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Implementing remotely a single-qubit rotation operation by three-qubit entanglement*

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We investigate the problem of quantum remote implementation of a single-qubit rotation operation using three-qubit entangled state. Firstly, we utilize the entanglement property of maximally entangled Greenberger–Horne–Zeilinger (GHZ) state to design a theoretical scheme for implementing the operation remotely with unit fidelity and unit probability. Then, we put forward two schemes for conclusive implementing the non-local single-qubit rotation with unit fidelity by employing a partially entangled pure GHZ state as quantum channel. The features of these schemes are that a third side is included, who may participate the process of quantum remote implementation as a supervisor. Furthermore, when the quantum channel is partially entangled, the third side can rectify the state distorted by imperfect quantum channel. In addition to the GHZ class state, the W class state can also be used to remotely implement the same operation probabilistically. The probability of successful implementation using the W class state is always less than that using the GHZ class state.

Keywords: single-qubit rotation, remote implementation, GHZ class state, W class state
PACC: 0365, 4250

1. Introduction

Entanglement is the key physical resource in quantum information processing. Many novel results have been obtained by using quantum non-locality of entangled particle. In recent years, much attention has been paid to the issue of non-locality of quantum states.[1−8] In fact, an equally important issue is that of non-locality of quantum operations. This situation arises, for example, in the context of distributed quantum computation,[9] which can be viewed as a multiprocessor device where each processor contains only a small number of qubits and acts as a node of quantum communication network. To realize distributed quantum computation, not only quantum state but also quantum operation has to be transferred between the nodes of the network efficiently. Hence optimal remote implementation of quantum operations using local operations, classical communication and shared entanglement (LOCCSE) becomes an important problem in quantum computation and quantum information, and some fundamental researches on remote implementing quantum operations have been published.[10−17]

It is proved that any unitary operation on N qubits may be implemented exactly by composing single-qubit rotations and control-Not (CNOT) gates.[9] For this reason Eisert *et al*[11] and Collins *et al*[12] studied non-local CNOT gate first in 2000 and found that a non-local CNOT gate can be implemented by LOCCSE and consumes only one maximally entangled pair of qubits and two bits of classical communication. In 2001, Huelga *et al*[13] studied the remote implementation of a unitary transformation on a qubit and showed the existence of non-trivial protocols (i.e., using less resource than bidirectional state teleportation) which allow the perfect remote implementation of certain continuous sets of quantum operation. Thereafter, a series of researches of some special operations[14−17] and arbitrary operations[10,18] have been performed. However, all the previous works deal with the problem using entangled pair as quantum channel. In recent years, moving onwards from two-particle entangled state, much attention has been devoted to multiple-particle entangled state,[19,20] especially the Greenberger–Horne–

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Zeilinger (GHZ) state $|\mathrm{GHZ}\rangle = 1/\sqrt{2}(|000\rangle + |111\rangle)$ and the W state $|W\rangle = 1/\sqrt{3}(|001\rangle + |010\rangle + |100\rangle)$, which have also been shown to have advantages over the two-particle Bell states in their application to cloning,[21] teleportation,[22] and dense coding.[23] In Refs.[20,24], Dur *et al* pointed out that the GHZ and W states exhibit very different properties when subjected to physical processes like state loss, or white noise.

In this paper, we firstly utilize the entanglement property of GHZ class state to design some theoretical schemes for implementing a single-qubit rotation remotely. According to the schemes, a sender may teleport a "rotation" to one of two receivers, and all of them must collaborate, by exchanging classical information, in order to implement a single-qubit rotation remotely. We then study the consequences of replacing the three-qubit GHZ class states in the schemes with three-qubit W class states. These schemes are very simple and maybe useful to the future quantum computers.

