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Probabilistic teleportation of an arbitrary two-particle state by a partial three-particle entangled GHZ state and a two-particle entangled state*

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We propose a scheme to teleport an arbitrary unknown two-particle state by a non-maximally three-particle entangled Greenberger–Horne–Zeilinger (GHZ) state and a non-maximally two-particle entangled state from a sender to either one of two receivers. We probabilistically teleport the arbitrary unknown two-particle state by performing two Bell-state measurements on the sender's side, introducing two appropriate unitary transformations on the receiver's side conditioned on a Hadamard operation and a projection measurement on the other possible receiver's side.

Keywords: entangled state, teleportation, Hadamard operation
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In recent years, people have paid more and more attention to quantum teleportation and have proposed many teleportation schemes[1−15]. Li *et al*,[1] and Shi and Tomita[2] have proposed schemes to teleport a single qubit by a non-maximally entangled pair and a three-particle entangled W state, respectively. On the other hand, multi-particle quantum teleportation may be extremely useful in quantum computation and quantum communication, so it is important to consider how to realize multi-particle teleportation. Recently, Shi *et al*,[3] and Lu and Guo[4] have proposed schemes to teleport a two-particle entangled pair by a non-maximally three-particle entangled Greenberger–Horne–Zeilinger (GHZ) state and two EPR pairs, respectively. Gu *et al*[5] have put forward a scheme to teleport an arbitrary and unknown two-particle state by two non-maximally entangled pairs. Zheng *et al*,[6] and Liu and Guo[7] have proposed two schemes to teleport an unknown three-particle entangled W state and an unknown three-particle entangled GHZ state both by three non-maximally entangled pairs.

While all schemes of teleportation mentioned above are meant for use of a sender to one receiver, the following schemes are for a sender to either one of two receivers. Gorbachev and Trabilko[8] have proposed a scheme to teleport an entangled pair by three-particle entanglement. Dai *et al*[9] have proposed a scheme to teleport an unknown three-particle entangled GHZ state by a non-maximally three-particle entangled GHZ state and a non-maximally three-particle entangled W state. In addition, Dai *et al*[10,11] have proposed two schemes to teleport an arbitrary two-particle state by two non-maximally three-particle entangled W states, as well as the combination of a non-maximally three-particle entangled GHZ state and a non-maximally three-particle entangled W state.

In this paper, we propose a scheme to teleport an arbitrary unknown two-particle state from a sender to either one of two receivers, who are spatially separated, by using a non-maximally three-particle entangled GHZ state and a non-maximally two-particle entangled state as the quantum channel.

We suppose that the sender Alice wants to transmit an arbitrary and unknown two-particle state to either one of two receivers Bob and Charlie. The arbitrary unknown two-particle state that will be teleported can be expressed as

$$|\Psi\rangle_{12} = (x_0|00\rangle + x_1|01\rangle + x_2|10\rangle + x_3|11\rangle)_{12}, \quad (1)$$

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where the coefficients x_0, x_1, x_2 and x_3 satisfy $|x_0|^2 + |x_1|^2 + |x_2|^2 + |x_3|^2 = 1$, and $\{|00\rangle_{12}, |01\rangle_{12}, |10\rangle_{12}, |11\rangle_{12}\}$ is the basis of a four-dimensional Hilbert space. Without loss of generality, the quantum channel, composed of a non-maximally three-particle entangled GHZ state and a non-maximally two-particle entangled state, is as follows:

$$|\Psi\rangle_{345} = (a|000\rangle + b|111\rangle)_{345}, \tag{2}$$

$$|\Psi\rangle_{67} = (c|00\rangle + d|11\rangle)_{67}, \tag{3}$$

where the coefficients a and b satisfy $|a|^2+|b|^2 = 1$, and $|a| \geq |b|$; coefficients c and d satisfy $|c|^2 + |d|^2 = 1$ and $|c| \geq |d|$. The non-maximally three-particle entangled GHZ state and the non-maximally two-particle entangled state are shared by the sender and two receivers who are spatially separated. Particles 3 and 6 belong to the sender Alice, while particles 4 and 5 belong to the receivers Bob and Charlie, respectively. Therefore, the state of the whole system is given by

$$|\Psi\rangle = |\Psi\rangle_{12} \otimes |\Psi\rangle_{345} \otimes |\Psi\rangle_{67}. \tag{4}$$

