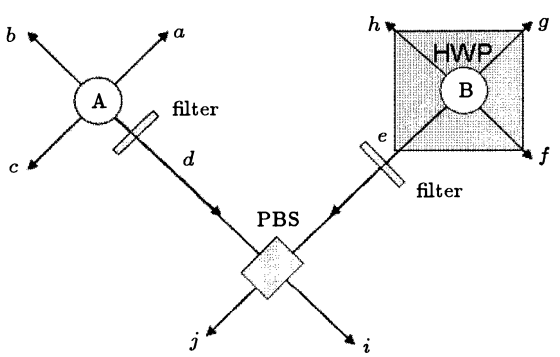


Fig.1. The four-particle sources A and B, pumped by short pulses, each emits four photons in the superposition $\alpha|HHHH\rangle + \beta|VVVV\rangle$. By using HWP in the four particles of source B we can obtain the state $\beta|0000\rangle + \alpha|1111\rangle$. The letters a, b, c, $\cdots$, and j represent beams taken by the particles. All beams have the same polarization. The beams d and e are mixed by a 50–50 PBS where the path lengths of photons each have been adjusted so that they can arrive simultaneously. The i and j represent two photons from different routes. To erase these opportunities to identify the trigger source we place narrow filters in the source beams i and j.

The beams d and e are mixed by a 50–50 polarizing beam splitter (PBS)[27] where the path lengths of photons each have been adjusted so that they can arrive simultaneously. The PBS performs the following operation:

$$\begin{aligned}&|0\rangle_d|0\rangle_e \to |0\rangle_i|0\rangle_j,\\ &|0\rangle_d|1\rangle_e \to |0\rangle_j|1\rangle_j,\\ &|1\rangle_d|0\rangle_e \to |0\rangle_i|1\rangle_i,\\ &|1\rangle_d|1\rangle_e \to |1\rangle_i|1\rangle_j.\end{aligned} \tag{5}$$

After the translation, the initial state becomes

$$\begin{aligned}|\psi\rangle =&\alpha\beta|000000\rangle_{abcfgh}|00\rangle_{ij}\\ &+ \alpha\beta|111111\rangle_{abcfgh}|11\rangle_{ij}\\ &+ \alpha^2|000111\rangle_{abcfgh}|01\rangle_{jj}\\ &+ \beta^2|111000\rangle_{abcgh}|01\rangle_{ii}.\end{aligned} \tag{6}$$

Since

$$\begin{aligned}&\alpha\beta|000000\rangle_{abcfgh}|00\rangle_{ij} + \alpha\beta|111111\rangle_{abcfgh}|11\rangle_{ij}\\ =&\frac{1}{\sqrt{2}}\alpha\beta(|000000\rangle + |111111\rangle)_{abcfgh}\\ &\otimes\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{ij}\\ &+\frac{1}{\sqrt{2}}\alpha\beta(|000000\rangle - |111111\rangle)_{abcfgh}\\ &\otimes\frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)_{ij},\end{aligned} \tag{7}$$

thus

$$\begin{aligned}|\psi\rangle =&\alpha\beta|00000000\rangle_{abcfghij} + \alpha^2|00011101\rangle_{abcfghjj}\\ &+ \beta^2|11100001\rangle_{abcfghii} + \alpha\beta|11111111\rangle_{abcfghij}\\ =&\frac{1}{\sqrt{2}}\alpha\beta(|000000\rangle + |111111\rangle)_{abcfgh}\\ &\otimes\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{ij}\\ &+\frac{1}{\sqrt{2}}\alpha\beta(|000000\rangle - |111111\rangle)_{abcfgh}\\ &\otimes\frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)_{ij}\\ &+\alpha^2|00011101\rangle_{abcfghjj} + \beta^2|11100001\rangle_{abcfghii}.\end{aligned} \tag{8}$$

To erase these opportunities of identifying the trigger source, we place narrow filters in the source beams i and j, and use different Bell states $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ or $\frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$ to measure the photons in the paths i and j (the trigger particles).[28] If

the state is $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$, the six photons are located in $\frac{1}{\sqrt{2}}(|000000\rangle + |111111\rangle)$. If the state is $\frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$, the six photons are located in $\frac{1}{\sqrt{2}}(|000000\rangle - |111111\rangle)$. If there are two photons in path i or in path j, then the scheme is unsuccessful. The probability of success is $2|\alpha\beta|^2$.