We will restrict our attention to the rotations about the $\hat{Z}$ axis

$$U_\varphi \equiv \exp(-\mathrm{i}\varphi\sigma_z) = \begin{pmatrix} \mathrm{e}^{-\mathrm{i}\frac{\varphi}{2}} & 0 \\ 0 & \mathrm{e}^{\mathrm{i}\frac{\varphi}{2}} \end{pmatrix} \tag{1}$$

for an arbitrary angle $\varphi \in (0, 2\pi]$. Note that

$$U_\pi = \begin{pmatrix} -\mathrm{i} & 0 \\ 0 & \mathrm{i} \end{pmatrix} = \mathrm{i}Z,$$

$$U_{\pi/4} = \mathrm{e}^{-\mathrm{i}\pi/8}\begin{pmatrix} 1 & 0 \\ 0 & \mathrm{e}^{\mathrm{i}\pi/4} \end{pmatrix} = \mathrm{e}^{\mathrm{i}\pi/8}T,$$

and

$$U_{\pi/4}^2 = \mathrm{e}^{\mathrm{i}\pi/4}\begin{pmatrix} 1 & 0 \\ 0 & \mathrm{i} \end{pmatrix} = \mathrm{e}^{\mathrm{i}\pi/4}S.$$

The Pauli operator Z, $\pi/8$ gate T, and phase gate S are some of the most important essential gates.

Consider that Alice, Bob and Cindy are localized in different space domains, and are able to perform any local operations. They also have additional resources, namely they share an entangled state, and they are able to communicate classically.

Suppose Alice possesses a device which is able to perform U_φ, whereas Bob has a qubit in the unknown state $|B\rangle_B = \alpha|0\rangle_B + \beta|1\rangle_B$ and Cindy has $|C\rangle_C = \gamma|0\rangle_C + \eta|1\rangle_C$. Their goal is that Bob or Cindy, say Bob, ends up with the processed state

$$U_\varphi|B\rangle_B = \alpha\mathrm{e}^{-\mathrm{i}\frac{\varphi}{2}}|0\rangle_B + \beta\mathrm{e}^{\mathrm{i}\frac{\varphi}{2}}|1\rangle_B, \tag{2}$$

which is supervised by the other, say Cindy.

2. Implementing remotely a single-qubit rotation by GHZ class state

2.1. Scheme for remotely implementing a single-qubit rotation by GHZ state

In order to complete the above transformation, an ancillary three-qubit entangled GHZ state $|\mathrm{GHZ}\rangle_{abc} = 1/\sqrt{2}(|000\rangle_{abc} + |111\rangle_{abc})$ is prepared and shared by Alice with a, Bob with b, and Cindy with c, and the following operations have to be performed.

Step 1 Taking B as the control qubit and b the target qubit, Bob applies a local CNOT operation on B and b to give

$$\begin{aligned} |B\rangle_B|\mathrm{GHZ}\rangle_{abc} \rightarrow & 1/\sqrt{2}[(\alpha|000\rangle_{Bac} + \beta|111\rangle_{Bac})|0\rangle_b \\ & + (\sigma_X)_a(\sigma_X)_c(\alpha|000\rangle_{Bac} \\ & + \beta|111\rangle_{Bac})|1\rangle_b]. \end{aligned} \tag{3}$$

Here, and in what follows, all the mid-states have been normalized. Now Bob performs a computation basis measurement on qubit b. After that, he will broadcast the result of his measurement, so that the state of qubits B, a and c can be transformed (according to the two possible results $|0\rangle_b$ and $|1\rangle_b$, the corresponding transformations are $I_a \otimes I_c$ and $(\sigma_X)_a \otimes (\sigma_X)_c$, respectively) to a common form

$$1/\sqrt{2}(\alpha|000\rangle_{Bac} + \beta|111\rangle_{Bac}). \tag{4}$$

Step 2 Alice applies the operation U_φ onto her qubit a followed by a Hadamard transformation. After that, Alice measures the state of qubit a. When the result is $|1\rangle_a$, the state is

$$\begin{aligned} & 1/2(\alpha\mathrm{e}^{-\mathrm{i}\frac{\varphi}{2}}|00\rangle_{Bc} - \beta\mathrm{e}^{\mathrm{i}\frac{\varphi}{2}}|11\rangle_{Bc}) \\ = & 1/2(\sigma_Z)_B(\alpha\mathrm{e}^{-\mathrm{i}\frac{\varphi}{2}}|00\rangle_{Bc} + \beta\mathrm{e}^{\mathrm{i}\frac{\varphi}{2}}|11\rangle_{Bc}). \end{aligned} \tag{5}$$

Bob needs to perform σ_Z on qubit B to obtain

$$1/2(\alpha\mathrm{e}^{-\mathrm{i}\frac{\varphi}{2}}|00\rangle_{Bc} + \beta\mathrm{e}^{\mathrm{i}\frac{\varphi}{2}}|11\rangle_{Bc}). \tag{6}$$

Otherwise, no operation is performed on it. Considering this state, we can see that, at this moment, Bob cannot obtain the processed state of Eq.(2) without the cooperation of Cindy.