First of all, Alice must carry out two Bell-measurements on particles (1, 3) and particles (2, 6), respectively. After the two Bell-state measurements, all possible 16 results are enumerated as follows:

$$\begin{aligned}
&{}_{26}\langle\Phi^{\pm}|_{13}\langle\Phi^{\pm}|\Psi\rangle \\
=&\frac{1}{2}[x_0ac|000\rangle \pm^2 x_1ad|001\rangle \\
&\pm^1 x_2bc|110\rangle \pm^2 (\pm^1)x_3bd|111\rangle]_{457},
\end{aligned} \tag{5}$$

$$\begin{aligned}
&{}_{26}\langle\Psi^{\pm}|_{13}\langle\Phi^{\pm}|\Psi\rangle \\
=&\frac{1}{2}[x_0ad|001\rangle \pm^2 x_1ac|000\rangle \\
&\pm^1 x_2bd|111\rangle \pm^2 (\pm^1)x_3bc|110\rangle]_{457},
\end{aligned} \tag{6}$$

$$\begin{aligned}
&{}_{26}\langle\Phi^{\pm}|_{13}\langle\Psi^{\pm}|\Psi\rangle \\
=&\frac{1}{2}[x_0bc|110\rangle \pm^2 x_1bd|111\rangle \\
&\pm^1 x_2ac|000\rangle \pm^2 (\pm^1)x_3ad|001\rangle]_{457},
\end{aligned} \tag{7}$$

$$\begin{aligned}
&{}_{26}\langle\Psi^{\pm}|_{13}\langle\Psi^{\pm}|\Psi\rangle \\
=&\frac{1}{2}[x_0bd|111\rangle \pm^2 x_1bc|110\rangle \\
&\pm^1 x_2ad|001\rangle \pm^2 (\pm^1)x_3ac|000\rangle]_{457},
\end{aligned} \tag{8}$$

where the Bell-states $|\Phi^{\pm}\rangle_{ij}$ and $|\Psi^{\pm}\rangle_{ij}$ are defined as

$$|\Phi^{\pm}\rangle_{ij} = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)_{ij}, \tag{9}$$

$$|\Psi^{\pm}\rangle_{ij} = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)_{ij}, \tag{10}$$

and where $\pm^1$ and $\pm^2$ correspond to the superscripts for the Bell-states composed of particles (1, 3) and particles (2, 6), respectively. For example, if Alice's measurement results are $|\Phi^+\rangle_{13}$ and $|\Psi^-\rangle_{26}$, i.e., the corresponding superscripts are + and −, then the state of particles (4, 5, 7), as shown by Eq.(6), will be

$$\begin{aligned}
{}_{26}\langle\Psi^-|_{13}\langle\Phi^+|\Psi\rangle =&\frac{1}{2}[x_0ad|001\rangle - x_1ac|000\rangle \\
&+ x_2bd|111\rangle - x_3bc|110\rangle]_{457}.
\end{aligned} \tag{11}$$

If Bob wants to receive the teleported state $|\Psi\rangle_{12}$ in Eq.(1), particle 7 belongs to the receiver Bob. After the operations, Alice should inform Bob of her measurement results via classical communication, and ask Charlie to carry out a Hadamard operation on particle 5, then make a projection measurement on particle 5. The Hadamard operation is of the form

$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \tag{12}$$

$$H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle), \tag{13}$$

and after the Hadamard operation on particle 5, we will have

$$\begin{aligned}
&\frac{1}{2\sqrt{2}}[(x_0ad|01\rangle - x_1ac|00\rangle \\
&+ x_2bd|11\rangle - x_3bc|10\rangle)_{47}|0\rangle_5 \\
&+ (x_0ad|01\rangle - x_1ac|00\rangle - x_2bd|11\rangle \\
&+ x_3bc|10\rangle)_{47}|1\rangle_5)].
\end{aligned} \tag{14}$$

Charlie measures particle 5. If Charlie obtains the results of $|0\rangle_5$, the state of particle 4 and 7 become

$$\frac{1}{2\sqrt{2}}(x_0ad|01\rangle - x_1ac|00\rangle + x_2bd|11\rangle - x_3bc|10\rangle)_{47}. \tag{15}$$

Otherwise, if Charlie obtains the results of $|1\rangle_5$, the state of particle 4 and 7 will be

$$\frac{1}{2\sqrt{2}}(x_0ad|01\rangle - x_1ac|00\rangle - x_2bd|11\rangle + x_3bc|10\rangle)_{47}. \tag{16}$$