In order to generate the desired six-photon entangled states

$$\frac{1}{\sqrt{2}}(|000000\rangle + |111111\rangle)$$

and

$$\frac{1}{\sqrt{2}}(|000000\rangle - |111111\rangle),$$

all ways one might in principle identify true photons must be erased. For emitting a group of four-polarization-entangled photons, the source must include a parent particle at the beginning whose momentum is definite to some extent. That parent subsequently decays into four daughters. The momentum conservation produces the desired entanglement. But particles produced in this way in general have correlations in polarization, energy, and time. Any of them may be in principle exploited to identify the true sibling and hence the source of the trigger particle. And thereby the entanglement is prevented from forming. However, polarization correlations cannot be exploited if all particles from the two sources simply carry the same polarization. Energy correlations cannot be exploited if all eight particles carry the same energy or, more generally, if the energy correlations of true particles (emitted by the same source) are indistinguishable from a mixed particle (one particle from each source). Similarly, temporal correlations cannot be exploited if all eight particles are produced or detected at the same time or, more generally, if the temporal correlations of true group and of mixed group are indistinguishable from each other.

Can the scheme just outlined really be realized with existing laboratory techniques? Clearly, it depends on whether the following three necessary requirements are fulfilled: (I) availability of four-particle eight-beam entanglement sources A and B with the intensities so sufficient that occasionally both of them can emit four photons, and (II) realizable techniques for ensuring that the source A or B of the trigger is unknowable, and (III) feasibility of two sources A and B to generate the same polarization-entangled state $\alpha|HHHH\rangle+\beta|VVVV\rangle$. The four-particle eight-beam entanglement has already been demonstrated in the laboratory using a parametric down-conversion source (the polarizations of the photons are identical). Because of the PBS and the narrow filters the sources of the photons i and j are essentially unknowable. The requirements I and II have been achieved in laboratories. One could, in principle, fulfill the requirement III. With recent progress in the techniques and reforming equipment, the requirement should be met in the near future.

3. Scheme of the six-particle entanglement from five-particle entanglement

Recently, experimental demonstration of five-photon entanglement has been performed by Pan and his co-workers.[29] Now with the above technique we propose a scheme for observing the six-particle entanglement which is obtained from the five-particle entanglement.

Consider the arrangement as shown in Fig.2. Assume that a five-particle entangled state and a Bell state are given, the initial product state is

$$|\Phi\rangle = (\alpha|00000\rangle + \beta|11111\rangle)_{abcde} \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{fg}, \tag{9}$$

where $|\alpha|^2 + |\beta|^2 = 1$, $(\alpha, \beta \in c)$. Consequently, we have the results

$$\begin{aligned}|\Phi\rangle =& \frac{\alpha}{\sqrt{2}}|0000000\rangle_{abcdefg} + \frac{\beta}{\sqrt{2}}|1111100\rangle_{abcdefg} \\ &+ \frac{\alpha}{\sqrt{2}}|0000011\rangle_{abcdefg} + \frac{\beta}{\sqrt{2}}|1111111\rangle_{abcdefg}.\end{aligned} \tag{10}$$

Here the subscripts a, b, c, $\cdots$, and g represent the corresponding beams. All beams have the same polarization.

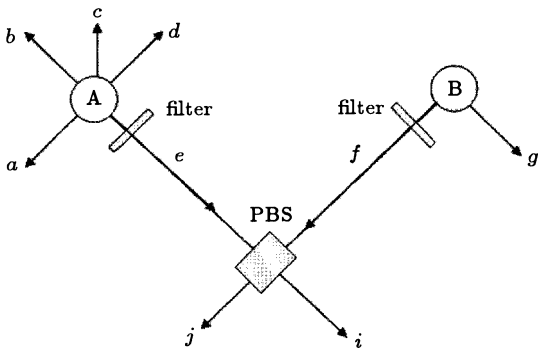


Fig.2. The source A emits five particles in the superposition $\alpha|00000\rangle+\beta|11111\rangle$. The source B emits two polarization-entangled photons which are in the state $\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)$. The letters a, b, c, $\cdots$, and j represent beams taken by the particles. All beams have the same polarization. The beams e and f are mixed by a 50/50 PBS where the path lengths of photons each have been adjusted so that they arrive simultaneously. The i and j represent two photons from different routes. To erase these opportunities to identify the trigger source we place narrow filters in the source beams i and j.