Step 3 Cindy should measure her qubit c on the bases of $1/\sqrt{2}(|0\rangle + |1\rangle)_c$ and $1/\sqrt{2}(|0\rangle - |1\rangle)_c$, and

transfer the result of her measurement to Bob. Here the state of Eq.(6) can be written as

$$\begin{aligned}&1/2(\alpha e^{-i\frac{\varphi}{2}}|00\rangle_{Bc}+\beta e^{i\frac{\varphi}{2}}|11\rangle_{Bc})\\=&1/\sqrt{8}(\alpha e^{-i\frac{\varphi}{2}}|0\rangle_B+\beta e^{i\frac{\varphi}{2}}|1\rangle_B)1/\sqrt{2}(|0\rangle+|1\rangle)_c\\&+(\sigma_Z)_B1/\sqrt{8}(\alpha e^{-i\frac{\varphi}{2}}|0\rangle_B\\&+\beta e^{i\frac{\varphi}{2}}|1\rangle_B)1/\sqrt{2}(|0\rangle-|1\rangle)_c.\end{aligned}\tag{7}$$

As soon as Bob is informed of Cindy's result, he can perform an appropriate unitary transformation (according to the two possible result $1/\sqrt{2}(|0\rangle+|1\rangle)_c$ and $1/\sqrt{2}(|0\rangle-|1\rangle)_c$, the corresponding transformations are I_B and$(\sigma_Z)_B$) on qubit B to obtain the state of Eq.(2).

Summing up all conditions (8 kinds in all), we obtain that the probability of successful implementation is $P=(1/\sqrt{8})^2\times 8=1$. We have thus shown how to implement a single-qubit rotation operation remotely with unit fidelity and unit probability.

2.2. Schemes for remotely implementing a single-qubit rotation by general GHZ state

In realistic conditions, instead of maximally entangled states, qubits a, b and c may share partially entangled GHZ state. This means that the quantum channel will be imperfect and the scheme described above will no longer work well. To complete the non-local single-qubit rotation operation via a partially entangled channel, we have to correct the state distorted by the imperfect quantum channel. Here we provide the following strategy (scheme 2.2 and scheme 2.3). We suppose that the ancillary entangled state shared by Alice, Bob and Cindy is $|\mathrm{GHZ}'\rangle_{abc}=(a|000\rangle+b|111\rangle)_{abc}$ (where $|a|^2+|b|^2=1,|a|>|b|$). The initial state of the whole system is $|B\rangle_B|\mathrm{GHZ}'\rangle_{abc}$.

Step 1–Step 2 The first two steps of the two schemes are exactly the same as those shown in Section 2.1. After this, the state $|B\rangle_B|\mathrm{GHZ}'\rangle_{abc}$ becomes

$$1/\sqrt{2}(a\alpha e^{-i\frac{\varphi}{2}}|00\rangle_{Bc}+b\beta e^{i\frac{\varphi}{2}}|11\rangle_{Bc})$$

(in step 1, if Bob′s measurement result is $|0\rangle_b$), (8a)

or

$$1/\sqrt{2}(b\alpha e^{-i\frac{\varphi}{2}}|00\rangle_{Bc}+a\beta e^{i\frac{\varphi}{2}}|11\rangle_{Bc})$$

(in step 1, if Bob′s measurement result is $|1\rangle_b$). (8b)

Suppose Bob's measurement result is $|0\rangle_b$ in step 1. Different from the state of Eq.(6), the above state includes the parameters of the imperfect quantum channel, a and b, and the state of particles B and c have been distorted by the imperfect channel. In order to distill state (2) from state (8), we must give some evolution to rectify the distorted state.