After the measurements, Charlie informs Bob of his measurement result via classical communication. Secondly, Bob needs to establish a correspondence so that the coefficients x_i $(i = 0, 1, 2, 3)$ can correspond to $|00\rangle_{47}$, $|01\rangle_{47}$, $|10\rangle_{47}$, and $|11\rangle_{47}$, respectively. This can be realized by performing the unitary transformation U_1 on particle 4 and 7. The form of U_1 is

$$U_1 = (|0\rangle\langle 0| + |1\rangle\langle 1|)_4 \otimes (|0\rangle\langle 1| + |1\rangle\langle 0|)_7, \tag{17}$$

where $|0\rangle\langle 0|+|1\rangle\langle 1|$ is the identity operator (expressed as I) and $|0\rangle\langle 1| + |1\rangle\langle 0|$ is Pauli operator (expressed as σ_x). That is, the unitary transformation U_1 will transform the state shown in Eq.(15) into

$$|\Psi\rangle_{47} = \frac{1}{2\sqrt{2}}(x_0 ad|00\rangle - x_1 ac|01\rangle + x_2 bd|10\rangle - x_3 bc|11\rangle)_{47}. \quad (18)$$

Bob then introduces an auxiliary particle A with its initial state $|0\rangle_A$ and makes another unitary transformation U_2 on particles (4, 7). In order to reincarnate the state of particles (1, 2) under the basis $\{|000\rangle_{47A}, |010\rangle_{47A}, |100\rangle_{47A}, |110\rangle_{47A}, |001\rangle_{47A}, |011\rangle_{47A}, |101\rangle_{47A}, |111\rangle_{47A}\}$, the unitary transformation U_2 may take the form of the following 8×8 matrix:

$$U_2 = \begin{pmatrix} A_1 & A_2 \\ A_2 & -A_1 \end{pmatrix}, \quad (19)$$

where A_i $(i = 1, 2)$ is a 4×4 matrix and may be written as

$$A_1 = \mathrm{diag}(a_0, a_1, a_2, a_3), \quad (20)$$

$$A_2 = \mathrm{diag}\left(\sqrt{1-a_0^2}, \sqrt{1-a_1^2}, \sqrt{1-a_2^2}, \sqrt{1-a_3^2}\right), \quad (21)$$

where a_i ($i = 1, 2$ and $|a_i| \leq 1$) depends on the state of particles (4, 7). For example, Bob introduces an auxiliary particle A with its initial state $|0\rangle_A$, the state described in Eq.(18) becomes

$$|\Psi\rangle_{47} \otimes |0\rangle_A = \frac{1}{2\sqrt{2}}(x_0 ad|00\rangle - x_1 ac|01\rangle + x_2 bd|10\rangle - x_3 bc|11\rangle)_{47} \otimes |0\rangle_A. \quad (22)$$

If we choose

$$(a_0, a_1, a_2, a_3) = \left(\frac{b}{a}, -\frac{bd}{ac}, 1, -\frac{d}{c}\right), \quad (23)$$

the unitary transformation U_2 will transform the state in Eq.(22) into

$$\begin{aligned} &\frac{bd}{2\sqrt{2}}(x_0|00\rangle + x_1|01\rangle + x_2|10\rangle + x_3|11\rangle)_{47}|0\rangle_A \\ &+ \frac{1}{2\sqrt{2}}(\sqrt{a^2-b^2}\; x_0|00\rangle \\ &- \sqrt{(ac)^2-(bd)^2}\; x_1|01\rangle \\ &+ \sqrt{c^2-d^2}\; x_3|11\rangle)_{47}|1\rangle_A. \end{aligned} \quad (24)$$

Table 1 shows all kinds of different unitary transformations U_1 and the coefficients a_i $(i = 0, 1, 2, 3)$ of unitary transformations U_2 performed by Bob on the states of particles 4 and 7, respectively. We must mention that $\pm^1$ and $\pm^2$ in Table 1 correspond to the superscripts for the Bell-state composed of particles (1, 3) and (2, 6), respectively. $+$ and $-$ in $\pm_5$ correspond to the measurement results $|0\rangle_5$ and $|1\rangle_5$ after the Hadamard operation $H_5^{(\pm)}$ on particle 5.

Table 1. All possible unitary transformation U_1 and the coefficients a_i $(i = 0, 1, 2, 3)$ for the unitary transformation U_2 performed by Bob.