The beams e and f are mixed by a 50/50 PBS where the path lengths of photons each have been adjusted so that they can arrive simultaneously.

Then the following operation is carried out via PBS

$$
\begin{aligned}
&|0\rangle_e|0\rangle_f \rightarrow |0\rangle_i|0\rangle_j,\\
&|1\rangle_e|0\rangle_f \rightarrow |0\rangle_i|1\rangle_i,\\
&|0\rangle_e|1\rangle_f \rightarrow |0\rangle_j|1\rangle_j,\\
&|1\rangle_e|1\rangle_f \rightarrow |1\rangle_i|1\rangle_j,
\end{aligned}
\tag{11}
$$

where i and j represent two photons from different routes.

The above operation can transform the initial state into the following form

$$
\begin{aligned}
|\psi\rangle =&\frac{1}{\sqrt{2}}(\alpha|0000000\rangle+\beta|1111111\rangle)_{abcdijg}\\
&+\frac{\alpha}{\sqrt{2}}|0000011\rangle_{abcdjjg}\\
&+\frac{\beta}{\sqrt{2}}|1111010\rangle_{abcdiig}.
\end{aligned}
\tag{12}
$$

For the first term on the right-hand side of Eq.(12), we have

$$
\begin{aligned}
&(\alpha|0000000\rangle+\beta|1111111\rangle)_{abcdijg}\\
=&\frac{1}{\sqrt{2}}(\alpha|000000\rangle+\beta|111111\rangle)_{abcdij}\\
&\otimes\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)_g\\
&+\frac{1}{\sqrt{2}}(\alpha|000000\rangle-\beta|111111\rangle)_{abcdij}\\
&\otimes\frac{1}{\sqrt{2}}(|0\rangle-|1\rangle)_g.
\end{aligned}
\tag{13}
$$

So the initial state is

$$
\begin{aligned}
|\psi\rangle =&\frac{1}{2}(\alpha|000000\rangle+\beta|111111\rangle)_{abcdij}\\
&\otimes\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)_g\\
&+\frac{1}{2}(\alpha|000000\rangle-\beta|111111\rangle)_{abcdij}\\
&\otimes\frac{1}{\sqrt{2}}(|0\rangle-|1\rangle)_g\\
&+\frac{\alpha}{\sqrt{2}}|0000011\rangle_{abcdjjg}\\
&+\frac{\beta}{\sqrt{2}}|1111010\rangle_{abcdiig}.
\end{aligned}
\tag{14}
$$

We use the different states $\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$ or $\frac{1}{\sqrt{2}}(|0\rangle-|1\rangle)$ to measure the photon in the path g (the trigger particles). It is obviously that if there is one photon in each of the paths a, b, c, d, i and j, the six photons are located in non-maximally entangled state. If the measured state of the photons of the path g is $\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$, the six photons of the paths a, b, c, d, i and j are located in the state $\alpha|000000\rangle+\beta|111111\rangle$. Otherwise, the measured state is $\frac{1}{\sqrt{2}}(|0\rangle-|1\rangle)$, and it will be located in the state $\alpha|000000\rangle-\beta|111111\rangle$.

If there are two photons or no photon in the path g, then the execution of state is unsuccessful. Obviously the probability of six-particle non-maximally entangled state in this scheme is 1/2.

With the above techniques the six-particle polarization-entanglements (analogous to the two-particle Bell states) would easily be produced if five-photon polarization entanglements can be the source.

4. Conclusions

In summary, by using this scheme, in principle, an n-particle entanglement can be obtained from both (I)the n-1-particle entanglement which is then combined with a Bell state and (II) two $\frac{n}{2}+1$-particle entanglements for even integer n or two $\frac{n+1}{2}$-particle

entanglements for odd integer n. But the precondition for realizing the above scheme is that (I) or (II) can be the particle source. What remains to be solved will come true based on improving the technology and reforming equipment. However, this general scheme and the procedures are a realizable method. The realization of six-particle entanglement in laboratory will pave the way for the study of many novel quantum phenomena and their applications. The relevant studies may include: (A) demonstration of GHZ correlations;[1] (B) demonstration of entangled entanglement,[30] and (C) multi-particle quantum communication schemes.[31−35]

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