Step 3 (for scheme 2.2) The goal of scheme 2.2 is to modify Cindy's measurement part so that after Cindy carries out her specific measurement and communicates her result, the state of Bob will be given by (2).

Cindy now prepares an ancilla qubit c' in the state

$$|\chi\rangle_{c'}=|0\rangle_{c'}+|1\rangle_{c'}.\tag{9}$$

The combined state of the three qubits is given by

$$\begin{aligned}|\psi\rangle_{Bcc'}=&1/\sqrt{2}(a\alpha e^{-i\frac{\varphi}{2}}|00\rangle_{Bc}\\&+b\beta e^{i\frac{\varphi}{2}}|11\rangle_{Bc})(|0\rangle_{c'}+|1\rangle_{c'})\\=&1/\sqrt{8}\{[U_\varphi(\alpha|0\rangle+\beta|1\rangle)_B](a|00\rangle+b|11\rangle)_{cc'}\\&+[(\sigma_Z)_BU_\varphi(\alpha|0\rangle+\beta|1\rangle)_B](a|00\rangle-b|11\rangle)_{cc'}\\&+[U_\varphi(\alpha|0\rangle+\beta|1\rangle)_B](a|01\rangle+b|10\rangle)_{cc'}\\&+[(\sigma_Z)_BU_\varphi(\alpha|0\rangle+\beta|1\rangle)_B]\\&\times(a|01\rangle-b|10\rangle)_{cc'}\}.\end{aligned}\tag{10}$$

Now measurement on Cindy's side takes place in two steps. The first measurement projects the state onto either of the subspaces spanned by $\{|00\rangle,|11\rangle\}$ or $\{|01\rangle,|10\rangle\}$. Thus this measurement has two possible outcomes that occur with equal probability. Suppose the result is in the subspace spanned by $\{|00\rangle,|11\rangle\}$. Cindy now performs an optimal positive operator value measure (POVM),[9] which provides the most general physically realizable measurement in quantum mechanics. The generalized measurement can distinguish conclusively between the two nonorthogonal states $(a|00\rangle+b|11\rangle)$ and $(a|00\rangle-b|11\rangle)$. Having the measurement result on qubit c and c', Bob can transform the state of his qubit into the state of Eq.(2). Because there is always a possibility of an inconclusive result in the state discrimination procedure, the total probability of successful implementation is $2|b|^2$. Note that for $b=1/\sqrt{2}$, which corresponds to a maximally entangled channel, the proposal is always successful with certainty as there need not be any inconclusive result since in this case one discriminates between two orthogonal states.

The previous proposal suggested change in the type of measurement by Cindy. In the following proposal Cindy has to remove the channel dependence to recover the states before performing her local operation to complete the remote implementation of single-qubit rotation.

Step 3 (for scheme 2.3) An auxiliary qubit with the original state $|0\rangle_{c'}$ is introduced, and a collective unitary transformation on the basis $\{|00\rangle, |10\rangle, |01\rangle, |11\rangle\}_{cc'}$ is constructed.[3] For the state of Eq.(8a) and Eq.(8b), the transformation can be taken as

$$T_1^a = \begin{pmatrix} b/a & 0 & \sqrt{1-(b/a)^2} & 0 \\ 0 & 1 & 0 & 0 \\ \sqrt{1-(b/a)^2} & 0 & -b/a & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{11a}$$

and

$$T_1^b = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & b/a & 0 & \sqrt{1-(b/a)^2} \\ 0 & 0 & -1 & 0 \\ 0 & \sqrt{1-(b/a)^2} & 0 & -b/a \end{pmatrix}. \tag{11b}$$

Cindy applies T_1^a on qubits c and c', and state (8a) and 8(b) are transformed to the result

$$\frac{b}{\sqrt{2}}(\alpha e^{-i\frac{\varphi}{2}}|00\rangle_{Bc} + \beta e^{i\frac{\varphi}{2}}|11\rangle_{Bc})|0\rangle_{c'} + \frac{\alpha}{\sqrt{2}}\sqrt{a^2-b^2}e^{-i\frac{\varphi}{2}}|00\rangle_{Bc}|1\rangle_{c'}. \tag{12a}$$

and

$$\frac{b}{\sqrt{2}}(\alpha e^{-i\frac{\varphi}{2}}|00\rangle_{Bc} + \beta e^{i\frac{\varphi}{2}}|11\rangle_{Bc})|0\rangle_{c'} + \frac{\beta}{\sqrt{2}}\sqrt{a^2-b^2}e^{i\frac{\varphi}{2}}|11\rangle_{Bc}|1\rangle_{c'}. \tag{12b}$$