States of particles 4, 5 and 7	U_1	a_0	a_1	a_2	a_3
$H_5^{(\pm)}({}_{26}\langle\Phi^\pm|_{13}\langle\Phi^\pm|\Psi\rangle)$	$I_4 \otimes I_7$	$\frac{bd}{ac}$	$\pm^2\frac{b}{a}$	$\pm_5(\pm^1)\frac{d}{c}$	$\pm_5(\pm^2)(\pm^1)1$
$H_5^{(\pm)}({}_{26}\langle\Psi^\pm|_{13}\langle\Phi^\pm|\Psi\rangle)$	$I_4 \otimes (\sigma_x)_7$	$\frac{b}{a}$	$\pm^2\frac{bd}{ac}$	$\pm_5(\pm^1)1$	$\pm_5(\pm^2)(\pm^1)\frac{d}{c}$
$H_5^{(\pm)}({}_{26}\langle\Phi^\pm|_{13}\langle\Psi^\pm|\Psi\rangle)$	$(\sigma_x)_4 \otimes I_7$	$\pm_5\frac{d}{c}$	$\pm_5(\pm^2)1$	$\pm^1\frac{bd}{ac}$	$\pm^2(\pm^1)\frac{b}{a}$
$H_5^{(\pm)}({}_{26}\langle\Psi^\pm|_{13}\langle\Psi^\pm|\Psi\rangle)$	$(\sigma_x)_4 \otimes (\sigma_x)_7$	$\pm_5 1$	$\pm_5(\pm^2)\frac{d}{c}$	$\pm^1\frac{bd}{ac}$	$\pm^2(\pm^1)\frac{b}{a}$

Finally, Bob measures the state of auxiliary particle A. If the result is $|0\rangle_A$, we realize the quantum teleportation is successful with the probability of $|bd|^2/8$. Otherwise, teleportation fails. It can be easily verified that 32 kinds of states, which Bob can obtain, have the same results. Therefore the total probability of successful teleportation is $4|bd|^2$.

Similarly, if Charlie wants to receive the teleported state $|\Psi\rangle_{12}$ in Eq.(1), particle 7 belongs to the receiver Charlie. First, Alice should inform Charlie of her measurement results via classical communication, and asks Bob to carry out a Hadamard operation on particle 4. Then Charlie performs unitary transformations on particles (5, 7). We can see that particle

4 and particle 5 are in the same state in each term in Eq.(5), and the same case of particles 4 and 5 in Eqs.$(6-8)$. So we can easily obtain all possible U_1 and the coefficients a_i $(i = 0, 1, 2, 3)$ for the unitary transformation U_2 performed by Charlie, only by exchanging the subscripts 5 and 4 in Table 1. Certainly, Charlie will successfully receive the teleported state $|\Psi\rangle_{12}$ in Eq.(1) with the probability of $4|bd|^2$. That is to say that Charlie can do the same thing as Bob does especially with the same unitary transformation.

From the above analysis, we can see that, in each case, the total probability of successful teleportation for each receiver is $4|bd|^2$. The probability of successful teleportation increases with entanglement degree of the quantum channel. If the quantum channel composed of two maximally entangled states, i.e. $|a| = |b| = |c| = |d| = \frac{1}{\sqrt{2}}$, the total probability would equal 1. Obviously, the unitary transformation made by each receiver in this situation is reduced to the unitary U_1 on particles (4, 7) or particles (5, 7), and the auxiliary particle A and unitary transformation U_2 are not needed.

In summary, we have proposed a scheme for teleporting an unknown and arbitrary two-particle state from a sender to either one of two receivers by using a non-maximally three-particle entangled GHZ state and a non-maximally two-particle entangled state as the quantum channel. Compared with previous schemes, our scheme has the following advantages. First, the probability of teleportation is greatly increased by changing the quantum channel with the aid of a Hadamard operation and a projection measurement. In this scheme, if Bob wants to receive the teleported state $|\Psi\rangle_{12}$ in Eq.(1) , no matter which state is particle 5 in, the scheme will be successful, that is, it is without fail on particle 5. If Charlie wants to receive the teleported state $|\Psi\rangle_{12}$ in Eq.(1), it is also without fail on particle 4. In other schemes[10,11] that teleport an arbitrary two-particle state, there is failure on some particles in the process of teleportation. Second, no matter who receives the state in Eq.(1), all operations and results corresponding to each operation are the same in form, especially the unitary transformation. Third, this scheme can be directly generalized to teleport an arbitrary and unknown two-particle state from a sender to any one of N receivers by using a non-maximally $(N+1)-$particle entangled GHZ state and a non-maximally two-particle entangled state.

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