After these operations, a measurement on the auxiliary qubit follows. If the result is $|1\rangle_{c'}$, the correction fails and no information about the initial state is left. If $|0\rangle_{c'}$ is obtained, the correction is successfully accessed. Then Cindy and Bob have the state

$$(\alpha e^{-i\frac{\varphi}{2}}|00\rangle_{Bc} + \beta e^{i\frac{\varphi}{2}}|11\rangle_{Bc}). \tag{13}$$

The probability of success for having the above state either from Eq.(12a) or form Eq.(12b) is $p = |b|^2/2$, hence the total probability of success is $2|b|^2$. It is apparent that the state of Eq.(13) is exactly the same as that of Eq.(6). Hence, following the third step we have given after Eq.(6), the non-local single-qubit rotation operation using the partially entangled GHZ state can be easily completed.

The features of the schemes presented in Section 2 are that a third side (Cindy) is included, who may participate in the process of quantum remote implementation as a supervisor. Without the cooperation (permission) of Cindy, Bob cannot obtain the "rotation" from Alice by himself. What is more, when the quantum channel is partially entangled, Cindy can correct the state distorted by imperfect quantum channel after carrying out her specific operations and communicating her result.

3. Implementing remotely a single-qubit rotation by W class state

3.1. Scheme for remote implementing a single-qubit rotation by W state

Now we again consider the quantum remote implementation of single-qubit rotation operation scheme, but instead of Alice, Bob and Cindy sharing a three-qubit entangled GHZ state, they share a three-qubit entangled W state $|W\rangle_{abc} = 1/\sqrt{3}(|001\rangle + |010\rangle + |100\rangle)_{abc}$, where, qubits a, b and c belong to Alice, Bob and Cindy, respectively.

Step 1 If Bob wants to obtain the "rotation", he applies a local CNOT operation on his qubits with the B acting as the control qubit to give

$$\begin{aligned}|B\rangle_B|W\rangle_{abc} \to & 1/\sqrt{3}(\sigma_X)_a[\alpha|0\rangle_B(|11\rangle + |00\rangle)_{ac} \\ & + \beta|1\rangle_B|10\rangle_{ac}]|0\rangle_b \\ & + 1/\sqrt{3}[\alpha|0\rangle_B|00\rangle_{ac} \\ & + \beta|1\rangle_B(|01\rangle + |10\rangle)_{ac}]|1\rangle_b. \end{aligned} \tag{14}$$

Then Bob does the computation basis measurement on qubit b. If qubit b is projected to the state, for example, $|1\rangle_b$, then qubits B, a and c are projected to the state

$$1/\sqrt{3}[\alpha|0\rangle_B|00\rangle_{ac} + \beta|1\rangle_B(|01\rangle + |10\rangle)_{ac}]. \tag{15}$$

Step 2 After having obtained the classical information about the result of measurement from Bob, Alice performs the operation of the second part of the scheme, which is exactly the same as the one shown in Section 2.1. After that, state of Eq.(15) will become

$$1/\sqrt{6}[(\alpha e^{-i\frac{\varphi}{2}}|0\rangle_B + \beta e^{i\frac{\varphi}{2}}|1\rangle_B)|0\rangle_c + \beta e^{i\frac{\varphi}{2}}|1\rangle_B|1\rangle_c]. \tag{16}$$

Step 3 Alice asks Cindy to do the projection measurement on qubit c in the basis $\{|0\rangle, |1\rangle\}$. If Cindy obtains the result of $|0\rangle_c$, the qubit B will be projected to the state

$$\alpha e^{-i\frac{\varphi}{2}}|0\rangle_B + \beta e^{i\frac{\varphi}{2}}|1\rangle_B. \tag{17}$$

It is the same as the state $U_\varphi|B\rangle_B$. The probability of success is $\frac{1}{6}\times(1+\beta^2)\times\frac{1}{(1+\beta^2)}\times 2=\frac{1}{3}$. But if Cindy obtains the state $|1\rangle_c$, qubit B will be projected to the state $\beta e^{i\frac{\varphi}{2}}|1\rangle_B$, then this scheme fails.

If qubit b is projected to the state $|0\rangle_b$, by using unitary operation σ_X, Bob can obtain the state of Eq.(17) with probability $\frac{1}{6}\times(1+\alpha^2)\times\frac{1}{(1+\alpha^2)}\times 2=\frac{1}{3}$ following the same way. The total probability of this scheme is $\frac{1}{3}\times 2=\frac{2}{3}$.

In Section 2.1, we can teleport a "rotation" to one of two receivers with 100% probability using the GHZ state. But if we use the W state, we can only teleport a "rotation" probabilistically. We can also use another way to remotely implement a single-qubit rotation by the state $|W\rangle_{abc}$. If Bob wants to obtain the "rotation", he can ask Cindy do the projection measurement on qubit c firstly in the basis $\{|0\rangle,|1\rangle\}$,

$$\begin{aligned}&1/\sqrt{3}(|001\rangle+|010\rangle+|100\rangle)_{abc}\\ =&1\sqrt{3}[|00\rangle_{ab}|1\rangle_c+(|01\rangle+|10\rangle)_{ab}|0\rangle_c].\end{aligned}\tag{18}$$

Obviously, if Cindy obtains the result $|0\rangle_c$, qubits a and b will be projected to maximally two-qubit entangled state, then Alice can teleport the "rotation" to Bob with 100% probability by the scheme shown in Ref.[14]. The total probability of success is also 2/3.

3.2. Schemes for remote implementing a single-qubit rotation by general W state

Next, we consider how to use the general three-qubit W state to realize the quantum remote control of single-qubit rotation. Scheme 3.2 and scheme 3.3 are presented as follows. The general W state is

$$|W'\rangle=(a|001\rangle+b|010\rangle+c|100\rangle)_{abc},\tag{19}$$

where $|a|^2+|b|^2+|c|^2=1, |a|>|b|>|c|$. The initial state of the whole system is $|B\rangle_B|W'\rangle_{abc}$.

Step 1–Step 3 The first three steps of the two schemes are exactly the same as the ones shown in Section 3.1. After this, Bob will obtain

$$1/\sqrt{2}(c\alpha e^{-i\frac{\varphi}{2}}|0\rangle_B+b\beta e^{i\frac{\varphi}{2}}|1\rangle_B)$$

(in step 1, if Bob's measurement result is $|0\rangle_b$), (20a)

or

$$1/\sqrt{2}(a\alpha e^{-i\frac{\varphi}{2}}|0\rangle_B+c\beta e^{i\frac{\varphi}{2}}|1\rangle_B)$$

(in step 1, if Bob's measurement result is $|1\rangle_b$), (20b)

But Bob finds that they cannot yet evolve the state $|B\rangle_B|W'\rangle_{abc}$ to the state of Eq.(2) which they need. In order to obtain processed state $U_\varphi|B\rangle_B$, Bob must correct the distorted state.

Step 4 (for scheme 3.2) Suppose Bob's measurement result is $|1\rangle_b$ in the step 1. Bob prepares an ancilla qubit b' in a state $|0\rangle_{b'}$. Thus the combined state of the two qubits that Bob holds is given by

$$|\psi\rangle_{Bb'}=\frac{1}{\sqrt{2}}U_\varphi(a\alpha|00\rangle+c\beta|10\rangle)_{Bb'}.\tag{21}$$

Bob now performs a CNOT operation on his two qubits states (with the control being qubit B), thus transforming it into the state

$$\begin{aligned}|\psi'\rangle_{Bb'}=&1/\sqrt{8}\{(a|0\rangle+c|1\rangle)_B[U_\varphi(\alpha|0\rangle+\beta|1\rangle)_{b'}]\\&+(a|0\rangle-c|1\rangle)_B[Z_{b'}U_\varphi(\alpha|0\rangle+\beta|1\rangle)_{b'}]\}.\end{aligned}\tag{22}$$

From Eq.(22) it is clear that an optimal POVM, which can conclusively distinguish between the two nonorthogonal states $(a|0\rangle+c|1\rangle)$ and $(a|0\rangle-c|1\rangle)$, will give the desired result. Note that the POVM can be carried out on any one of the two qubits that Bob holds. In step 1, if Bob's measurement result is $|0\rangle_b$, it can be discussed in the same way. Summing up all cases, we obtain the total probability of obtaining state of Eq.(2) is $2|c|^2$. Bob can also do the following collective unitary transformation to obtain the state of Eq.(2).

Step 4 (for scheme 3.3) If Bob obtains the state of Eq.(20a), he can do the following unitary transformation on qubit B and an ancilla qubit $|0\rangle_{b'}$ in the basis $\{|00\rangle,|10\rangle,|01\rangle,|11\rangle\}_{Bb'}$. The unitary transformation matrices for Eqs.(20a) and (20b) are respectively

$$T_2^a=\begin{pmatrix}1&0&0&0\\0&c/b&0&\sqrt{1-(c/b)^2}\\0&0&-1&0\\0&\sqrt{1-(c/b)^2}&0&-c/b\end{pmatrix}\tag{23a}$$

and

$$T_2^b=\begin{pmatrix}c/a&0&\sqrt{1-(c/a)^2}&0\\0&1&0&0\\\sqrt{1-(c/a)^2}&0&-c/a&0\\0&0&0&-1\end{pmatrix},\tag{23b}$$

then we obtain

$$c/\sqrt{2}(\alpha e^{-i\frac{\varphi}{2}}|0\rangle_B + \beta e^{i\frac{\varphi}{2}}|1\rangle_B)|0\rangle_{b'} + \alpha/\sqrt{2}\sqrt{b^2-c^2}e^{-i\frac{\varphi}{2}}|0\rangle_B|1\rangle_{b'}. \quad (24a)$$

and

$$c/\sqrt{2}(\alpha e^{-i\frac{\varphi}{2}}|0\rangle_B + \beta e^{i\frac{\varphi}{2}}|1\rangle_B)|0\rangle_{b'} + \beta/\sqrt{2}\sqrt{a^2-c^2}e^{i\frac{\varphi}{2}}|1\rangle_B|0\rangle_{b'}, \quad (24b)$$

i.e., Bob can obtain the processed state $U_\varphi|B\rangle_B$ with certain probability. This probability is also $2|c|^2$. So we conclude that a general W state can be used to realize remote implementation of single-qubit rotation probabilistically. Besides, we also want for Cindy to correct the state distorted by the imperfect quantum channel, but we find that the distorted state cannot be rectified if Cindy follows the same procedure as in schemes 2.2 and 2.3.

4. Conclusion

In conclusion, we have investigated the problem of implementing remotely a single-qubit rotation operation via three-qubit entanglement controlled channel. Two classes of channel have been considered to do the job. We show how the operation can be implemented remotely with unit fidelity and unit probability by using a three-qubit maximally entangled GHZ state as quantum channel. By performing the local operations, projective measurements and classical communications, Alice can remotely "rotate" Bob's qubit. We also put forward two schemes for probabilistic implementing the non-local quantum single-qubit rotation with unit fidelity by employing a partially entangled pure GHZ state as quantum channel. The features of these schemes are that a third side (Cindy) is included, who may participate in the process of quantum remote implementation as a supervisor. Without the permission of Cindy, Bob cannot obtain the "rotation" from Alice by himself. Furthermore, when the quantum channel is partially entangled, Cindy can rectify the state distorted by imperfect quantum channel by carrying out her specific operations and communicating her result. In addition to the GHZ class state, the W class state can also be used to implement remotely the same operation probabilistically. The probability of successful implementation using the W class state is always less than that using the GHZ class state. In 1999, the first three-photon entangled GHZ state was experimentally realized[25,26] and a scheme how to produce the W state has been reported in Ref.[27], so our schemes maybe implementable in practice.

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